Fuzzy-Based Prediction of Spatio-Temporal Distribution of Wet Muck in Block Cave Mine of PT Freeport Indonesia

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Abstract. Mud rushes, or wet muck spills, are hydro-geotechnical challenges in block cave mines where wet muck spills out of drawpoints formed by the accumulation of fine materials and water in drawbells. The purpose of this paper is to share the results of the development of an improved predictive tool that can be used to manage wet muck spills. The tool was developed based on the hybrid modeling of wet muck distribution using fuzzy logic and fuzzy number operations. The fuzzy logic operations were applied to model the spatial distribution of wet muck classes, providing the spatial model of drawpoint status based on five contributing factors, i.e. the height of draw, the water content, the grain size of the fine material, rainfall, and no-mucking days. The fuzzy number operations were used in accordance with the mass balance principle to estimate the temporal distribution of wet muck that forms a mud deposit consisting of fine materials and water in a drawbell. The mass balance principle was expressed using the fuzzy ordinary differential equation, including the uncertainty of joining variables. A wet muck spill event at the Deep Ore Zone (DOZ) block cave mine of PT Freeport Indonesia was utilized as a case study as well as to validate the proposed method. The fuzzy-based approach shows promising results in predicting wet muck spill events.

Keywords: block cave; fuzzy differential equation; fuzzy logic; mud rush; predictive tool; wet muck.

1 Introduction

Mud rushes, or wet muck spills, are hydro-geotechnical challenges in block cave mines caused by a sudden rush of wet muck formed by the accumulation of fine materials and water in a drawbell from the corresponding drawpoint. Mud rushes are critical to safety and production. Several authors have written about mud rushes, including Butcher, et al. in 2005 [1] and 2007 [2], who stated two main contributing aspects for a mud rush to occur, i.e. fine materials and water. Wet muck spills and their related aspects in block cave mines of PT
Freeport Indonesia (PTFI) have been reported, documented, and published by previous authors [3-14]. Wet muck studies at PTFI have also been conducted [15-17]. PTFI currently uses the wet muck classification developed by Samosir, et al. [7] to define the status of drawpoints (see Figure 1). The classification consists of nine combinations of three classes of fine materials (grain sizes) and three classes of water content (wetness) to identify one of three predicted status of the corresponding drawbell: low risk/no issues, medium risk/needs attention, or high risk/danger.

<table>
<thead>
<tr>
<th>Wetness / Water Content</th>
<th>Grain size ≥ 5 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M ≥ 70 %</td>
</tr>
<tr>
<td></td>
<td>(coarser grain)</td>
</tr>
<tr>
<td></td>
<td>30 % &lt; M ≤ 70 %</td>
</tr>
<tr>
<td></td>
<td>(medium grain)</td>
</tr>
<tr>
<td></td>
<td>M ≤ 30 %</td>
</tr>
<tr>
<td></td>
<td>(finer grain)</td>
</tr>
<tr>
<td>Dry</td>
<td>&lt; 8.5 %</td>
</tr>
<tr>
<td>Moist</td>
<td>8.5 % - 11.0 %</td>
</tr>
<tr>
<td>Wet</td>
<td>&gt; 11 %</td>
</tr>
<tr>
<td></td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>C1</td>
</tr>
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<td></td>
<td>A2</td>
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<td>C2</td>
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<td>A3</td>
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<tr>
<td></td>
<td>B3</td>
</tr>
<tr>
<td></td>
<td>C3</td>
</tr>
</tbody>
</table>

Low risk: mucking is operated using any loader
Medium risk: mucking is operated using any loader with close supervision
High risk: mucking has to be operated using a remote loader

Figure 1 Wet muck classes according to [7] and mucking operation procedures.

Over time, cave materials tend to become more porous, the quantity of finer materials increases and surface water reaches the extraction level faster. Accordingly, wet muck spills are more likely to occur. Therefore, the classification of wet muck presented by [7] should be modified to obtain more robust wet muck spill potential predictions. The purpose of this paper is to share the development of an improved predictive tool for spatio-temporal wet muck distribution, including additional wet muck contributing factors besides fine materials (grain size) and water content (wetness). The improved predictive tool uses hybrid modeling, i.e. it combines spatial and temporal modeling.

2 Approach and Method

The spatial modeling was carried out using the fuzzy logic operation (FLO) based on the presumed contributing factors of wet muck occurrences that were later assigned as the inputs. FLO enables and allows for combining expert judgement or opinions with uncertainties to map the relations between the presumed contributing factors and wet muck occurrences. The spatial FLO model was used to estimate the distribution of wet muck in drawbells at specific times to determine the status of drawpoints. Hence, it basically provides qualitative measures of the drawpoints that preliminarily indicate the necessary conditions for mud rush to occur. The temporal modeling was conducted based on the mass balance principle, which expresses the difference between mass moving into a drawbell and mass drawn from the drawbell through the
corresponding drawpoint within a certain time interval. The mass balance principle can be described using a mathematical expression involving fuzzy number operations (FNO), which form fuzzy ordinary difference equations (FODEs). These are best suited to be used to model state variables that are present in a range of validity, such as the wet muck classes presented by [7], expressing the variation of the fine materials and water content data observed at a drawpoint. The accumulation of fine materials and water may result in the development of mud deposits in a drawbell. By solving the FODE, the quantity of mud deposits in the drawbell within a certain time interval can be estimated. The procedure can be applied for all drawbells, leading to the spatial distribution of the estimated mud deposits in all drawbells within a certain time interval. Hence, it basically provides a quantitative measure of drawpoints that indicates sufficient conditions for mud rush to occur.

