

Study on The Breaker Height of Water Wave Equation Formulated Using Weighted Total Acceleration Equation (Kajian Teknis)

Syawaluddin Hutahaean

Ocean Engineering Program, Faculty of Civil and Environmental Engineering
Bandung Institute of Technology - Bandung 40132, Indonesia, Email: syawaluddin@ocean.itb.ac.id

Abstract

This research is the continuation of previous research conducted by the author, where in this research a wave equation is developed using total acceleration equation for a function $f(x, z, t)$ and formulated using a complete velocity potential. Based on the wave equation, shoaling-breaking model is developed and the breaker height produced by the model is examined. There is a conformity between breaker height a model and breaker height produced by the previous research.

Keywords: Complete potential velocity, weighted total acceleration, breaker height

Abstrak

Penelitian ini merupakan kelanjutan dari penelitian sebelumnya yang dilakukan oleh penulis, dimana pada penelitian tersebut dikembangkan suatu persamaan gelombang dengan menggunakan total acceleration equation untuk suatu fungsi $f(x, z, t)$ dan dirumuskan dengan menggunakan velocity potential yang lengkap. Berdasarkan persamaan gelombang tersebut dikembangkan model shoaling-breaking dan diteliti breaker height yang dihasilkan model. Diperoleh kesesuaian antara breaker height model yang digunakan dengan breaker height hasil penelitian terdahulu.

Kata-kata kunci: Potensial kecepatan yang lengkap, percepatan total terbobot, breaker height

1. Introduction

Hutahaean (2019a) developed weighted total acceleration for total derivative function $f(x, z, t)$ where there is weighting coefficient at the differential time t term. The equation was done for the calculation of wave constant, i.e. wave number k and wave constant G . Using the equation, a shoaling breaking model was developed and it is obtained that wave-constant G should be multiplied by a coefficient to obtain a breaker height that is in line with the result of laboratory research from previous researchers.

Hutahaean (2019b) developed weighted total acceleration for total derivative in a function $f(x, z, t)$ where there is weighting coefficient at the differential time t terms and differential at the vertical z axis. Then the equation was done in the Euler momentum equation and is used to calculate wave number k and wave constant G using a complete wave flow potential of Laplace equation solution of Dean (1991). In this research, wave profile in the deep water is compared with Wilson criteria (1963), whereas the maximum wave height in a wave period is compared with Wiegel equation (1949-1964). The comparison shows a very good conformity. Review on the result of the research is presented in section II.

In this research, a further study is done, i.e. on breaker height produced by the model, where the breaker height

model is compared with the average values of breaker height from five breaker height index (BHI) from Komar and Gaughan (1972), Larson, M. and Kraus, N.C. (1989), Smith and Kraus (1990), Gourlay (1992) and Rattana Pitikon and Shibayama (2000), whereas breaker depth is compared with breaker height from SPM (1984). The formulation of shoaling and breaking is written in section III and IV.

2. Review on the Explored Wave Theory

Hutahaean (2019b) developed water wave theory using a complete velocity potential equation, i.e.

$$\Phi(x, z, t) = G(\cos kx + \sin kx) \cosh k(h + z) \sin \sigma t \quad (1)$$

Where x is horizontal axis, z is vertical axis, $\sigma = \frac{2\pi}{T}$ is

angular frequency, whereas T is wave period, k is wave number and G is wave constant.

Equations produced in that research and those that will be studied against the shoaling and breaking phenomenon are among others water wave surface equation, wave number k equation and wave constant G equation.

2.1 Waterwave surface equation

Using the velocity potential equation, water surface equation is formulated. To simplify the writing, the

following is defined $c_0 = \frac{gk}{\gamma\sigma} \cosh k(h + \eta)$

$c_1 = \sin kx$, $c_2 = \cos kx$ and $c_3 = \cos \sigma t$

$$\eta(x, t) = c_0 \left((c_2 + c_1) + (c_1 - c_2) \frac{\partial \eta}{\partial x} \right) c_3 \quad (2)$$

Equation (2) is differentiated against horizontal- x axis

$$\frac{\partial \eta}{\partial x} = c_0 k \left((-c_1 + c_2) + (c_2 + c_1) \frac{\partial \eta}{\partial x} \right) c_3 \quad (3)$$

Equation (2) is water wave surface equation that was used to calculate water surface elevation where $\frac{\partial \eta}{\partial x}$ in

