Dynamical analysis of a predator-prey model arising from palm tree plantation

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Abstract

Palm oil industry has become an issue that has caught the attention of the world community in recent years. From an economic point of view, this industry is very influential in developing and spurring economic growth in rural areas. In this paper, a predator-prey dynamical model representing the interaction between palm leaf, caterpillar and predator is discussed here. The caterpillar life-cycle starts from eggs, larvae, pupas and the adult moths, and only the larvae interact with the predator. With a given threshold level of the leaves for survival and productivity, the critical level of predators is shown. Further, the dynamical analysis is discussed analytically and numerically. Bifurcation diagrams and sensitivity analysis of each compartment were also obtained to see the effect of changing parameters on the dynamics. The results explain that the increase of larvae predators can reduce the number of larvae pests that eat palm oil leaves, but they need to be controlled to maintain the balance of the ecosystem.

Keywords: predator-prey model, sensitivity analysis, biological control, numerical simulation
2010 MSC: 93A30, 37N25, 91B78, 74H60, 49Q12

1. INTRODUCTION

Over the past decade, palm oil has become a major source in the food and energy industry in Indonesia and has contributed significantly to the international trade [1], [2]. In addition, palm oil has become a major commodity both domestically and internationally, and a major product in the plantation sector in Indonesia. Large-scale production of palm oil requires extensive land conversion, which has an impact on primary forest loss. One of the problem in the field is the use of pesticide that have an impact on the environment. Pests and diseases are the main obstacles that can reduce production significantly. The emergence of pest attacks on plants is caused by various factors. Therefore, controlling pest attacks has to be done with less effect on the environment.

For many years, the use of large scale insecticide has been done for controlling insect pests in oil palm plantations in Indonesia [3]. Unfortunately, heavy use of insecticides has side effect to non-targeted pests, including insect predators, parasitoids and pollinators [4]. In addition, uncontrolled and massive use of insecticides may even cause pest outbreaks either through resistance or resurgence mechanism [5], [6], [7]. The latter case, was observed in 2012 with the outbreak of a new caterpillar, Pseudoresia desmierdechenoni. This outbreak has caused a substantial loss of oil palm in Batubara, North Sumatra [8]. Continuous reappearance of nettle caterpillars in plantations in Indonesia indicates the loss of effective beneficial insects that has naturally controlled the pest within the economic threshold.

These caterpillar feed on the leaves of either young or old palm and become defoliator of oil palm, and significantly cause the decrease of production. The long life cycle of the insects, which range from 86 to 109 days, directly contribute to the massive leaf consumption and give affect to the loss in the production [9].

The use of biological control by natural predator is being chosen for maintaining sustainable crop production [10]. This strategy is expected to improve the damage in the diversity in agricultural fields [11], [12], [13]. The release of natural predators in the fields have been shown to be environmentally safe and economically sound for an alternative replacement of chemical pest control [14].

In this study the dynamics of nettle caterpillar population, its predator and leaves of palm oil were investigated through a mathematical model. The dynamics of the caterpillar population, its predator, and the leaf surface area of oil palm are represented in the form of compartments and expressed in an ordinary...
We also analyzed the equilibrium points, their stability condition and bifurcation. At the end of this study, numerical simulations and parameter sensitivity analysis are shown to provide biological interpretation of the constructed model.

2. **Mathematical Model**

In constructing this model, we assume that the entire life cycle of a caterpillar is influenced by food availability (leaf surface area) and its interaction with the predator as follows.

i There are six compartments in this model; moths, caterpillar eggs, caterpillars, pupae, leaf surface area, and caterpillar predators.

ii In the moth compartment the population increases due to a transition from pupa to moth and decrease due to natural death.

iii In the egg compartment, the population increases from the moths.

iv In the caterpillar compartment, it increases due to a transition from egg to caterpillar and decreases due to a transition from caterpillar to pupae, and from predation by predators.

v In the pupa compartment, the population increases due to transition from caterpillar to pupa and decreases due to transition into moth.

vi Leaf surface area increases due to the natural growth of leaf surface area and decreases due to natural death (damage) and consumption by caterpillars.

vii In caterpillar predator compartment, the population increases due to natural logistical birth rate and food conversion by preying on caterpillars, and decreases due to natural death.

The interaction dynamic is represented in the following diagram.