2.1 Spatial Distribution of Wet Muck and Qualitative Measure of Drawpoints

The current determination of drawpoint status follows the recommendations of [7] and distinguishes only two wet muck contributing factors, i.e. fine materials and water content. On the other hand, according to [10, 11, 13, 16 and 17], wet muck occurrences may be strongly correlated with presumed contributing factors such as fine materials, degree of saturation, no or less mucking, rainfall intensity, height of draw (HoD) and highly altered ore (HALO) content. To determine the relation between the expected contributing factors and wet muck occurrences, principal component analysis, correlation analysis, and expert opinion surveys were conducted. Accordingly, it can be presumed that five primary contributing factors may affect wet muck occurrences, i.e. fine materials, water content, HoD, rainfall, and no mucking days. The last three factors listed are additional factors introduced in this paper, because these factors were considered by [3, 4], and [17] to be strongly related to wet muck occurrences. The five contributing factors were assigned as the input while the output is the status or the qualitative measure of the corresponding drawpoint expressed linguistically: low, medium, or high risk, which preliminarily indicates the necessary conditions for mud rush to occur.

2.1.1 Method of Fuzzy Logical Operation (FLO)

Fuzzy logic was introduced by [18]. It allows for a definitive solution for unstructured, complex, and uncertain problems using linguistic expressions during the assessment of factors with uncertainties. A fuzzy set \(A\) may be defined in Eq. (1) as follows:

\[
A = \{ x, \mu_A(x) | x \in X \}
\]
where $X = \{x\}$ is a finite set of values and $\mu_A(x)$ is a membership function of $x$ in $A$ describing the variable membership assigned to $A$. This quantifies the influence of variable $x$ on the predicted phenomenon. For this paper, fuzzy sets were used to determine the class or measure of wet muck contributing factors, i.e. HoD, grain size, water content, daily rainfall, and no-mucking days, which are given in fuzzy membership functions (FMFs) as shown in Figure 2.

### Table: Crisp Boundary of Wet Muck Contributing Factors

<table>
<thead>
<tr>
<th>Crisp Boundary of Wet Muck Contributing Factors</th>
<th>Fuzzy Membership Function of Wet Muck Contributing Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HoD (m)</strong></td>
<td><img src="image" alt="HoD Graph" /></td>
</tr>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td><strong>Grainsize of Fine Material (%)</strong></td>
<td><img src="image" alt="Grainsize Graph" /></td>
</tr>
<tr>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td><strong>Water Content / Wetness (%)</strong></td>
<td><img src="image" alt="Water Content Graph" /></td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td><strong>Daily Rainfall (mm/day)</strong></td>
<td><img src="image" alt="Daily Rainfall Graph" /></td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td><strong>No-Mucking Days (days)</strong></td>
<td><img src="image" alt="No-Mucking Days Graph" /></td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

Figure 2 Crisp boundary and FMF of wet muck contributing factors.
A fuzzy set (FS) and FMF were created for each of the five contributing factors in the input space. The mapping of the input space to the output space was performed according to [19], as schematically depicted in Figure 3. The inference process was performed by FLO through a series of rules combining the FMFs of each fuzzy set using the AND operator, resulting in the output in the form of the FMFs of the drawpoint status. The distribution of wet muck in a drawbell at a certain time period can be analyzed in all active drawbells, resulting in the spatio-temporal distribution of wet muck, which reveals the status of the drawpoints.

### 2.2 Temporal Distribution of Wet Muck and Quantitative Measure of Drawpoints

In this study, the mud deposit development in a drawbell was conceptualized as follows. Wet muck was defined as a mixture formed by fine materials and water in a block cave mine as fine materials absorb water until reaching its liquid limit in accordance with [14]. With an increasing number of no-mucking days and under an increasing quantity of rainfall, the mixture then begins to accumulate in drawbells, forming mud deposits. Furthermore, it may clog and therefore prevent other materials and water from passing through the corresponding drawpoints, which may result in increasing mud deposits. Increasing rainfall and no-mucking days result in an increasing mass of materials and water in the drawbells. This causes the materials and mud deposits in the drawbells to consolidate and, accordingly, this tends to increase the saturation degree. Eventually, it can change the status of the drawpoints from moist to wet. According to [16], changes in drawpoint status can occur within a minimum of 14 no-mucking days. If the full saturation degree is exceeded, then the fine materials cease to absorb water. Subsequently, finer materials may be diluted in the water, forming suspended solids in the water at the top of the mud deposits. Less concentrated suspended solids in water may float higher due to the
buoyancy effect. As more water with a low suspended solid concentration or clear water at the top of the mud deposits continues to develop due to no or low mucking under medium to high rainfall, the water may spill over into nearby drawbells, forming a cluster of inter-connected over-saturated wet drawbells.