(2) is calculated with (3). η In $c_0 = \frac{gk}{\gamma\sigma} \cosh k(h + \eta)$

is calculated with (4),

$$\eta(x, t) = A(\cos kx + \sin kx) \cos \sigma t \quad (4)$$

Whereas $\frac{\partial \eta}{\partial x}$ in (3) is calculated with,

$$\frac{\partial \eta}{\partial x} = Ak(-\sin kx + \cos kx) \cos \sigma t \quad (5)$$

2.2 Wave number k , wave constant G and wave amplitude function equations

Hutahaean (2019b) formulated a weighted total acceleration equation for a function $f(x, z, t)$ in the

form of, $\frac{Df}{dt} = \gamma \frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + \gamma_z w \frac{\partial f}{\partial z}$

u is the velocity of fluid particle in the horizontal- x direction, w is the velocity of fluid particle in the vertical- z direction, t is time, γ and γ_z is weighting coefficient.

Equation to calculate wave number k or dispersion equation in the form of,

$$\gamma^2 \sigma^2 \left(1 - \frac{kA}{2} \right) + \frac{\gamma^2 \sigma^2}{4} (1 - \gamma_z) kA = gk \left(1 - \frac{kA}{2} \right)^2 \quad (6)$$

Where k is the wave number whose values is sought,

$\sigma = \frac{2\pi}{T}$ is angular frequency, T is wave period, g is gravitational velocity, A is wave amplitude.

Weighting coefficient value for time- t (γ) differential

$$\frac{1}{2} - \gamma + \frac{1}{2} \gamma^2 = \frac{\epsilon}{\sigma \delta t} (1 + \gamma) \quad (7)$$

Where for $\delta t = \frac{2\epsilon}{\sigma}$, ϵ is a very small number, $\gamma = 3$ is

obtained for all wave period and for all values of ϵ . Weighting coefficient value for differential against vertical- z (γ_z) axis.

$$\begin{aligned} & -\frac{\gamma^2}{2} c_1 + \gamma - \frac{c_1}{2} - \frac{1}{2} (-\gamma c_1 - c_1) \\ & - \left(\gamma + 1 + \frac{1}{2} \right) c_2 \gamma_z + \frac{c_1}{2} \gamma_z^2 = 0 \end{aligned} \quad (8)$$

$c_1 = c_2 = \cosh(2.415\pi) = \sinh(2.415\pi)$, for $\gamma = 3$, $\gamma_z = 1.630$ is obtained. The constant 2.415 is obtained from the adjustment of breaker depth that was done in Chapter V.

Wave amplitude function equation

$$A = \frac{2Gk}{\sigma\gamma} \cosh(2.415\pi) \left(1 - \frac{kA}{2} \right) \quad (9)$$

In a deep water where wave amplitude A is a known input, the calculation of wave constant G is done with (9),

$$G = \frac{\sigma\gamma A}{2k \cosh(2.415\pi) \left(1 - \frac{kA}{2} \right)} \quad (10)$$

2.3 The result of equation in deep water

In the case that convective equation is ignored then (6) becomes,

$$\gamma^2 \sigma^2 = gk \left(1 - \frac{kA}{2} \right) \quad (11)$$

This equation is a quadratic equation for wave number k , where there is an equation root if the value of determinant D is bigger than or equal to zero. The wave amplitude is maximum in the equation if determinant $D = 0$, where the following is obtained

$$A_{max} = \frac{g}{2\gamma^2 \sigma^2} \quad (12)$$

A_{max} in (11) is wave amplitude maximum for a wave period

Deep water depth h_0 is obtained with the following equation

$$h_0 = \frac{1}{k} \left(2.415\pi - \frac{kA}{2} \right) \quad (13)$$

Table 1 shows the result of the calculation for a number of wave periods T , where A_{max} is used as the wave amplitude. Wave height H , is calculated with equation $H = \eta_{max} - \eta_{min}$, where η_{max} is wave crest elevation and η_{min} is wave trough elevation, both of which are obtained from water surface equation (2), using $\cos \sigma t = 1$.

