On a territorial competition between rhinoceros sondaicus and bos javanicus at ujung kulon national park

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Abstract

Abstract. In this paper, we consider the interaction between Rhinoceros Sondaicus, known as Javan Rhino, and Bos Javanicus, known as Javan Bull, at Ujung Kulon National Park. For years, the population of Javan Rhinos never exceeds their estimated carrying capacity, despite their vast habitat and there is no natural predator in the area. Both species naturally consume different food resources, hence there is no direct competition on food resource between the two species. This stagnant growth of Javan Rhino is suspected due to the territorial competition between the two species, in which the rapid growth of Javan Bull may reduce the territory of Javan Rhino and consequently reduce the carrying capacity of Javan Rhino. We construct a mathematical model of territorial competition between two species and show that domination of one species can lower the carrying capacity of the second species.

Keywords: Territorial competition, Rhinoceros Sondaicus, Bos Javanicus, Ujung Kulon National Park.

1. Introduction

Rhinoceros Sondaicus, known as Javan Rhino, is categorized as one of the critically endangered animals in the world by International Union for Conservation of Nature Red List Category Criteria [4], [2]. It is classified as a critically endangered species because of its number that never exceeds 250 mature individuals, and its subpopulation that never reaches 50 individuals, and its continuous decline [14]. Javan Rhino formerly has been inhabited Bangladesh, Myanmar, Thailand, Laos, Cambodia, Vietnam, southern China, Sumatra and Java [8], but in the latest report, IUCN listed all Javan Rhino as extinct, except the one in Ujung Kulon National Park, Indonesia. Thus, the only existed Javan Rhino is at Ujung Kulon National Park, Indonesia. The reason behind this continuous decline was due to the excessive demand for rhino horn and other traditional medicine products [6], which eventually caused excessive hunting for some period.

Ujung Kulon National Park is located at Ujung Kulon peninsula in the most western tip of Java, Indonesia. It was a habitat for the Javan Rhinos and was chosen as a UNESCO World Heritage in 1992. The population of Javan rhinoceros here was only 25 around 1967 [13], and increased in number to 60 rhinos in 1986 [3], and afterwards, its number never exceeds 60 (Figure 1). The habitat alone has a total area of 300 km^2, consisted of 180 km^2 optimum habitat and 120 km^2 suboptimum habitat [7]. It means if the density per km^2 for a rhinoceros is about 2,6 km^2 when its habitat is near water ponds [1] (it is assumed that the space needed for a javan rhino is the same as the space needed for Black Rhino), the carrying capacity of the Javan Rhino is about 115 in Ujung Kulon National Park. Thus, despite the high carrying capacity of Ujung Kulon National Park, the Javan Rhinos never reach the carrying capacity.

For years, people try to understand the phenomenon of the decrease of rhino population. Hariyadi [9] found that the palm *Arenga obtusifolia* and the vine *Merremia peltata* dominate the national park and reduce the Javan Rhinos food. Fernando [5] found that the genetic diversity of the Javan Rhinoceros was low, and hence, decreased the Javan Rhinos survival. Khan [10] suggested that Bos Javanicus (Bulls) are competing with Javan Rhinos for food, since they change from grazers to browsers as grazing land become scarce [11]. So far there is not enough observation data which can be used to support the construction of the model. In this paper, the competition between Javan Rhinos and Bulls is explored.

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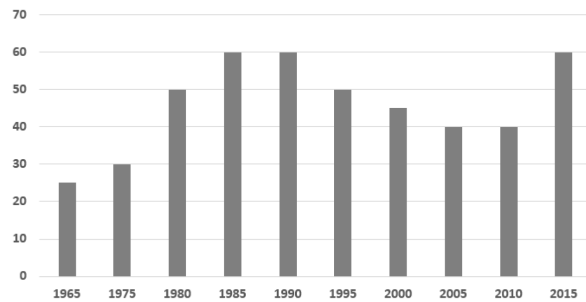

Fig. 1: Population of Javan Rhino between 1965 and 2015, based on available census data [12].

2. Formulation of the Model

For several decades, Ujung Kulon National Park has been declared as a natural reserve and highly protected from poaching. Both Javan Rhinos and Bos Javanicus are listed as endangered species. Naturally, rhinos and bulls consume different food sources, but with the domination of bulls in number, as stated in [11], direct competition may occur. Hence, instead of logistic competition, it is more realistic to view the competition as a territorial competition, in the sense that each species occupies separate territory, which may change in size as time evolves. We model one kind of interaction between Javan Rhinos and Javan Bulls. In this model, under normal circumstances, it is assumed that Javan Rhinos and Bulls do not compete over the same food resource. We would like to know what happens when they compete over the same region. It means that if in the area, Javan Bulls are dominating in size, then the Javan Rhinos will decrease in numbers, and vice versa. The general model is as follows.