The wet muck spill potential was determined based on the findings in [14] and was defined as the driving force that may cause mud deposits and the water in a wet drawbell to rush over or to spill from the corresponding drawpoint under necessary and sufficient conditions. It can also be defined in terms of the quantity of mud deposits and water in a wet drawbell. Accordingly, it can be attributed to the mass balance within the corresponding drawbell. It is actually a temporal change between the materials entering the drawbell and the materials drawn from the corresponding drawpoint. It includes the temporal changes between water entering the drawbell and water flowing from the corresponding drawpoint as well as water spilling into nearby drawbells. No or low mucking will lead to the accumulation of mass in a drawbell and, accordingly, it will increase the wet muck spill potential.

The quantity of materials and water within block caves behind or above drawpoints or the extraction level cannot be fixed. Therefore, all computations of mass balance in a drawbell were carried out using all information and data measured at the drawpoint and collected in relation to the drawpoint. Water and fine materials in a drawbell can be analyzed using relative quantities, such as the water content and the fine material fractions, which are determined through observation and visual inspection according to the wet muck classification developed in [7].

### 2.2.1 Mathematical Model of Wet Muck Variation

Mathematical modeling of wet muck variations was developed in [14] based on the mass balance principle, which is used to express the accumulation of wet muck in a drawbell within a certain time interval. The temporal change in the mass of fine materials in a drawbell can be expressed in Eq. (2) as:

\[
\frac{\partial M_f}{\partial t} = M_{in} c_{in} - M_{out} c_{out}
\]

where \( M_f \), \( M_{in} \), \( M_{out} \), \( c_{in} \), and \( c_{out} \) are mass of fine materials (ton); estimated total mass of materials, including fine materials entering the drawbell (ton/day); estimated total mass of materials leaving the corresponding drawpoint in terms of mucking productivity (ton/day); mass fraction of fine materials to total mass of materials entering the drawbell based on geological mapping (%); and mass fraction of fine materials to total mass of materials leaving the drawpoint (%)
based on the observation at the drawpoint, respectively. If the mass fraction of fine materials entering the drawbell can be assumed to be equal to the observed mass fraction of fine materials at the corresponding drawpoint, Eq. (2) can be simplified into Eq. (3) as:

\[
\frac{\partial M_f}{\partial t} = \left( M_{in} - M_{out} \right) c_{out}
\]  

(3)

To describe the temporal change in the mass of water in the drawbell, it is necessary to assume that (1) fine materials are the only water-absorbing materials, (2) water absorbed in boulder and rock mass is negligible, and (3) the loss of water due to evaporation is also negligible. Accordingly, the temporal change in the mass of absorbed water in a dry or moist drawbell can be mathematically expressed in Eq. (4) as:

\[
\rho_w \frac{\partial V_{aw}}{\partial t} = \left( M_{in} - M_{out} \right) c_{out} \theta_{out}
\]

(4)

where \( \theta_{out}, V_{aw} \) and \( \rho_w \) are observed water content at the drawpoint in terms of mass fraction (%), volume of absorbed water (\( m^3 \)), and specific mass of water (ton/\( m^3 \)), respectively. In Eq. (4), water is absorbed by fine materials. There is still no water at the top of the mud deposits for a dry or moist drawbell. Furthermore, the temporal change in the mass of water in a saturated, wet drawbell with a potentially limited amount of water above the mud deposits can be mathematically expressed in Eq. (5) as:

\[
\rho_w \frac{\partial V_{w}}{\partial t} = \left( M_{in} - M_{out} \right) c_{out} \theta_{out} + \left( Q_{in} - Q_{out} \right) \rho_w
\]

(5)

where \( V_w, Q_{in}, \) and \( Q_{out} \) are volume of water (\( m^3 \)), rate of water entering the drawbell (\( m^3/day \)), and rate of water passing through the corresponding drawpoint (\( m^3/day \)), respectively. Moreover, the temporal change in the mass of water in a saturated, wet drawbell in a cluster of inter-connected, over-saturated wet drawbells can be expressed in Eq. (6) as:

\[
\rho_w \frac{\partial V_{w}}{\partial t} = \left( M_{in} - M_{out} \right) c_{out} \theta_{out} + \left( Q_{in} - Q_{out} \pm Q_{so} \right) \rho_w
\]