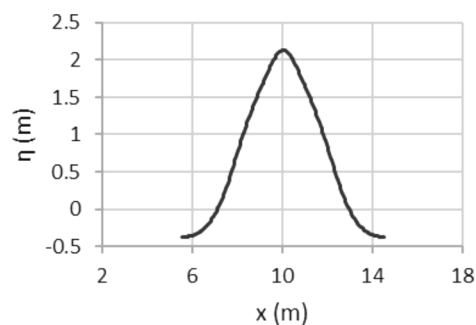


Figure 1. The profile of a wave with wave period 8 sec., in one wave length

Wave-steepness $\frac{H}{L} = 0.28$, where considering the calculation used maximum wave Amplitude A that was calculated using (12), then wave steepness is critical

Table 1. Characteristic of wave profile in deep water

T (sec)	H (m)	L (m)	$\frac{H}{L}$	$\frac{H}{A}$	$\frac{\eta_{max}}{H}$
6	1,409	5,026	0,28	2,865	0,851
7	1,918	6,842	0,28	2,865	0,851
8	2,506	8,936	0,28	2,865	0,851
9	3,171	11,309	0,28	2,865	0,851
10	3,915	13,962	0,28	2,865	0,851
11	4,737	16,894	0,28	2,865	0,851
12	5,638	20,105	0,28	2,865	0,851
13	6,617	23,595	0,28	2,865	0,851
14	7,674	27,365	0,28	2,865	0,851
15	8,81	31,413	0,28	2,865	0,851

Table 2. Type of wave based on Wilson criteria (1963)

Wave Type	$\frac{\eta_{max}}{H}$
Airy waves	< 0.505
Stoke's waves	< 635
Cnoidal waves	$0.635 < \frac{\eta_{max}}{H} < 1$
Solitary waves	= 1

wave steepness. The critical wave steepness is bigger than the criteria of Michell (1893) i.e. $\frac{H}{L} = 0.142$. The comparison between wave height and wave amplitude is $\frac{H}{A} = 2.865$ which is bigger than 2. Therefore, the relation between wave height and wave amplitude is $H = 2A$ cannot be determined. The obtained Wilson parameter is $\frac{\eta_{max}}{H} = 0.851$. Based on Wilson criteria (1963), Table 2, the value of the parameter shows that the wave profile has a cnoidal wave type, with wave profile presented in Figure 1. for wave period $T = 8$ sec.

2.4 Comparison with Wiegel equation

Using data from an observation, Wiegel (1949-1964) formulated the relation between wave period T and maximum wave height H_{max} in a wave period, i.e.

$$T_{Wieg} = 15.6 \sqrt{\frac{H_{max}}{g}} \quad (14)$$

The comparison was done to calculate T_{Wieg} in (14) using wave height from the result of a calculation using a model, and as an input in the model is wave period T and wave amplitude calculated with (12), so that wave height that was obtained is wave height maximum H_{max} in the related wave period. The calculation was done using two γ values as presented in Table 3.

Table 3 shows that at $\gamma = 3$, T_{Wieg} that was obtained is almost equal to T which is a wave period to calculate H_{max} with model. Whereas for $\gamma = 2.97102$, it can be stated that $|T - T_{Wieg}|$ is equal to zero. From this comparison, a conclusion can be made that the values of γ , γ_z and equations formulated in this research

Table 3. The comparison with wiegel equation.

T (sec)	$\gamma = 3.0$ $\gamma_z = 1.63164$		$\gamma = 2.97102$ $\gamma_z = 1.60095$	
	H_{max} (m)	T_{Wieg} (sec)	H_{max} (m)	T_{Wieg} (sec)
6	1,40943	5,91305	1,45118	6
7	1,9184	6,89858	1,97523	7,00002
8	2,50569	7,88412	2,57992	8,00007
9	3,1713	8,86969	3,26522	9,00007
10	3,91518	9,85522	4,03119	10,0002
11	4,73746	10,8408	4,87774	11,0002
12	5,63796	11,8264	5,80504	12,0003
13	6,61695	12,8121	6,81286	13,0004
14	7,67409	13,7976	7,90155	14,0006
15	8,80954	14,7831	9,07066	15,0006

correspond with the result of Wiegel (1949-1964) research which is the result of an observation.

3. Governing Equations of Shoaling-Breaking Model

3.1 Energy conservation equation

a. Mass conservation of a potential velocity component

Equation 1 was done at a characteristic point, i.e. a point where $\cos kx = \sin kx = \cos \sigma t = \sin \sigma t$. And as function x , $\cos kx$ is used,

$$\Phi(x, z, t) = 2G \cos kx \cosh k(h+z) \sin \sigma t \quad (15)$$

$$u = -\frac{\partial \Phi}{\partial x} = 2Gk \sin kx \cosh k(h+z) \sin \sigma t - 2 \frac{\partial G}{\partial x} \cos kx \cosh k(h+z) \sin \sigma t \quad (16)$$

