Efficient CFO Compensation Method in Uplink OFDMA for Mobile WiMax

Lakshmanan Muthukaruppan1,*, Parthasharathi Mallick2, Nithyanandan Lakshmanan3 & Sai Krishna Marepalli1

1School of Electronics Engineering, Katpadi – Tiruvalam Road, VIT University, Vellore-632 014, Tamilnadu, India
2School of Electrical Engineering, VIT University, Katpadi – Tiruvalam Road, Vellore-632 014, Tamilnadu, India
3Department of Electronics and Communication, Pondicherry Engineering College, ECR Road and V.S.R. Street, Puducherry – 605 014, Tamilnadu, India
*E-mail: mlakshmanan@vit.ac.in

Abstract. Mobile WiMax uses Orthogonal Frequency Division Multiple Access (OFDMA) in uplink where synchronization is a complex task as each user presents a different carrier frequency offset (CFO). In the Data Aided Phase Incremental Technique (DA-PIT) estimation is performed after FFT operation to use the received frequency domain pilot subcarrier information. As estimation is done in the presence of noise, there exists some offset error, which is called residual frequency offset (RFO). The Simple Time Domain Multi User Interference Cancellation scheme (SI-MUIC) is a time domain approach which takes a longer time delay to compensate the CFO effect for the last user. Decorrelation-Successive Interference Cancellation (DC-SC) and Integrated Estimation and Compensation (IEC) are frequency domain approaches that compensate the CFO effect with a more complex method for ICI cancellation. The Modified Integrated Estimation and Compensation technique (Modified IEC) is proposed for better residual CFO compensation. The proposed technique has better performance due to its efficient suppression of ICI and MUI. The difference between the CFOs of two OFDMA symbols lies within the range of RFO that is not considered in the conventional compensation techniques, such as the SI-MUIC, DC-SC and IEC compensation techniques.

Keywords: carrier frequency offset; mobile WiMax; OFDMA; synchronization; uplink.

1 Introduction

OFDMA has emerged as one of the prime multiple access schemes for future broadband wireless networks with variable data rates and quality of service requirement. Mobile WiMax [1] is one of the future broadband wireless networks that employ OFDMA in uplink. In OFDMA, the subcarriers are divided into different groups. Each group of subcarriers is assigned to a user based on different subcarrier assignment schemes. OFDMA is sensitive to CFO due to variations in frequency at the transmitter and receiver or due to Doppler
shifts. It is more sensitive to CFO as it is affected by the CFO of more than a single user. Thus in OFDMA, synchronization [2-5] is more difficult as each user presents a different CFO.

The carrier frequency offset results in intersymbol interference (ISI) and multiuser interference (MUI). ICI is caused by the leakage or interference between the user’s own subcarriers, whereas MUI is caused by leakage or interference from other users’ subcarriers. Hence synchronization becomes a crucial task for uplink in Mobile WiMax. The synchronization process involves the estimation of CFO for all active users. The estimated CFOs [6,7] are used to compensate the effect caused by it. There is a need to design a synchronization method that can produce a good performance.

This paper deals with various techniques to compensate the CFO effect on the received signal in the time and frequency domain. SI-MUIC is a time domain approach that takes a longer time delay to compensate the CFO effect for the last user, since the last user can be processed when all other users have been demodulated. DC-SC and IEC [8] are frequency domain approaches to compensate the CFO effect on the received signal. In DC-SC, the decorrelation method reduces ICI, while the successive interference cancellation method reduces MUI. The main issue in DC-SC is that there will be an error in the estimated CFO, called residual frequency offset. IEC is an iterative [9] approach of DC-SC, where the compensated output is fed back to the estimation stage for correcting the residual frequency offset. DC-SC and IEC have a more complex method for ICI cancellation.