(6)

where \( Q_{so} \) is the rate of water spilling into/from the surrounding nearby drawbells (\( m^3/day \)). The quantity of water above mud deposits is difficult to estimate because some of the related governing parameters cannot be fixed easily or clearly. Among the quantities in Eqs. (2) to (6), \( Q_{in} \) and \( Q_{so} \) are difficult to determine and include high uncertainties. \( Q_{out} \) can be roughly estimated using flow measurement at the drawpoints. For an over-saturated drawbell, the following characteristics can be observed during visual inspection:
(1) the class of wet muck (Figure 1) according to [7] falls into C2 or C3; (2) the average water content at the corresponding drawpoint exceeds the liquid limit; (3) the HoD value is usually high; (4) no-mucking days increase; (5) rainfall is high or excessive; (6) clear water flows from the corresponding drawpoint; and (7) over-saturated drawbells are usually present as a cluster when an already over-saturated drawbell spills into nearby surrounding drawbells. The temporal change of the volume of clear water above the mud deposits in a saturated wet drawbell in a cluster of inter-connected over-saturated wet drawbells can be mathematically expressed in Eq. (7) as:

$$\frac{\partial V_{cw}}{\partial t} = V_{db} - \left( M_{in} - M_{out} \right) \left( \frac{1}{\rho_r} + \frac{c_{out}}{\rho_f} + \frac{c_{out}}{\rho_w} \right)$$

where $V_{cw}$, $V_{db}$, $\rho_r$, and $\rho_f$ are volume of clear water above the mud deposits (m$^3$), volume of a drawbell (m$^3$), specific mass of rock (ton/m$^3$), and specific mass of fine materials (ton/m$^3$), respectively.

In order for the quantitative measure of the drawpoints to indicate the sufficient conditions for mud rush to occur, criteria need to be set for: (1) dry drawbells, (2) moist drawbells, (3) wet drawbells without spillover water, and (4) wet drawbells in a cluster of inter-connected, over-saturated, wet drawbells. These criteria will be given in the case study. The criteria of each drawbell are required to establish a pair of equations that can be used to estimate the temporal changes of wet muck in terms of the temporal changes in the mass of fine materials and water in the drawbells.

### 2.2.2 Fuzzy Number Operation (FNO) Method

Eqs. (2) to (7) can be best expressed using FODE with FNO, since all quantities on the right hand side of Eqs. (2) to (7) involve uncertainties, whose values are given in certain ranges of validity. For example, the fraction of fine materials and water content are given in the form of variations within certain ranges, which are observed at a drawpoint based on the wet muck classes presented in [7]. A fuzzy number is expressed as a fuzzy set that defines a fuzzy interval in a real number set with an ambiguous limit, usually represented by two end points, $a$ and $c$, as well as a peak point, $b$. In this study, $a$ was set as the minimum value, $c$ at the maximum value, and $b$ was set at the most likely value. Among the various shapes that represent fuzzy numbers, the triangular fuzzy number (TFN) is the simplest and most widely used. It was chosen here following [20], as it provides good results. It is represented by three points, namely $\mathcal{A}([a, b, c])$, which can be interpreted as an FMF of a fuzzy set. FNO is usually performed
by using the extension principle introduced in [21]. In this study, the arithmetic FNO formulation was adopted from [22]. Suppose there are two fuzzy sets given as \( \tilde{A} = [a_1, b_1, c_1] \) and \( \tilde{B} = [a_2, b_2, c_2] \) with \( c > b > a \). Then, the FNO for addition or substraction can be expressed in Eq. (8) as follow:

\[
\begin{align*}
\hat{A}(+) \hat{B} &= [a_1 + a_2, b_1 + b_2, c_1 + c_2]; \\
\hat{A}(-) \hat{B} &= [a_1 - c_2, b_1 - b_2, c_1 - a_2]; \\
-(\hat{B}) &= [-c_2, -b_2, -a_2] 
\end{align*}
\]  

(8)

The FMF for TFN represented by three points \( \tilde{A} = [a, b, c] \) is expressed in Eq. (9) as:

\[
\mu_a(x) = \begin{cases} 
0 & \text{for } x \leq a \\
\frac{x - a}{b - a} & \text{for } a < x \leq b \\
\frac{c - x}{c - b} & \text{for } b < x \leq c \\
0 & \text{for } x > c 
\end{cases}
\]  

(9)

The FNO for multiplication or division is more complex. Two TFNs, as given above, were used to illustrate the FNO for multiplication. The result of FNO for multiplication between the TFNs of positive real numbers can be expressed in Eq. (10) as:

\[
\hat{A}(x) \hat{B} = [a_1 a_2, b_1 b_2, c_1 c_2] 
\]  

(10)

The FNO for division can be treated similarly to the FNO for multiplication but using an inverse value instead. Following [22], the FMF of a TFN associated with the results of a multiplication operation can be expressed in Eq. (11) as follow:

\[
\mu(x) = \begin{cases} 
-\frac{(a_1 b_2 + a_2 b_1 - 2a_1 a_2) + \sqrt{(a_1 b_2 - a_2 b_1)^2 + 4(b_1 - c_1)(b_2 - a_2)x}}{2(b_1 - a_1)(b_2 - a_2)} & \text{for } a_1 a_2 \leq x \leq b_1 b_2 \\
-\frac{(c_1 b_2 + c_2 b_1 - 2c_1 c_2) + \sqrt{(c_1 b_2 - c_2 b_1)^2 + 4(b_1 - c_1)(b_2 - c_2)x}}{2(b_1 - c_1)(b_2 - c_2)} & \text{for } b_1 b_2 < x \leq c_1 c_2 \\
0 & \text{for } x > c_1 c_2 
\end{cases}
\]  