$$\begin{aligned} \frac{\partial u}{\partial x} &= 2Gk^2 \cos kx \cosh k(h+z) \sin \sigma t \\ &+ 2 \frac{\partial Gk}{\partial x} \sin kx \cosh k(h+z) \sin \sigma t \\ &+ 2k \frac{\partial G}{\partial x} \sin kx \cosh k(h+z) \sin \sigma t \\ &- 2 \frac{\partial^2 G}{\partial x^2} \cos kx \cosh k(h+z) \sin \sigma t \end{aligned} \quad (17)$$

$$w = -\frac{\partial \Phi}{\partial z} = -2Gk \cos kx \sinh k(h+z) \sin \sigma t \quad (18)$$

$$\frac{\partial w}{\partial z} = -2Gk^2 \cos kx \sinh k(h+z) \sin \sigma t \quad (19)$$

Substitute (16) and (19) to the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \text{ and it is done at the characteristic point,}$$

$$G \frac{\partial k}{\partial x} + 2 \frac{\partial G}{\partial x} k - \frac{\partial^2 G}{\partial x^2} = 0 \quad (20)$$

Considering that in G there is a wave energy, this equation can also be called energy conservation equation. The existence of the term $\frac{\partial^2 G}{\partial x^2}$ has caused

difficulties in using conservation equation (20). To get rid of the term $\frac{\partial^2 G}{\partial x^2}$, Hutahaeen (2019c) conducted an approximation where (18) is differentiated against horizontal- x axis, then the terms $\frac{\partial^3 G}{\partial x^3}$ and $\frac{\partial^2 k}{\partial x^2}$ are regarded as very small number that can be ignored, whereas $\frac{\partial^2 G}{\partial x^2}$ is substituted with (3), and a new conservation equation is obtained, i.e.

$$3 \frac{\partial G}{\partial x} \frac{\partial k}{\partial x} + 2kG \frac{\partial k}{\partial x} + 4k^2 \frac{\partial G}{\partial x} = 0 \quad (21)$$

This equation is an approximation equation. To obtain a better equation, a complete velocity potential equation (1) is used at the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \text{ and is done in the following section.}$$

b. Mass Conservation of a complete potential velocity

From (1),

$$\begin{aligned} u = -\frac{\partial \Phi}{\partial x} = & -Gk \cosh k(h+z)(-\sin kx + \cos kx) \sin \sigma t \\ & -\frac{\partial Gk}{\partial x} \cosh k(h+z)(-\sin kx + \cos kx) \sin \sigma t \\ & -\frac{\partial G}{\partial x} k \cosh k(h+z)(-\sin kx + \cos kx) \sin \sigma t \\ & -\frac{\partial^2 G}{\partial x^2} \cosh k(h+z)(\cos kx + \sin kx) \sin \sigma t \end{aligned} \quad (22)$$

$$\begin{aligned} w = -\frac{\partial \Phi}{\partial z} = & -Gk \sinh k(h+z)(\cos kx + \sin kx) \sin \sigma t \\ \frac{\partial w}{\partial z} = & -Gk^2 \sinh k(h+z)(\cos kx + \sin kx) \sin \sigma t \end{aligned} \quad (23)$$

(22) and (23), are substituted to continuity equation $\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$ and are done at the characteristic point, and the following is obtained

$$\frac{\partial^2 G}{\partial x^2} = 0 \quad (24)$$

Substitute (24) to (18), energy conservation equation as well as mass conservation equation were obtained in other form, i.e.

$$G \frac{\partial k}{\partial x} + 2 \frac{\partial G}{\partial x} k = 0 \quad (25)$$

This is exact equation, not an approximation equation as in (20).

3.2 Wave amplitude function equation

Hutahaeen (2019d) shows that wave constant, i.e. wave number k and wave constant G can be obtained by using one velocity potential component where either using the cosine component or sine component, similar wave constants equations will be obtained. By substituting one velocity potential component at the surface kinematic boundary condition and integrating the formed equation against time- t , Hutahaeen (2019b) obtained wave amplitude function equation, i.e.

$$A = \frac{2Gk}{\sigma \gamma} \cosh(2.415\pi) \left(1 - \frac{kA}{2}\right) \quad (26)$$

This equation is differentiated against horizontal- x axis.

$$\frac{\partial A}{\partial x} - \frac{2}{\sigma \gamma} \cosh(2.415\pi) \left(1 - \frac{kA}{2}\right) \left(k \frac{\partial G}{\partial x} + G \frac{\partial k}{\partial x}\right) = 0$$