Thus, an efficient CFO compensation technique in uplink OFDMA for mobile WiMAX with a reduced number of computations is proposed in this paper. The simulation results show that the proposed Modified IEC compensation technique has a better bit error rate (BER) performance over the existing IEC, SI-MUIC (2 it), DC SC, and SI-MUIC (1 it) compensation techniques due to the efficient suppression of ICI and MUI, since it compensates the residual CFO on the received symbol using the CFO of the previous symbol after FFT operation. Thus, the proposed Modified IEC compensation technique with DA-PIT estimator is better at estimating CFO.

The rest of this paper is organized as follows. The second section includes the system model and describes the effect of carrier frequency offset on the received signal. The third section describes the DA-PIT method for estimating the CFO, the time domain approach for compensation and the frequency domain approach for compensation. The proposed compensation method is explained in the fourth section. The fifth section gives the simulation results followed by the conclusion.
2 System Model

An OFDMA system in uplink OFDMA for mobile WiMAX is considered with K users and N subcarriers. The N subcarriers are divided among K users by a block subcarrier assignment scheme. The base station is assumed to know the subcarrier assignment scheme for each active user.

Let \( N_i \) be the set of subcarriers allocated to the \( i^{th} \) user with no subcarrier in common between two users such that in Eq. (1).

\[
\bigcup_{i=1}^{K} N_i = \{0,1,2, \ldots, N - 1\}
\]  

(1)

The baseband signal model of the uplink OFDMA system is shown in Figure 1.

Each user modulates his subcarriers using independent and identically distributed information symbols drawn from finite constellations.

\[
x_{i}(n) = \sum_{n \in I_{i}} d_{i}(n)e^{j \pi n k / N} ; \quad -N_g \leq k \leq N - 1
\]  

(2)

where \( I_{i} \) is the set of subcarriers assigned to a selected user and \( d_{i}(n) \) is the symbol transmitted over the \( n^{th} \) subcarrier. Assume K users are simultaneously active in a system and are transmitting data to the base station receiver. Each stream propagates through a multipath channel with impulse response. Here \( h_{i} = [h_{i}(0), h_{i}(1), \ldots, h_{i}(L - 1)]^{T} \) represents L tapped multipath channel impulse response for the \( i^{th} \) user and arrives at the base station with timing offset \( \theta_{i} \) and frequency offset \( \epsilon_{i} \).

At the receiver, the cyclic prefix is removed and the received signal in the discrete time domain can be written as in Eq. (3).
Efficient CFO Compensation Method in Uplink OFDMA …

\[ y(n) = \sum_{i=1}^{K} \sum_{l=0}^{L-1} x_i(n-l) e^{j2\pi n \epsilon_i / N} * h_i(l) + w_i(n) \]  (3)

where \( y(n) = \{y(0), y(1), ..., y(N-1)\} \) represents the received signal and 
\( w(n) \) is complex additive white Gaussian noise with variance \( \sigma^2 \) and 
\( \epsilon_i = \frac{\Delta f}{f_{sub}} \)
is the normalized CFO for the \( i \)th user.

A perfect timing synchronization is considered and CFO is estimated for mobile
users at the base station.

Then, IFFT is applied to the received signal \( y(n) \) and the output at the \( k \)th
subcarrier in the frequency domain can be given as Eq. (4):

\[ Y(k) = \sum_{i=1}^{K} \sum_{u \in N_i} X_i(u) H_i(u) C(u, k, \epsilon_i) + W_i(k) \]  (4)

where \( X_i(u), H_i(u), W_i(k) \) are the frequency domain representations of \( x_i(n), \\
h_i(n), w_i(n) \) respectively.