![Figure 1: Compartment diagram of a predator-prey model described interactions between nettle caterpillar, predator (Sycanus sp) and palm tree leaves.](image-url)

Based on the assumption above, we constructed a deterministic model of nettle caterpillar, its predator, and palm tree leaves in the following ordinary differential equations.
\[
\begin{align*}
\frac{dM}{dt} &= \alpha P - \beta M \\
\frac{dE}{dt} &= \sigma M - \delta E \\
\frac{dL}{dt} &= \delta E - \theta L - aL(\frac{S_0}{\pi} - S) - cLR \\
\frac{dP}{dt} &= \theta L - \alpha P \\
\frac{dS}{dt} &= \pi - \eta S - dL \\
\frac{dR}{dt} &= \tau R(1 - bR) + cLR - \mu R,
\end{align*}
\]
with the corresponding state variables and parameters are given in Table 1.

### Table 1: Variable and parameter values with their description used in the simulation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Estimated Value</th>
<th>Unit</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>M(t)</td>
<td>Number of moth</td>
<td>-</td>
<td>moth</td>
<td>-</td>
</tr>
<tr>
<td>E(t)</td>
<td>Number of egg</td>
<td>-</td>
<td>egg</td>
<td>-</td>
</tr>
<tr>
<td>L(t)</td>
<td>Number of larva</td>
<td>-</td>
<td>larva</td>
<td>-</td>
</tr>
<tr>
<td>P(t)</td>
<td>Number of pupa</td>
<td>-</td>
<td>pupa</td>
<td>-</td>
</tr>
<tr>
<td>S(t)</td>
<td>Leaf surface area</td>
<td>-</td>
<td>area</td>
<td>-</td>
</tr>
<tr>
<td>R(t)</td>
<td>Number of larvae’s predator</td>
<td>-</td>
<td>predator</td>
<td>-</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Transition rate from pupa to moth</td>
<td>0.025</td>
<td>(\frac{1}{day})</td>
<td>[15]</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Moth natural death rate</td>
<td>(\frac{1}{13})</td>
<td>(\frac{1}{day})</td>
<td>[15]</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>Egg production rate by a moth</td>
<td>(\frac{1}{7})</td>
<td>(\frac{1}{egg\cdot day})</td>
<td>[15]</td>
</tr>
<tr>
<td>(\delta)</td>
<td>Transition rate from egg to larva</td>
<td>(\frac{1}{50})</td>
<td>(\frac{1}{day})</td>
<td>[15]</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Transition rate from larva to pupa</td>
<td>(\frac{1}{50})</td>
<td>(\frac{1}{day})</td>
<td>[15]</td>
</tr>
<tr>
<td>(\eta)</td>
<td>Growth rate of leaf surface area</td>
<td>0.005</td>
<td>(\frac{1}{day})</td>
<td>assumed</td>
</tr>
<tr>
<td>(\tau)</td>
<td>Predator natural growth rate</td>
<td>0.02</td>
<td>(\frac{1}{day})</td>
<td>[16]</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Predator natural death rate</td>
<td>(\frac{1}{83})</td>
<td>(\frac{1}{day})</td>
<td>[16]</td>
</tr>
<tr>
<td>(a)</td>
<td>Interaction coefficient between (L) and (S)</td>
<td>0.08</td>
<td>(\frac{1}{area\cdot day})</td>
<td>assumed</td>
</tr>
<tr>
<td>(b)</td>
<td>Increase of natural carrying capacity of (R)</td>
<td>0.01</td>
<td>(\frac{1}{predator\cdot day})</td>
<td>assumed</td>
</tr>
<tr>
<td>(c)</td>
<td>Interaction coefficient between (L) and (R)</td>
<td>0.001</td>
<td>(\frac{1}{predator\cdot day})</td>
<td>assumed</td>
</tr>
<tr>
<td>(d)</td>
<td>Consumption rate of (L) on (S)</td>
<td>0.02</td>
<td>(\frac{1}{larva\cdot day})</td>
<td>assumed</td>
</tr>
<tr>
<td>(e)</td>
<td>Interaction coefficient between (L) and (R)</td>
<td>0.001</td>
<td>(\frac{1}{larva\cdot day})</td>
<td>assumed</td>
</tr>
</tbody>
</table>

We normalize the System (1) by the carrying capacity \(S_0 = \frac{\pi}{\eta}\) of the leaf as follow

\[
\begin{align*}
\frac{dL}{dt} &= \delta E - \theta L - aL(1 - S) - cLR \\
\frac{dS}{dt} &= \eta(1 - S) - dL.
\end{align*}
\]