Substitute $\frac{\partial G}{\partial x} = -\frac{G}{2k} \frac{\partial k}{\partial x}$ from (22)

$$\frac{\partial A}{\partial x} - \frac{2}{\sigma \gamma} \cosh(2.415\pi) \left(1 - \frac{kA}{2}\right) \left(-k \frac{G}{2k} \frac{\partial k}{\partial x} + G \frac{\partial k}{\partial x}\right) = 0 \quad (27)$$

3.3 Wave number conservation equation

From the method of variable separation in the completion of Laplace equation, where velocity potential is considered as a multiplication of three functions, i.e. $\Phi(x, z, t) = X(x)Z(z)T(t)$, where $X(x)$ is just a function of x , $Z(z)$ is just a function of z and $T(t)$ is just a function of time t , where in (1), $Z(z) = \cosh(h+z)$, then the following should apply,

$$\frac{\partial k(h+z)}{\partial x} = 0 \quad (28)$$

This equation is a wave number conservation equation. Equation (28) is done at $z = \eta(x, t)$ and done at the characteristic point, then the wave number equation becomes,

$$\frac{\partial k(h+\frac{A}{2})}{\partial x} = 0 \quad (29)$$

4. Shoaling-Breaking Equation

A wave moving from deeper water to shallower water will experience a change in its constant, i.e. $\frac{\partial G}{\partial x}$, $\frac{\partial k}{\partial x}$ and $\frac{\partial A}{\partial x}$. To calculate the change in the three constants, three equations are needed. Those equations are energy conservation equation, wave amplitude function equation and wave number conservation equation. The three equations formed linear simultaneous equation with variables $\frac{\partial G}{\partial x}$, $\frac{\partial k}{\partial x}$ and $\frac{\partial A}{\partial x}$ that can be completed using Gauss elimination method or the like. Bearing in mind that the form of the three equations is simple linear equation so that from the three equations, equations for $\frac{\partial G}{\partial x}$, $\frac{\partial k}{\partial x}$ and $\frac{\partial A}{\partial x}$, so that matrix operation is no longer needed to complete it. In the following sections, calculation method will be formulated with the two methods.

4.1 Using matrix operation

As has been stated that the three change equations of the wave constant will form a system of linear equation with variables $\frac{\partial G}{\partial x}$, $\frac{\partial k}{\partial x}$ and $\frac{\partial A}{\partial x}$. The equation can be written in the form of matrix as follows.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix} \quad (30)$$

Where $x_1 = \frac{\partial G}{\partial x}$; $x_2 = \frac{\partial k}{\partial x}$; $x_3 = \frac{\partial A}{\partial x}$

$$G_{x+\delta x} = G_x + \delta x \frac{\partial G}{\partial x} \quad (31)$$

$$k_{x+\delta x} = k_x + \delta x \frac{\partial k}{\partial x} \quad (32)$$

$$A_{x+\delta x} = A_x + \delta x \frac{\partial A}{\partial x} \quad (33)$$

Next is a discussion on matrix coefficients in (30)

a. Energy equation

Energy conservation equation (23), is written as

$$2k \frac{\partial G}{\partial x} + G \frac{\partial k}{\partial x} = 0 \quad (34)$$

Matrix coefficients of this equation are,

$$a_{11} = 2k ; a_{12} = G ; a_{13} = 0 ; b_1 = 0$$

b. Wave amplitude function equation

From (24) i.e.

$$\frac{\partial A}{\partial x} - \frac{2}{\sigma \gamma} \cosh(2.4\pi) \left(1 - \frac{kA}{2}\right) \left(k \frac{\partial G}{\partial x} + G \frac{\partial k}{\partial x}\right) = 0 \quad (35)$$

Matrix coefficients of this equation are,

$$a_{21} = -\frac{2}{\sigma \gamma} \cosh(2.415\pi) \left(1 - \frac{kA}{2}\right) k$$

$$a_{22} = -\frac{2}{\sigma \gamma} \cosh(2.415\pi) \left(1 - \frac{kA}{2}\right) G$$

$$a_{23} = 1 ; b_2 = 0$$

c. Wave number conservation equation

Wave number conservation equation (36) is,

$$\frac{\partial k \left(h + \frac{A}{2}\right)}{\partial x} = 0 \quad (36)$$

Matrix coefficients in this equation are,

$$a_{31} = 0 ; a_{32} = \left(h + \frac{A}{2}\right) ; a_{33} = \frac{k}{2} ; b_3 = -k \frac{\partial h}{\partial x}$$

4.2 Simple equation

From the linear equation, simple equation can be formed, so that in the calculation the completion of simultaneous linear equations operation is not needed.