\( C \) represents the leakage of subcarrier \( u \) on subcarrier \( k \) due to frequency offset 
\( \epsilon_i \) and is given as Eq. (5):

\[ C(u, k, \epsilon_i) = e^{j\pi(u-k-\epsilon_i)^2 / N} \sin(\pi(u-k-\epsilon_i) / N) 
\]  (5)

The IFFT of \( L \) tapped multipath channel response is given as Eq. (6):

\[ H_i(k) = \sum_{l=0}^{L-1} h_i(l) e^{j2\pi l / N} \]  (6)

Assume \((k \in N_i)\), \( k \)th subcarrier information for \( i \)th user can be written as in Eq. (7).

\[ Y_i(k) = X_i(k) H_i(k) C(u, k, \epsilon_i) + \sum_{u \in N_i, u \neq k} X_i(u) H_i(u) C(u, k, \epsilon_i) + \\
\sum_{j \neq i}^{K} \sum_{u \in N_j} X_j(u) H_j(u) C(u, k, \epsilon_j) + W_i(k) \]  (7)

In Eq. (7) the first term represents the desired signal affected by its multipath
channel and leakage from its own subcarrier. The second term represents ICI
due to the leakage from the user’s own subcarriers. The third term represents
MUI caused by the leakage from other users’ subcarriers.
3 Related Works

3.1 CFO Estimation

The pilot subcarriers are inserted into every M OFDMA symbol by active users during data transmission. The phase rotation between the first and the second OFDMA symbol due to the frequency offset \( \varepsilon \) is given as in Eq. (8):

\[
\theta_2 = 2\pi \frac{N_s}{N} \varepsilon
\]

(8)

where \( N_s = N + N_g \)

The phase rotation between the first and the third OFDMA symbol is given as in Eq. (9).

\[
\theta_3 = 2\pi \frac{2N_s}{N} \varepsilon
\]

(9)

The rotation in Eq. (9) is twice that of Eq. (8). Hence, CFO estimation is more accurate in Eq. (9) than Eq. (8). The estimation range is reduced to half using Eq. (9) and there will be no problem when CFOs are small. The pilot subcarrier in the first and the \( M^{th} \) OFDMA symbol can be used if the channel and CFOs remain constant for the \( M \) OFDMA symbols. It is important to note that there must be the same pilot information at the same location in the first and the \( M^{th} \) OFDMA symbol.

3.1.1 Data Aided Phase Incremental Technique (DA-PIT)

The DA-PIT estimator [11] uses pilot subcarriers that are included in the data frame to estimate CFO. In OFDMA systems, each user inserts the pilot with data subcarriers into the first and the \( M^{th} \) OFDMA symbol of their data frame. The pilot subcarrier location is assumed to be known at the receiver. Once the FFT is performed, the pilot subcarriers are separated and the post FFT correlation [12] at the \( p^{th} \) pilot subcarrier is shown in Eq. (10).

\[
C_i(p) = Y_m(p) Y_{m+M-1}^*(p)
\]

(10)

where \( Y_m(p) \) and \( Y_{m+M-1}(p) \) are the \( p^{th} \) pilot subcarrier information extracted from the \( m^{th} \) and the \( (m+M-1)^{th} \) OFDMA symbol.

Using (10), CFO can be estimated as in Eq. (11).

\[
\varepsilon_i = \frac{N}{2\pi(N_g)(M-1)} \text{arg} \left( \sum_p C_i(p) \right)
\]

(11)

where \( \text{arg}(.) \) gives the phase shift between the \( m^{th} \) and the \( (m+M-1)^{th} \) OFDMA symbol. From Eq. (11) it can be noted that the number of computations will not increase with \( M \). In DA-PIT, CFO estimation is done once for every \( M \).
OFDMA symbol. It is required to estimate CFO for each OFDMA symbol for fast varying channels. DA-PIT can be applied when CFO is identical for M OFDMA symbols.

### 3.2 Time Domain Compensation Technique

#### 3.2.1 Simple Time Domain Multi-user Interference Cancellation (SI-MUIC)

In a conventional single FFT receiver, one FFT block is used to demodulate all users at the same time. It is important to note that in the scenarios where multiple CFOs are involved, a conventional single FFT receiver is not efficient as it can be aligned to one user at a given time and the remaining users are misaligned. In a multiple FFT receiver structure, each active user is assigned with one OFDMA demodulation block so that the CFO effect can be compensated independently in the time domain.