(11)
2.2.3 Numerical Model of Wet Muck Variations and Discrete Solution

A discrete solution refers to a change in the quantity of a state variable within a certain time interval. For example, Eq. (3) expresses the change in the mass of fine materials in a certain time interval, which is a function of several quantities on the right-hand side that are considered fuzzy sets because they involve uncertainties. Eqs. (2) to (7) involve arithmetic operations of fuzzy sets and therefore the discrete solution is found by using FNO. For example, Eqs. (3) and (4) are valid for a dry drawpoint and are given in the form equations with the FNO expressed in Eqs. (12) and (13) as follows:

\[
\frac{\partial \hat{M}_f}{\partial t} = \left[ M_{in}\left[ a_1, b_1, c_1 \right] \left( \Delta M_{out}\left[ a_2, b_2, c_2 \right] \hat{c}_{out}\left[ a_3, b_3, c_3 \right] \right) \right] \\
= \Delta \hat{M}_f\left[ a_m, b_m, c_m \right] \\
\frac{\partial \hat{V}_{aw}}{\partial t} = \left[ M_{in}\left[ a_1, b_1, c_1 \right] \left( \Delta M_{out}\left[ a_2, b_2, c_2 \right] \hat{\theta}_{out}\left[ a_4, b_4, c_4 \right] \right) \right] \\
= \Delta \hat{M}_w\left[ a_w, b_w, c_w \right] \\
\]

The fuzzy set of the change in the mass of mud deposits in a certain time interval was defined based on the simplified approach as a combination of the fuzzy set of the change in the mass of fine materials and the fuzzy set of the change in the mass of water in the time interval. It can be expressed by using FNO for addition between TFNs as in Eq. (14):

\[
\Delta \hat{M}_d\left[ a_d, b_d, c_d \right] = \hat{M}_f\left[ a_m, b_m, c_m \right] \oplus \hat{M}_w\left[ a_w, b_w, c_w \right] \\
\]

\[
\hat{M}_{in}\left[ a_1, b_1, c_1 \right], \hat{M}_{out}\left[ a_2, b_2, c_2 \right] \]
are the fuzzy sets of materials or rock entering a drawbell and materials or rock leaving the corresponding drawpoint. 
\[
\hat{c}_{out}\left[ a_3, b_3, c_3 \right], \hat{\theta}_{out}\left[ a_4, b_4, c_4 \right] \
\]
are the fuzzy sets of fraction of fine materials and water content, observed at a drawpoint according to [7]. The specific mass of water \((\rho_w)\) was assumed to be a constant despite having small variations. Eqs. (12) to (13) involve FNO for subtraction and multiplication, and the solution is based on Eqs. (8) and (10). \( \Delta \hat{M}_f \left[ a_{af}, b_{af}, c_{af} \right] \oplus \Delta \hat{M}_w \left[ a_{aw}, b_{aw}, c_{aw} \right] \)
\( \Delta \hat{M}_d \left[ a_d, b_d, c_d \right] \) are the fuzzy sets of change in the mass of fine materials, water, and mud deposits, respectively, in a certain time interval. For real positive fuzzy sets expressed by \( \hat{A}\left[ a, b, c \right] \) with \(c>b>a\), the FMF of the corresponding mud deposit fuzzy set can be determined by using Eqs. (9) or (11). Then, this
solution scheme can be applied to other drawpoints, such as moist drawpoints, saturated wet drawpoints, and saturated wet drawpoints in a cluster of interconnected over-consolidated drawbells. Finally, the distribution of wet muck in a drawbell within a certain time interval can be applied to all active drawbells, leading to the spatio-temporal distribution of wet muck, which expresses the wet muck spill potential in terms of the quantity of mud deposits and water in the corresponding drawbells.

3 Case Study

3.1 Block Cave Overview

PTFI operates copper and gold mining in the Erstberg Mining District in the province of Papua, Indonesia. It is located in the Sudirman Mountains, which has an extremely rugged topography, at an elevation that ranges from 3000 to 4500 masl. It lies on the collisional boundary of the Australian and Indo-Pacific plates, within the Tertiary Papuan/Irian fold belt. The geology of the region consists of typically subduction-related arc systems with the Jurassic to Tertiary age sedimentary facies of the Kembelangan Group and the New Guinea Limestone Group. The block cave is located at the Erstberg East Skarn System (EESS) [23]. The production level is about 1200 meters below the surface and has column heights of up to 500 meters.

The annual rainfall is about 5500 mm with the highest recorded daily rainfall at 110 mm. Structures, sediment, intrusive rock contacts, fractured and karstic limestone, old block caves, and hydraulic properties within the cave line have been recognized as control mechanisms for water occurrence in the EESS mining complex. There are three types of water in the area: groundwater, direct surface recharges into the cave, and water from old mines situated higher up.

According to [12], fine-grained and clayey materials are readily available from areas dominated by breccia rock types. As the HoD exceeds 100 meters, materials within various skarns also break down to create additional wet muck fine material sources. Following the wet muck spill event on April 18th, 2011, detailed observations were conducted by PTFI and by [16]. Wet muck properties obtained from the detailed observations by [16] include:

1. The only materials that absorbed water within the cave were the fine materials.
2. No-mucking periods longer than 4 days had an influence on wet muck occurrence.
3. No-mucking periods longer than 14 days caused the rocks in the drawbells to consolidate, tended to increase the saturation degree, and could change the status of drawpoints from dry to moist.