Here all the states are in normalized form representing the frequency per unit size of leaf. All other equations in (1) remain the same, with the nonlinear coefficients are multiplied by \(S_0\). As a biological consequence, it is natural to assume that \(\sigma > \mu\).
2.1. Equilibria and their stability

The System (2) has four possible equilibria \( (M^*, E^*, L^*, P^*, S^*, R^*) \) as follow.

\[
X_1 = (0, 0, 0, 0, 1, 0) \\
X_2 = (0, 0, 0, 0, 1, \frac{\tau - \mu}{\tau b}) \\
X_3 = \left( \frac{(\sigma - \beta)\theta^2\eta}{a\beta^2d}, \frac{(\sigma - \beta)\sigma^2\eta}{a\beta^2d\delta}, \frac{(\sigma - \beta)\theta\eta}{a\beta d}, \frac{(\sigma - \beta)\sigma\eta}{a\beta d\alpha}, \frac{\alpha\beta - (\sigma - \beta)\theta}{a\beta} \right) \\
X_4 = \left( M = \frac{\psi\theta\eta}{\omega^2}, \frac{\psi\sigma\theta\eta}{\omega^2}, \frac{\psi\theta\eta}{\omega^2}, 1 - \frac{d\mu}{\eta}, \frac{(\sigma - \beta)\eta\theta - a\beta d\mu}{\beta c\eta} \right),
\]

where \( \psi = \sigma\beta\theta - \beta\sigma\beta\theta + \beta\sigma\mu - \beta\sigma\tau \) and \( \omega = (ab\tau + c\eta)\beta \).

In the absence of predator, the equilibrium \( X_3 \) exists if

\[
1 < \frac{\sigma}{\beta} < 1 + \frac{a}{\theta}. \tag{7}
\]

Further, the condition for coexistence \( X_4 \) is given by

\[
1 + \frac{c(\tau - \mu)}{\theta \tau b} < \frac{\sigma}{\beta} < 1 + \frac{a}{\theta} + \frac{c(\tau - \mu)}{\theta \tau b} + \frac{c\eta c}{b\theta \tau d}. \tag{8}
\]

Note that the parameter \( \tau \), which represents the growth rate of predator in the absence of larva, is a critical parameter which will be discussed in the next section. The coexistence \( X_4 \) is reduced to \( X_3 \) when \( \tau = \mu - \frac{c\eta\theta(\sigma - \beta)}{a\delta^2} \). Further \( X_3 \) becomes unstable for \( \tau > \mu \). The trivial equilibria \( X_0 \) and \( X_1 \) (for \( \tau > \mu \)) are always unstable.

3. Dynamical analysis

In order to understand the significant role of the predator in controlling the larvae, we first consider the basic structure of the interaction between the M-E-L-S, i.e. when no predators involve in the interaction. The level sets of \( S^* \) for variation of \( (\sigma, a) \) are shown in Figure 2(a).

The role of predator naturally improve the survivability of the leaf surface \( S \), as shown in Figure 2(b). Evaluation of the Jacobian at the equilibrium \( X_1 \) gives the characteristic polynomial

\[
P(\lambda) = (\lambda + \eta)(\lambda^4 + (\alpha + \delta + \theta + \beta)\lambda^3 + (\theta \beta + \theta \alpha + \delta \alpha + \delta \theta + \beta \alpha + \delta \beta)\lambda^2 + \delta \alpha \beta + \beta \alpha + \delta \theta + \beta \alpha + \delta \beta \lambda - \alpha \delta \theta (\sigma - \beta)). \tag{9}
\]

Evaluation of the Hurwitz Matrix of the forth order polynomial in (9) gives the stability of the zero equilibrium \( X_1 \) when \( \sigma \leq \beta \). This is the case when the production rate of egg is less than or equal to the death rate of Moth, then none of moth will survive and the leaves grow naturally.