After compensation, the output of the OFDMA demodulator for the $i^{th}$ user is given as

$$
Y_i(k) = A_u F_N \left( e^{-j \frac{2\pi f_m}{N}} \right) r(n)
$$

$$
Y_i(k) = X_i(k) H_i(k) + \sum_{j=1, j \neq i}^{K} X_j(u) H_j(u) C_{e_j} + W_i(k)
$$

(12) (13)

where $A_u$ is the diagonal matrix to select the subcarriers of a particular user, $F_N$ is the FFT matrix, the exponent term $e^{-j \frac{2\pi f_m}{N}}$ is the time domain frequency offset correction factor, and $r(n)$ is the received time domain signal after the removal of the cyclic prefix. The diagonal elements of $A$ are taken as one for the subcarriers of a particular user and the remaining elements are taken as zero.

From Eq. (13) it is observed that the attenuation and self-interference factors are eliminated for the $i^{th}$ user, provided that its CFO value is exactly predicted leaving out MUI. A time domain MUI cancellation scheme is used in the multiple FFT receiver structure to eliminate MUI. Initially, received signals are sorted in the order of signal strength and the base station processes these signals from the strongest to the lowest power signal.

The received signal for the $i^{th}$ user is given as in Eq. (14).

$$
\tau_{i,j} = C_{-\epsilon_i} \left[ r(n) - \sum_{p_1=1}^{K} C_{\epsilon_{p_1}} r_{p_1,j} - \sum_{p_2=1}^{K} C_{\epsilon_{p_2}} r_{p_2,j-1} \right]
$$

(14)
where $r_{i,j}$ represents the feedback from the $i^{th}$ user and the $j^{th}$ iteration and $C_{-E_i}$ is the frequency offset correction factor given as $e^{-j2\pi E_i/N}$. The initial iteration of $r_{i,j}$ is taken as zero.

SI-MUI takes a longer time delay to compensate the CFO effect for the last user since the last user can be processed when all other users have been demodulated. It does not require channel information but it does require the correct CFO values of each user for compensation.

### 3.3 Frequency Domain Compensation Technique

Decorrelation-Successive Cancellation (DC-SC) compensates the effect of ICI and MUI [13]. In DC-SC the decorrelation method reduces ICI and the successive interference cancellation method reduces MUI.

In the decorrelation method, an interference matrix is constructed using the estimated CFO for a group of subcarriers that forms a block. The blocks are arranged in decreasing order of their average power. Then, the decorrelation is applied to all the subcarriers of the arranged blocks, starting from the block with the highest power, to reduce ICI.

Successive Interference Cancellation (SC) eliminates MUI from the subcarrier information without ISI of the decorrelation method. It needs correct data decisions in the demapper for accurate MUI cancellation [14]. This process continues for each subcarrier in all the blocks to eliminate MUI. MUI occurs mostly due to the block of neighbors. Hence, it is sufficient to eliminate MUI from neighboring blocks than from all the blocks, so computational complexity is reduced. The performance of DC-SC is very much dependent on an accurate estimation of CFO. As estimation is done in the presence of noise, there exists some offset error in some cases, called residual frequency offset (RFO).

The performance of Decorrelation-Successive Cancellation (DC-SC) is highly dependent on an accurate estimation of CFO. Even a 2% offset will introduce a subcarrier phase shift of $22^\circ$. Therefore, an iterative approach [8] is introduced in DC-SC, called Integrated Estimation and Compensation (IEC).