4. Water in a saturated drawbell could spill into surrounding drawbells after 6 days of no-mucking under cumulative rainfall that exceeded 160 mm, leading to clustering in the wet drawbells.

5. Each meter of HoD was predicted to contribute 4 to 5 meters movement of rocks.

6. The limit of rainfall was 80 mm/4 days.

7. A minimum of 20% of the fine material fraction fell within the grain size of sand.

8. The minimum saturation degree and water content were 80% and 10%, respectively.

9. The wet muck properties obtained from the detailed observations by [24] comprise the following average properties of wet muck: grain size: 22.72% of fine materials passed 200 mesh; wet unit weight: 2.41 ton/m$^3$; dry unit weight: 2.07 ton/m$^3$; specific gravity: 2.68; moisture content: 18%; plastic limit: 16.60%; and liquid limit: 21.25%.

10. The wet muck properties obtained from the desk study by [24], i.e. the limits of water content and fine materials, were similar to [7], except the water content for the middle class decreased to 8% based on [17].

3.2 Spatial Distribution of Wet Muck and Qualitative Measure of Drawpoints

Figure 2 provides an illustration of crisp, or numeric, boundaries and the FMF plots for each contributing factor. The fuzzy sets for HoD, fine materials, and water content were each divided into three FMFs (low, medium, and high), while the fuzzy sets for rainfall and no-mucking days were each divided into two FMFs (medium and high), as they have an influence on wet muck occurrence for medium or high values only. It was concluded that low rainfall and a high number of mucking days do not have significant effects on wet muck occurrence. The FMFs for each contributing factor are given in Eqs. (15) to (19b) as:

The FMFs of HoD:

$$
\mu(x) = \begin{cases} 
\frac{40 - x}{40 - 0} & \text{for } 0 \leq x \leq 40 \text{ and } 0 \text{ for } 40 \leq x \\
0 & \text{for } x \leq 15 \text{ and } \frac{x - 15}{50 - 15} & \text{for } 15 \leq x \leq 50 \\
\frac{85 - x}{85 - 50} & \text{for } 50 \leq x \leq 85 \text{ and } 0 \text{ for } 85 \leq x \\
\frac{x - 60}{100 - 60} & \text{for } 60 \leq x \leq 100 \text{ and } 1 \text{ for } 100 \leq x
\end{cases}
$$

(15)
The FMFs of fine materials:

\[
\mu(x) = \begin{cases} 
  40 - x & \text{for } 0 \leq x \leq 40 \text{ and } 0 \text{ for } 40 \leq x \\
  40 - 0 & \\
  0 & \text{for } x \leq 20 \text{ and } \frac{x - 20}{50 - 20} & \text{for } 20 \leq x \leq 50; \frac{80 - x}{80 - 50} & \text{for } 50 \leq x \leq 80 \text{ and } 0 \text{ for } 80 \leq x \\
  x - 60 & \text{for } 60 \leq x \leq 100 \text{ and } 1 \text{ for } 100 \leq x 
\end{cases}
\]  

(16)

The FMFs of water content:

\[
\mu(x) = \begin{cases} 
  1 & \text{for } x \leq 5; \text{ for } x \geq 9; \\
  1 - 2 \left( \frac{x - 5}{9 - 5} \right)^2 & \text{for } 5 \leq x \leq \frac{5 + 9}{2}, \frac{x - 9}{9 - 5} & \text{for } \frac{5 + 9}{2} \leq x \leq 9 \\
  \frac{- (x - 10)^2}{2} & \\
  2 \left( \frac{x - 10}{21.5 - 10} \right)^2 & \text{for } 10 \leq x \leq \frac{10 + 21.5}{2}; 2 \left( \frac{x - 21.5}{21.5 - 10} \right)^2 & \text{for } \frac{10 + 21.5}{2} \leq x \leq 21.5 
\end{cases}
\]  

(17a)

\[
\mu(x) = e^{2(1.5)}
\]  

(17b)

(17c)

The FMFs of daily rainfall:

\[
\mu(x) = \begin{cases} 
  1 & \text{for } x \leq 11.5; \text{ for } x \geq 40 \\
  1 - 2 \left( \frac{x - 11.5}{40 - 11.5} \right)^2 & \text{for } 11.5 \leq x \leq \frac{11.5 + 40}{2}, \frac{x - 40}{40 - 11.5} & \text{for } \frac{11.5 + 40}{2} \leq x \leq 40 \\
  0 & \text{for } x \leq 15; \text{ for } x \geq 116 \\
  2 \left( \frac{x - 15}{116 - 15} \right)^2 & \text{for } 15 \leq x \leq \frac{15 + 116}{2}; 2 \left( \frac{x - 116}{116 - 15} \right)^2 & \text{for } \frac{15 + 116}{2} \leq x \leq 116 
\end{cases}
\]  