$\frac{\partial G}{\partial x}$ in (35) is substituted with energy conservation equation (34),

$$\frac{\partial A}{\partial x} = \frac{G}{\sigma \gamma} \cosh(2.415\pi) \left(1 - \frac{kA}{2}\right) \frac{\partial k}{\partial x} \quad (37)$$

This equation is substituted to wave number conservation equation,

$$\left(h + \frac{A}{2}\right) \frac{\partial k}{\partial x} + k \frac{\partial h}{\partial x} + \frac{Gk}{2\sigma \gamma} \cosh(2.415\pi) \left(1 - \frac{kA}{2}\right) \frac{\partial k}{\partial x} = 0 \quad (38)$$

The calculation step is to calculate $\frac{\partial k}{\partial x}$ with (38), then $\frac{\partial A}{\partial x}$ is calculated with (37). Next, Taylor series is done to obtain new wave number and wave amplitude values.

$$k_{x+\delta x} = k_x + \delta x \frac{\partial k}{\partial x}$$

$$A_{x+\delta x} = A_x + \delta x \frac{\partial A}{\partial x}$$

The calculation of $\frac{\partial G}{\partial x}$ was done analytically. The energy conservation equation (24) can be written as,

$$\frac{1}{G} \frac{\partial G}{\partial x} = -\frac{1}{2k} \frac{\partial k}{\partial x} \quad (39)$$

Both terms of the equation are multiplied with dx and then are integrated against horizontal- x axis.

$$\int_x^{x+\delta x} \frac{\partial G}{G} = -\frac{1}{2} \int_x^{x+\delta x} \frac{\partial k}{k}$$

The following is obtained,

$$G_{x+\delta x} = e^{\left(\ln G_x - \frac{1}{2}(\ln k_{x+\delta x} - \ln k_x)\right)} \quad (40)$$

5. The Result of Shoaling-Breaking Model

5.1 Breaker height comparator

The result of breaker height model was calibrated against the average value of 5 (five) breaker height indexes. The breaker height index equations used as comparators are breaker height index (BHI) equations from Komar and Gaughan (1972), Larson, M. and Kraus, N.C. (1989), Smith and Kraus (1990), Gourlay (1992) and Rattana Pitikon and Shibayama (2000). Breaker depth is adjusted with the result of breaker depth from SPM (1984).

Komar and Gaughan (1972)

$$\frac{H_b}{H_0} = 0.56 \left(\frac{H_0}{L_0}\right)^{-\frac{1}{5}} \quad (41)$$

Larson and Kraus (1989),

$$\frac{H_b}{H_0} = 0.53 \left(\frac{H_0}{L_0}\right)^{-0.24} \quad (42)$$

Smith and Kraus (1990),

$$\frac{H_b}{H_0} = (0.34 + 2.74m) \left(\frac{H_0}{L_0}\right)^{-0.30+0.88m} \quad (43)$$

Gourlay (1992),

$$\frac{H_b}{H_0} = 0.478 \left(\frac{H_0}{L_0}\right)^{-0.28} \quad (44)$$

Rattana Pitikon and Shibayama (2000) :

$$\frac{H_b}{H_0} = (10.02m^3 - 7.46m^2 + 1.32m + 0.55) \left(\frac{H_0}{L_0}\right)^{-\frac{1}{5}} \quad (45)$$

H_0 is deep water wave height, L_0 is deep water wave length (calculated using linear wave theory,

$k_0 = \frac{\sigma^2}{g}$, $L_0 = \frac{2\pi}{k_0}$), m is bottom slope and H_b is breaker height.

5.2 Breaker depth comparator

As a comparator of the result of breaker depth calculation, breaker depth index from SPM (1984) is used, with breaker height as input that is obtained from a.

$$\frac{h_b}{H_b} = \frac{1}{b - \left(\frac{\alpha H_b}{g T^2}\right)} \quad \text{atau} \quad h_b = \frac{H_b}{b - \left(\frac{\alpha H_b}{g T^2}\right)} \quad (46)$$

$$\alpha = 43.75(1 - e^{-19.0m}) \quad b = \frac{1.56}{1 + e^{-19.5m}}$$

h_b breaker depth.

5.3 Breaker height calculation

The breaker height H_{br} calculation was done using water wave surface equation (2), where $\cos \sigma t = 1$ was used. As an input for the equation is wave parameter at breaker point, i.e. wave-number k_{br} , breaker depth h_{br} and breaker wave amplitude A_{br} . Considering that there are summation of two waves in (2) or (4), then breaker wave amplitude should be divided with 2, or $A = \frac{A_{br}}{2}$.

was used. The next input berikutnya is the value of wave constant G at breaker point. The adjustment with breaker height from breaker height index (BHI) equation, was done by multiplying wave constant G at breaker point with a coefficient c_G .