In the existing IEC, the compensated output is fed back to the estimation stage for the next iteration. Thus, the residual frequency offset is estimated in the next iteration and the effect is compensated. The iterative process [15] is repeated to provide better performance than DC-SC. The problem in IEC is that the difference between the CFOs of two OFDMA symbols does not lie within the range of RFO.
4 Modified Integrated Estimation and Compensation

Modified IEC is proposed to compensate residual CFO available in MDA-PIT estimation with better reduction in ICI and MUI, as shown in Figure 2.

\[ \hat{y}_{ci} = (\hat{C}_i)^{-1} y_{ci} \]  

where \( y_{ci} \) and \( \hat{y}_{ci} \) are the subcarrier information of the \( c^{th} \) block of the \( i^{th} \) user with and without ICI respectively.

\[ \hat{C}_i = C(u, k, \hat{\varepsilon}_i) \]  

ICI and phase rotation caused in the desired subcarrier due to frequency offset \( \hat{C}(k, k, \hat{\varepsilon}_i) \) are corrected. The subcarrier information \( \hat{y}_{ci} \) obtained from (15) is transmitted with MUI. The interference \( \rho_c(k) \) caused by the \( c^{th} \) block of the \( i^{th} \) user, \( N_i^c \) in the \( k^{th} \) subcarrier can be reconstructed with the knowledge of estimated CFO and the channel information is given as in Eq. (17)

\[ \rho_c(k) = \sum_{u \in N_i^c} H'(u)X'(u)C(u, k, \hat{\varepsilon}_i), \quad k \notin N_i^c \]  

where \( H'(u)X'(u) \) is the transmitted signal of the \( c^{th} \) block, \( C \) is the interference of the \( u^{th} \) subcarrier on the \( k^{th} \) subcarrier with estimated CFO \( \hat{\varepsilon}_i \).

Hence the received output \( \hat{Y}(k) \) on the \( k^{th} \) subcarrier from the \( c^{th} \) block after the removal of MUI is given as in Eq. (18).

\[ \hat{Y}(k) = Y(k) - \rho_c(k) \]  

where
\[ Y(k) = \hat{y}_{ci} \]

The limitation of Modified IEC is that if the CFO of the \( i^{th} \) user for \( j^{th} \) OFDMA symbol is larger than \( (j+1)^{th} \) OFDMA symbol and vice versa, the net CFO will be higher than the original CFO. Hence, the Modified IEC can be used for every M OFDMA symbol. After performing Modified IEC for the M OFDMA symbols, the CFO estimation is performed and this process is repeated for the next consecutive M OFDMA symbols. The CFO for the \( (j+1)^{th} \) OFDMA symbol is updated by subtracting the RFO of the \( j^{th} \) CFO from the \( (j-1)^{th} \) CFO.

The steps involved in Modified IEC are:
1. Update \( C_i(u, k, \hat{\epsilon}_i) \) using (5) with CFO of previous OFDMA symbol \( \hat{\epsilon}_i \).
2. Compensate ICI and MUI using (15) and (18) using \( C_i(u, k, \hat{\epsilon}_i) \) in Step 1.
3. Estimate RFO \( \hat{\epsilon}_i \) for K active users.
4. Compensate ICI and MUI using (15) and (18) with leakage across the subcarrier, \( C_i(u, k, \hat{\epsilon}_i) \) and \( \hat{\epsilon}_i \) from Step 3.
5. Update CFO for next OFDMA symbol.

5 Simulation Results

The performance of CFO compensation techniques has been analyzed through simulations using MATLAB for various signal to noise ratios (SNRs). SNR is defined as the ratio of signal power to noise power. A quasi-synchronous scenario was considered between the mobile and the base station and the simulations were performed under mobile multipath channel. The parameters listed in Table 1 were used for the simulation.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Simulation parameters of OFDMA in Uplink Mobile WiMAX.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>Number of active users (K)</td>
<td>4</td>
</tr>
<tr>
<td>Number of subcarriers (N)</td>
<td>512</td>
</tr>
<tr>
<td>CFO range</td>
<td>{-0.1, 0.1}</td>
</tr>
<tr>
<td>System bandwidth</td>
<td>20MHz</td>
</tr>
<tr>
<td>Subcarrier spacing</td>
<td>10.9375KHz</td>
</tr>
<tr>
<td>Subcarrier mapping</td>
<td>QPSK</td>
</tr>
<tr>
<td>Cyclic prefix length (Ng)</td>
<td>32</td>
</tr>
<tr>
<td>Perfect sequence length</td>
<td>512</td>
</tr>
<tr>
<td>Circular shift in time dimension</td>
<td>64</td>
</tr>
<tr>
<td>Circular shift in space dimension</td>
<td>64</td>
</tr>
</tbody>
</table>