(18a)

(18b)

The FMFs of no-mucking days:

\[
\mu(x) = \begin{cases} 
  1 & \text{for } x \leq 4.5; \text{ for } x \geq 10 \\
  1 - 2 \left( \frac{x - 4.5}{10 - 4.5} \right)^2 & \text{for } 4.5 \leq x \leq \frac{4.5 + 10}{2}, \frac{x - 10}{10 - 4.5} & \text{for } \frac{4.5 + 10}{2} \leq x \leq 10 \\
  0 & \text{for } x \leq 5; \text{ for } x \geq 19 \\
  2 \left( \frac{x - 5}{19 - 15} \right)^2 & \text{for } 5 \leq x \leq \frac{5 + 19}{2}; 2 \left( \frac{x - 19}{19 - 5} \right)^2 & \text{for } \frac{5 + 19}{2} \leq x \leq 19 
\end{cases}
\]  

(19a)

(19b)
The total number of FMFs is 13, which results in 108 rules consisting of FMF combinations using the AND operator. These were then submitted to experts to obtain approval through expert opinion surveys.

### 3.3 Temporal Distribution of Wet Muck and Quantitative Measure of Drawpoints

The purpose of the case study was to validate the proposed method by reconstructing the spill event that occurred on April 18th, 2011. If the proposed method is validated, then it can be used later on as a robust and effective predictive tool for the spatio-temporal distribution of wet muck.

The spatio-temporal materials, or rocks, entering each drawbell are not uniform in nature. In general, the quantity of materials and water that enter each drawbell are difficult to determine. For simplification, it was assumed that they were spatio-temporally uniform. Based on practical experience, rock entering drawbells is estimated to be within 9-18 inches/day or ranging from 148 to 351 tons/day. The quantity of materials that leaves a drawpoint can be estimated based on the mucking productivity and/or the HoD of the corresponding drawpoint.

### 4 Results and Discussion

#### 4.1 Spatial Distribution of Wet Muck and Qualitative Measure of Drawpoints

The drawpoints status, referring to Figures 4(a) and 4(b), represents the drawpoint’s condition at the extraction level, indicating qualitatively the potential of mud rush occurrence given in 3 classes of safety attributes. FLO with 108 rules was applied to more than 250 drawpoints, which then generated the status of the drawpoints, as summarized in Figures 5(a) and 5(b) according to [7] for comparison. Figures 5(a) and 5(b) were intended to provide the status of the drawpoints in a certain time interval as simply as possible, so that they can be quickly identified. The drawpoint status was given in relation to a column stating the position of the panel (P#mW or P#mE) and with a line indicating the corresponding drawpoint number (-n). For example, drawpoint P#2W-10 is located at panel P#2 west and numbered 10, while drawpoint P#2E-11 is located at panel P#2 east and numbered 11.

Following field investigations and studies, the wet muck spill event on April 18th, 2011 presumably occurred at drawpoint P#2W-10 or P#2E-11. Based on the simulation results using FLO, both drawpoints had the same high-risk status.
(Figure 5(a)) prior to the wet muck spill event. Hence, the FLO model can represent real events and almost precisely predict the status of the drawpoints. However, according to [7], both drawpoints had a low-risk status (Figure 5(b)). The model presented in [7] thus appears to underestimate the risk in comparison with the FLO model and was indicated to have a weakness by not including other aspects, such as HoD, rainfall, and no-mucking days, which also influence wet muck occurrences. Since there are additional factors that potentially influence wet muck occurrences, it is advisable to periodically update the wet muck classes presented in [7], as some wet muck contributing factors change over time in accordance with the progress of the block cave.

Figure 4  (a) Sketch of a block cave with drawbells and drawpoints at extraction level [25]; (b) sketch of a block cave with three levels, i.e. undercut, extraction and conveyor [26].
Figure 5  (a) The status of the active drawpoints before the wet muck spill event on 18 April 2011 according to the fuzzy-based approach, P#2W-10 and P#2E-11 indicate a high risk (red); (b) The status of the active drawpoints before the wet muck spill event on 18 April 2011 according to [7], P#2W-10 and P#2E-11 indicate a low risk (green).
4.2 Temporal Distribution of Wet Muck and Quantitative Measure of Drawpoints

In order for the quantitative measure of drawpoints to indicate the sufficient conditions for mud rush to occur, criteria need to be set for: (1) dry drawbells, (2) moist drawbells, (3) wet drawbells without spillover water, and (4) wet drawbells in a cluster of inter-connected, over-saturated, wet drawbells. In this study, the status of the drawpoints (the qualitative measure of the drawpoints) was used to indicate the necessary conditions for mud rush to occur by means of setting the criteria for the drawpoints, where a dry drawpoint is seen as low-risk, a moist drawpoint as medium-risk, and a saturated wet drawpoint as high-risk. The criteria for a saturated wet drawpoint were specifically added for the block cave, such as when the daily rainfall was greater than 27 mm and there have been more than 14 no-mucking days. Thus, a saturated wet drawpoint can be categorized as a saturated wet drawpoint in a cluster of inter-connected over-consolidated wet drawbells. Accordingly, establishing a couple of equations for a drawpoint becomes easier and can be summarized as follows: Eqs. (3) and (4) were used for low-risk, or dry, drawpoints; Eqs. (3) and (5) were used for medium-risk, or moist, drawpoints; Eqs. (3) and (6) were used for high-risk, or saturated, wet drawpoints; and Eqs. (3), (6), and (7) were used for high-risk, or saturated, wet drawpoints in a cluster of inter-connected over-consolidated wet drawpoints.