The adjustment of breaker depth with the result of SPM (1984) was done by changing deep water constant. The definition of deep water depth is $\tanh k \left(h_0 + \frac{A_0}{2} \right) = 1$ atau $k \left(h_0 + \frac{A_0}{2} \right) = \alpha \pi$. The deep water constant is $\alpha \pi$. Using the value of $\alpha = 2.415$, breaker depth h_b that corresponds to the breaker depth from SPM (1984) was obtained.

a. Breaker height model without adjustment, $c_G = 1$

Table 3 shows the result of breaker height calculation without correcting the value of G , or $c_G = 1$. The value of breaker height model is bigger than the value of breaker height from the average value of Breaker Height Index (BHI), but it can be stated that the difference is relatively small. Table 4 presents the comparison between breaker height model and breaker minimum and breaker height maximum of the five Breaker Height Index Equations and it is found that breaker height model is bigger than breaker height maximum, but with relatively small difference, with differences between 0.03 m-0.15 m.

b. Breaker height model with adjustment, $c_G = 0.878$

Furthermore, Table 6 shows the result of the calculation using coefficient correction $c_G = 0.878$, and the breaker height that is equal to the value of breaker height average from BHI equation. From the result of the comparison, it can be stated that water wave equations that were developed in this research including weighted total acceleration equation with its weighting coefficient, i.e. Y and Y_z provides the result of breaker height that corresponds to the result of physical model.

The wave profile at breaker point for wave period $T = 8$ sec is presented in Figure 2., where the calculation uses $c_G = 0.878$. It shows that in general, the form of the wave profile is cnoidal wave.

Table 4. The result of breaker height calculation without correction ($c_G = 1$).

T (sec)	H_0 (m)	H_{br} Model (m)	H_{br} (BHI) (m)	h_b (Model) (m)	h_b (SPM) (m)
6	1,409	1,838	1,615	2,019	2,019
7	1,918	2,502	2,198	2,749	2,748
8	2,506	3,269	2,871	3,59	3,59
9	3,171	4,137	3,634	4,543	4,543
10	3,915	5,106	4,486	5,612	5,609
11	4,737	6,179	5,428	6,789	6,787
12	5,638	7,353	6,46	8,079	8,077
13	6,617	8,629	7,582	9,483	9,479
14	7,674	10,008	8,793	10,998	10,993
15	8,81	11,49	10,094	12,624	12,62

Note : H_0 : deep water wave height

Table 5. The Comparison of breaker height model ($c_G = 1$), with breaker height maximum and minimum

T (sec)	H_0 (m)	H_{br-min} (m)	$H_{br-model}$ (m)	H_{br-max} (m)
6	1,409	1,482	1,839	1,809
7	1,918	2,017	2,502	2,463
8	2,506	2,635	3,268	3,216
9	3,171	3,335	4,136	4,071
10	3,915	4,117	5,106	5,026
11	4,737	4,981	6,179	6,081
12	5,638	5,928	7,353	7,237
13	6,617	6,958	8,63	8,494
14	7,674	8,069	10,009	9,851
15	8,81	9,263	11,49	11,308

Table 6. The result of the breaker height calculation using correction coefficient $c_G = 0.878$.

T (sec)	H_0 (m)	H_{br} Model (m)	H_{br} (BHI) (m)	h_b (Model) (m)	h_b (SPM) (m)
6	1,409	1,614	1,615	2,019	2,019
7	1,918	2,197	2,198	2,749	2,748
8	2,506	2,87	2,871	3,59	3,59
9	3,171	3,632	3,634	4,543	4,543
10	3,915	4,483	4,486	5,612	5,609
11	4,737	5,425	5,428	6,789	6,787
12	5,638	6,456	6,46	8,079	8,077
13	6,617	7,577	7,582	9,483	9,479
14	7,674	8,787	8,793	10,998	10,993
15	8,81	10,088	10,094	12,624	12,62

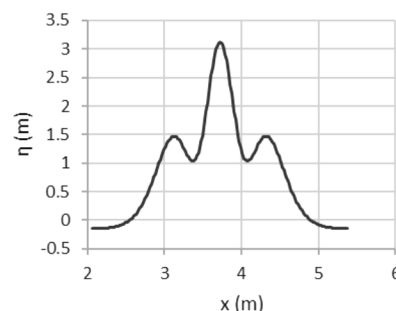


Figure 2. The wave profile at breaker point.