Figure 3 shows the BER performance of the Modified IEC, IEC, DC-SC and SI-MUIC (1 it) compensation techniques with DA-PIT estimator for \( L = 2 \). It is
observed that Modified IEC with DA-PIT estimator requires an SNR of 17.5dB, whereas the IEC, DC-SC and SI-MUIC (1 it) compensation techniques with DA-PIT estimator require an SNR of 20dB, 23dB and 26dB, respectively, to achieve a BER of $10^{-3}$. Thus, Modified IEC with DA-PIT estimator has better BER performance than IEC, DC-SC and SI-MUIC (1 it) with DA-PIT estimator for $L = 2$.

Figure 3 BER performance of Modified IEC, IEC, DC-SC and SI-MUIC for one iteration (1 it) with DA-PIT estimator for $L = 2$.

Figure 4 shows the BER performance of the Modified IEC, IEC, SI-MUIC (2 it) and DC-SC compensation techniques with DA-PIT estimator for $L = 2$. It is observed that Modified IEC with DA-PIT estimator requires an SNR of 17.5dB whereas IEC, SI-MUIC (2 it) and DC-SC compensation techniques with DA-
PIT estimator require an SNR of 20dB, 20.5dB and 23dB respectively, to achieve a BER of $10^{-3}$. Thus, Modified IEC with DA-PIT estimator has better BER performance than IEC, SI-MUIC (1 it) and DC-SC with DA-PIT estimator for $L = 2$. From Figures 3 and 4 it can be concluded that Modified IEC with DA-PIT estimator is better than IEC, DC-SC and SI-MUIC with DA-PIT estimator for one single iteration, because ICI and MUI are suppressed due to the iterative approach of IEC without precompensation.

Figure 5 shows the BER performance of Modified IEC, IEC and DC-SC with DA-PIT estimator for $L = 2$. It is observed that Modified IEC with DA-PIT estimator requires an SNR of 17.5dB whereas the IEC and DC-SC compensation techniques with DA-PIT estimator require an SNR of 20dB and 23dB respectively, to achieve a BER of $10^{-3}$. Thus, Modified IEC with DA-PIT estimator has better BER performance than IEC and DC-SC with DA-PIT estimator.

Thus, Modified IEC compensation with DA-PIT estimator outperforms DC-SC and IEC compensation with DA-PIT due to efficient suppression of ICI and MUI. Also, the difference between the CFOs of two OFDMA symbols lies within the range of RFO. Hence, the effect of ICI and MUI on the received symbol is pre-compensated using the CFO of the previous symbol.
6 Conclusion

In this paper, an efficient CFO compensation technique in uplink OFDMA for mobile WiMAX was proposed. The simulation results showed that the proposed Modified IEC compensation technique has 2.5dB, 3dB, 5.5dB and 8.5dB gain over the existing IEC, SI-MUIC (2 it), DC SC and SI-MUIC (1 it) compensation techniques to achieve a BER of $10^{-3}$. Thus, the Modified IEC compensation technique with DA-PIT estimator outperforms the SI-MUIC, DC-SC and IEC compensation techniques with DA-PIT estimator. Modified IEC compensation has better BER performance due to the efficient suppression of ICI and MUI, since it compensates the residual CFO on the received symbol using the CFO of the previous symbol after FFT operation. It can be concluded that the proposed Modified IEC compensation technique with DA-PIT estimator is better in estimating CFO.

References