The contours of the estimated mud deposits in the drawbells at two specific time intervals indicating quantitatively the potential for mud rush occurrence are spatially depicted by Figures 6(a) and 6(b). Referring to Figures 4(a) and 4(b), Figures 6(a) and 6(b) represent the condition of the drawbells regarding the mass distribution of mud deposits at the extraction level. The status of the drawpoints and the contours of the estimated mud deposits in the drawbells at a specific time interval can be spatially overlaid and given as in Figure 7, which provides information on the necessary conditions (status of the drawpoints) and the sufficient conditions (mass of the mud deposits) of the potential of mud rush occurrence.

Figure 6(a) shows that prior to the wet muck spill event on April 18th, 2011, there were four clusters of drawbells with a high estimated quantity of mud deposits, which were located at the north, the south, and the east boundaries of the cave, and in the middle of the extraction level. These four clusters were consistent with those that were observed to have excessive water contents. They were predicted to be a cluster of inter-connected over-consolidated wet drawbells. Drawpoints P#2W-10 and P#2E-11 were located within one of the clusters in the middle of the extraction level. The status of drawpoints P#2W-10 and P#2E-11 were consistent with the conditions of the corresponding
drawbells, which were predicted to be high-risk with a high estimated quantity of deposited mud. Therefore, the proposed method is accurate and robust in predicting the conditions of real drawbells.

Figure 6 Spatial distribution of the estimated mud deposits according to the fuzzy-based approach (a) during 5-12 April 2011; (b) during 12-19 April 2011.
The estimated quantity of deposited mud is given in units of volume so as to be compatible with the water measurements, which are also usually given in units of volume. In addition, the results of the spilled mud observations by PTFI are also given in terms of volume. From 05-12 April 2011, the quantity of deposited mud in drawbells P#2W-10 and P#2E-11 were estimated to have been between 2500-3000 m$^3$, while a week later, from 12-19 April 2011, it was predicted that it increased to between 3000-3500 m$^3$ (depicted in Figure 6(b)). During these intervals of time, daily rainfall was recorded in the range of 15-55 mm/day, which is considered high. Accordingly, based on the criteria of a saturated wet drawbell, drawpoints P#2W-10 and P#2E-11 can be classified as a cluster of inter-connected over-consolidated wet drawbells. This classification indicates that both drawbells were probably less mucked. Indeed, it has been reported that prior to the wet muck spill event, the drawpoints were not mucked for more than 14 days. High rainfall and less mucking have been predicted to be the main factors involved in the wet muck spills from the drawpoints.

Following the wet muck spill event on April 18$^{th}$, 2011, detailed observations were conducted, including measurements of the amount of spilled wet mud, which was estimated to be 2485 m$^3$ of dense mud and about 522 m$^3$ of washy mud, which is a total of around 3007 m$^3$ of mud. These measurements fall within the range of the estimated quantity of mud, at the lower boundary;
however, it is still acceptable due to the high level of uncertainties of the contributing factors involved.

Deviations between observations and predictions can be linked to the following: (1) the values of fine material fraction and water content obtained from the observations of the drawpoints before 18 April 2011 may not have been accurate, as they resulted from visual observations and inspections; (2) the quantity of materials that entered drawbells P#2W-10 and P#2E-11 could have been less than was predicted by the model; and (3) there are other potential reasons that are still unknown regarding the distribution of fine materials and water above the extraction level, which cannot yet be determined by the model. Therefore, the model should be fine-tuned based on further research and development in this field. The empirical relationship between the historical observed wet muck spill and the predicted quantity of deposited mud from all drawpoints should be examined for additional information, which can be used to improve the accuracy of the predictive tool.

5 Conclusion

A fuzzy-based predictive tool for the estimation of the spatio-temporal distribution of wet muck was developed, as described in this paper. It consists of a model used to determine drawpoint status and a model used for the estimation of the quantity of deposited mud in the corresponding drawpoints. The status of a drawpoint can be determined using a fuzzy logic operation, and is related to five wet muck contributing factors, i.e. HoD, fine material, water content, rainfall, and no-mucking days. The model used for the estimation of the quantity of deposited mud was numerically solved using fuzzy number operations in a system of fuzzy ordinary difference equations. Both models were then successfully used to reconstruct the wet muck spill event on April 18th, 2011 in a block cave. The reconstruction was also utilized to validate the proposed method. Further research on this topic should be conducted to obtain better prediction accuracy. The empirical relationship between the observed historical data and the predicted quantities in combination with the logical and numerical operations that have been proposed in this paper should be investigated further to develop an adaptive and robust predictive tool.

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