6. Conclusion

- At the breaker point, the model produce breaker height that close to the average breaker height of the five breaker height index equations from the previous researches, and so does the breaker depth produced by the model corresponds to the previous research.
- In general, it can be concluded that the model can estimate breaker height and breaker depth, and maximum wave height in a wave period in deep water. The model is simple in form, and easy to be used, no sophisticated numerical method needed, only with first-order Taylor series. However, a further research needs to be done to other phenomenon contained in a water wave.
- It can be stated that the model development is close to exact without disregarding the non-linear terms at the governing equations that are used. Likewise, at the velocity potential equation a complete solution is used so that the mass conservation equation is well maintained. In the formulation of water surface elevation equation, second order horisontal derivative of water surface equation is neglected. Therefore, the follow up of this model development is the formulation of water surface elevation equation without neglecting the second order derivative.
- The conformity of breaker height model with breaker height from breaker height index indicates that the shoaling that is produced is also good enough, so that it can be used to predict wave height at the coastal water. The application of wave height at the coastal water is among other for design the coastal structure, either the structure strength against wave force and its elevation. The next application is on the development of refraction-diffraction wave model with bathymetry where the shoaling-breaking model at the model of the result of this research can develop a refraction-diffraction model by bathymetry where breaking takes place automatically.

7. References

- Dean, R.G., Dalrymple, R.A. (1991). Water wave mechanics for engineers and scientists. Advance Series on Ocean Engineering.2. Singapore: World Scientific. ISBN 978-981-02-0420-4. OCLC 22907242.
- Gourlay, M.R. (1992). Wave set-up, wave run-up and beach water table: Interaction between surf zone hydraulics and ground water hydarulics. Coastal Eng. 17, pp. 93-144.
- Hutahaeen , S. (2019a). Application of Weighted Total Acceleration Equation on Wavelength Calculation. International Journal of Advance Engineering Research and Science (IJAERS). Vol -6, Issue-2, Feb-2019. ISSN-2349-6495(P)/2456-1908(O). <https://dx.doi.org/10.22161/ijaers.6.2.31>
- Hutahaeen , S. (2019b). Modified Momentum Euler Equatuin for Water Wave Modeling. International Journal of Advance Engineering Research and Science (IJAERS). Vol-6, Issue-10, Oct-2019. ISSN-2349-6495(P)/2456-1908(O). <https://dx.doi.org/10.22161/ijaers.6.10.21>
- Hutahaeen , S. (2019c). Water Wave Modeling Using Wave Constatnt G. International Journal of Advance Engineering Research and Science (IJAERS). Vol-6, Issue-5, May-2019. ISSN-2349-6495(P)/2456-1908(O). <https://dx.doi.org/10.22161/ijaers.6.5.61>
- Hutahaeen , S. (2019d). Water Wave Modeling Using Complete Solution of Laplace Equation. International Journal of Advance Engineering Research and Science (IJAERS). Vol-6, Issue-8, Aug-2019. ISSN-2349-6495(P)/2456-1908(O). <https://dx.doi.org/10.22161/ijaers.6.8.33>
- Komar, P.D. and Gaughan, M.K. (1972): Airy wave theory and breaker height prediction. Proc. 13rd Coastal Eng. Conf., ASCE, pp 405-418.
- Larson, M. And Kraus, N.C. (1989): SBEACH. Numerical model for simulating storm-induced beach change, Report 1, Tech. Report CERC 89-9, Waterways Experiment Station U.S. Army Corps of Engineers, 267 p.
- Rattanapitikon, W. And Shibayama, T.(2000). Vervication and modification of breaker height formulas, Coastal Eng. Journal, JSCE, 42(4), pp. 389-406.
- Smith, J.M. and Kraus, N.C. (1990). Laboratory study on macro-features of wave breaking over bars and artificial reefs, Technical Report CERC-90-12, WES, U.S. Army Corps of Engineers, 232 p.
- Wilson, B.W., (1963). Condition of Existence for Types of Tsunami waves, paper presented at XIII th General
- Wiegel,R.L. (1949). An Analysisis of Data from Wave Recorders on the Pacific Coast of tht United States, Trans.Am. Geophys. Union, Vol.30, pp.700-704.
- Wiegel,R.L. (1964). Oceanographical Engineering, Prentice-Hall, Englewoods Cliffs, N.J.