We then did some simulation scenarios. First, we set parameters on the equilibrium condition of the leaf surface area of less than 50% to see the impact on other dynamics in the absence of larval predators, especially on pests (larvae), see Fig. 3(a). Second, the parameters are conditioned on the equilibrium of the leaf surface area of more than 50% to see its impact on other dynamics in the absence of larval predators. The results of the second scenario simulation can be seen in Fig. 3(b). Both of them give different results. The population of moth, egg, larvae, and pupae in the first simulation is greater than the second simulation, which is caused by the influence of leaf consumption parameters by larvae. In the last one, the simulation is carried out by involving the presence of larval predators in equilibrium conditions of the leaf surface area which more than 50%. The last scenario simulation was shown in Fig. 4. The results showed that with the emergence of predator larvae, the population of moth, egg, larvae, and pupae decreased, causing an increase in the area of oil palm leaves. Therefore, the presence of predators can be used as a strategy in controlling larvae pests in oil palm plantations.
Figure 2: (a) Level Sets of $S^*$ for the case with no predator, with $\alpha = 0.025, \beta = 1/13, d = 0.01, \delta = 1/50, \eta = 0.005, \mu = 1/83, \theta = 1/50$. (b) Level Sets of $S^*$ for the case with predator, with $\alpha = 0.025, b = 0.01, \beta = 1/13, c = 0.001, d = 0.02, \delta = 1/50, e = 0.001, \eta = 0.005, \mu = 1/83, \tau = 0.015, \theta = 1/50$.

Figure 3: (a) Dynamical simulation for the case of no predator, $S^* < 0.5$, with $a = 0.08, \alpha = 0.025, \beta = 1/13, d = 0.01, \delta = 1/50, \eta = 0.005, \mu = 1/83, \sigma = 1/3, \theta = 1/50$. (b) Dynamical simulation for the case of no predator, and $S^* > 0.5$, with $a = 0.15, \alpha = 0.025, \beta = 1/13, d = 0.01, \delta = 1/50, \eta = 0.005, \mu = 1/83, \sigma = 1/3, \theta = 1/50$. 
Figure 4: Dynamical simulation for the case with predator, \( S^* > 0.5 \), with \( a = 0.08, \alpha = 0.025, \beta = 1/13, d = 0.01, \delta = \frac{1}{50}, \eta = 0.005, \mu = \frac{1}{50}, \sigma = 1/3, \theta = \frac{1}{50}, \tau = 0.02, b = 0.15, c = 0.02, e = 0.0004 \)

3.1. Bifurcation analysis at the co-existence equilibrium

The growth rate, \( \tau \), of the predator at the absence of prey is a critical parameter. The coexistence \( X_4 \) start to appear only if

\[
\tau > \mu - \frac{c \eta \theta (\sigma - \beta)}{ad\beta}.
\]  

A saddle-node bifurcation occurs at the parameter value \( \sigma = \beta \) as shown in Figure (5). Further, the interaction with the predator is investigated. First, we analyze bifurcation in cases without larval predators. The critical point for bifurcation is given in (10). We then set a critical parameter in this case by running the parameters around \( \tau = 0.01183935743 \) with \( a = 0.08, \alpha = 0.025, b = 0.15, \beta = 1/13, c = 0.02, d = 0.01, \delta = \frac{1}{50}, \eta = 0.005, \mu = \frac{1}{50}, \sigma = 1/3, \theta = \frac{1}{50}. \)

Bifurcation diagrams for the case of no predator are shown in Fig. (5). The \( \sigma \) parameter makes a change instability when it passes through \( \sigma = \beta = \frac{1}{13} \). As the \( \sigma \) parameter increases, the population of leaves and larvae will decrease and increase respectively. Whereas in the case of predators, the bifurcation diagram of the parameters can be seen in Fig. (5). It seems that the \( \tau \) parameter is very helpful in balancing the dynamic behavior of the model especially concerning larvae populations. Increasing \( \tau \) gives the effect of increasing the leaf population and also predators. Therefore, the population of moth, egg, larva, and pupae will decrease. Spreading larvae predators can be used as a strategy for oil palm farmers in tackling larvae pests that attack oil palm leaves and have an impact on productivity. However, these efforts must also be controlled such that the food chain in the ecosystem remains balanced.
Figure 5: Bifurcation diagrams of the steady state solution and their stability and without predator for $a = 0.08, \alpha = 0.025, \beta = \frac{1}{13}, d = 0.01, \delta = \frac{1}{30}, \eta = 0.005, \mu = \frac{1}{33}, \theta = \frac{1}{50}$. The blue solid and red dashed lines represent stable and unstable solutions, respectively.
Figure 6: Bifurcation diagrams of the steady state solution and their stability and with predator for $a = 0.08$, $\alpha = 0.025$, $\beta = 1/13$, $d = 0.01$, $\delta = \frac{1}{27}$, $\eta = 0.005$, $\mu = \frac{1}{3}$, $\sigma = 1/3$, $\theta = \frac{1}{3}$, $b = 0.15$, $c = 0.02$, $e = 0.0004$. The blue solid and red dashed lines represent stable and unstable solutions, respectively.
4. Sensitivity Analysis

A sensitivity analysis of six-dimensional system representing an interaction between moths(M), eggs(E), larvae(L), pupas(P), leaves(S) and predators(R) is discussed here. System (1) can rewrite as $X_t = F(X, p)$ where $X_t = \frac{dX}{dt}$, $X(t) = (M(t), E(t), L(t), P(t), S(t), R(t))^T$ and $p = (\alpha, \beta, \sigma, \delta, a, c, \eta, d, \tau, b, e, \mu)^T$.

To see the parameter sensitivity to the dynamics, we first define a new variable $Y(t)$ where $Y(X, t, p) = \partial_p X(t, p)$ which represents the change of solution $X(t)$ with respects to parameter $p$ [17], [18]. Then we get the derivative of $Y(t)$ with respect to $t$ as follows.

$$Y_t(X, t, p) = J(X, p)Y(X, t, p) + Q(X, p)$$  \hspace{1cm} (11)

where $J(X, p) = \partial_X F(X, p)$ is a $6 \times 6$ jacobian matrix and $Q(X, p) = \partial_p F(X, p)$ is a $6 \times 13$ matrix.

We then simulate a sensitivity analysis around the critical point of bifurcation, i.e. $\tau = 0.01183935743, a = 0.08, c = 0.025, b = 0.15, \beta = 1/13, c = 0.02, d = 0.01, \delta = \frac{1}{15}, e = 0.0004, \eta = 0.005, \mu = \frac{1}{15}, \sigma = 1/3,$ and $\theta = \frac{1}{15}$. The simulation results are presented in Fig. 7. Not all sensitivity results are displayed here because there are 6 variables and 13 parameters that produce 78 simulations. We are more focused on the variables of larvae, leaves, and predators with respect to the parameters $c$ (caterpillar predation rate), $d$ (rate of leaf area reduction), $\tau$ (predator natural growth rate) and $e$ (predator growth rate of consumption). Fig. 7 (a) and (b) are the sensitivity of the larval solution to the four parameters. While the sensitivity to the variable leaves can be seen in Fig. 7 (c) and (d). The sensitivity of predatory dynamics to the four parameters is shown by Fig. 7 (e) and (f).

![Graphs showing sensitivity analysis results](image-url)
Simulation results in Fig. 7 can be interpreted as follows. For larval dynamics (L), the most sensitive parameter is $d$, then followed by $\tau$, $e$ and $c$ which are all inversely proportional to the larval solution. As for leaf dynamics (S), the most sensitive parameter is $\tau$ which is directly proportional, then $e$ and $c$ are also directly proportional. While the parameter $d$ has a significant effect only at time $t = 0$ to about $t = 700$, after that, the change effect is very small. For predatory (R) dynamics, the most sensitive parameters are $\tau$ and $e$ which are both directly proportional, then $d$ and $c$ are both inversely proportional.

5. CONCLUSION

A predator-prey interaction with two levels of predation between palm leaf (prey), nettle caterpillar, and predator (Sycanus Sp) is represented in a system of differential equations as shown in Eq. (1). The equilibria and conditions of existence have been obtained as well as their local stability. Level sets of oil palm leaf at the equilibrium in the parameter space $(\sigma - a)$ are simulated for the cases without and with predators. With the appearance of the predator $(\tau > 0)$, the equilibrium leaf surface area $S$ increases to resolve the minimal threshold of survivability. Bifurcation analysis at the trivial equilibrium is shown at $\sigma - \beta$, and at $\tau > \mu - \frac{\text{ad}(\sigma - \beta)}{a d^2}$.

In the case of sensitivity analysis, very influential parameters are $d$ for larvae which is the consumption rate of larvae on $S$, and the predator natural growth rate $\tau$. From the model, it is concluded that control of caterpillar pests in oil palm plantations can be done by selecting a proper and effective predator.

ACKNOWLEDGMENT

This research was financially supported by ITB Research Grant 2020 through KK ITB research for the third author and partially supported by Indonesian Ministry of Research, Technology and Higher Education (Ristekdikti) through PMDSU program for the second author.

REFERENCES