TABLE I: Variables and parameters involved in the dynamical model

Variables & Parameters	Description	Dimension/value
R	Rhino pPopulation	No. of Rhinos
B	Bull population	No. of Bulls
C_{r0}	Carrying capacity of Rhinos in the absence of Bull	
C_{b0}	Carrying capacity of Bulls in the absence of Rhino	
μ_r	Growth rate of Rhino per year	$\frac{1}{2} \cdot \frac{2}{5} \cdot \frac{11}{5}$
μ_b	Growth rate of Bull per year	

$$\begin{aligned} \dot{R} &= \mu_r R \left(1 - \frac{R}{C_r(B,R)}\right), \\ \dot{B} &= \mu_b B \left(1 - \frac{B}{C_b(B,R)}\right), \end{aligned}$$

where R and B denote the density of Javan Rhinos and Bulls, respectively. The parameter μ_r and μ_b are the intrinsic growth rate of Javan Rhinos and Bulls, respectively. While, the functions $C_r(B,R)$ and $C_b(B,R)$ are the carrying capacity functions for Javan Rhinos and Bulls, respectively, that depend on the density of Javan Rhinos and Bulls, with the restriction:

$$\begin{array}{rl} C_r(B,R) > 0, & C_b(B,R) > 0, \\ \frac{\partial}{\partial B} C_r(B,R) < 0, & \frac{\partial}{\partial R} C_r(B,R) > 0, \\ \frac{\partial}{\partial B} C_b(B,R) > 0, & \frac{\partial}{\partial R} C_r(B,R) < 0. \end{array} \tag{1}$$

In the first model, we consider two logistic differential equation with constant carrying capacities for both species. In the second model, to take into account the territorial effect, we use a certain carrying capacity functions that satisfy (1).

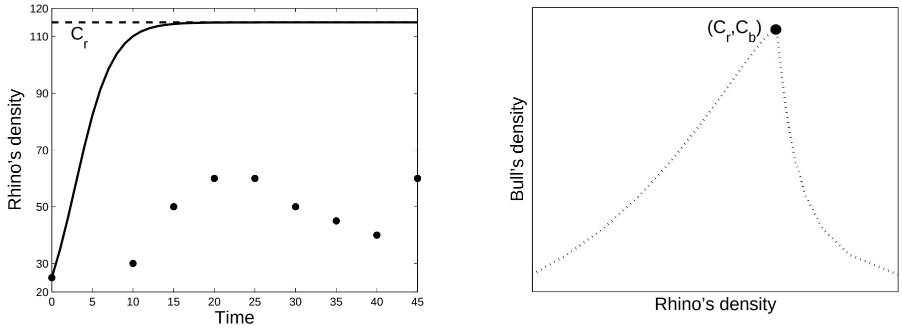

Fig. 2: Left: Simulated Rhinos' density (solid) tends to their carrying capacity (C_r) with their actual density (dot), see Figure 1. Right: The phase portrait of System 2 for two initial condition.

3. Analysis of The Model

3.1. Uncoupled Logistic Model

Let variables R and B denote the density of the rhinos and bulls at Ujung Kulon National Park, respectively, and the parameter μ_r and μ_b will be their intrinsic growth rate. If the parameter C_r and C_b are their carrying capacity, we can model their interaction with respect to time as follow.

$$\begin{cases} \dot{R} = & \mu_r R \left(1 - \dfrac{R}{C_r}\right), \\ \dot{B} = & \mu_b B \left(1 - \dfrac{B}{C_b}\right). \end{cases} \tag{2}$$

This system is an uncoupled logistic system that has an analytic solution:

$$\begin{pmatrix} R(t) \\ B(t) \end{pmatrix} = \begin{pmatrix} \frac{C_r R_0}{C_r - R_0 + R_0 e^{\mu_r t}} e^{\mu_r t} \\ \frac{C_b B_0}{C_b - B_0 + B_0 e^{\mu_b t}} e^{\mu_b t} \end{pmatrix}$$

for every initial condition $R(0) = R_0$ and $B(0) = B_0$. The system has four hyperbolic equilibria, i.e.

1) $(0,0)$ which is purely unstable,
2) $(C_r, 0)$ and $(0, C_b)$ that are both unstable saddles, and
3) (C_r, C_b) that is asymptotically stable.

Notice that in the long run, if the density of one of them is not zero, their density will asymptotically go to (C_r, C_b), as we would expect from an uncoupled logistic system (Figure 2). Figure 2 shows that uncoupled Model 2 is not suitable for simulating the reality in Ujung Kulon.

3.2. Model with Modified Carrying Capacity

In the next model, the relationship between their carrying capacity is taken into account. It is desireable to have Rhinos' (Bulls') carrying capacity that inversely proportional to Bulls' (Rhinos') density, i.e. the carrying capacity of Rhinos (Bulls) is decreasing when the Bulls' (Rhinos') density are increasing, vice versa. It means that the carrying capacity of Rhinos (Bulls) is a function of Bulls' (Rhinos') density.

In this paper, the chosen carrying capacities are

$$C_r = \frac{C_{r0}}{1 + \varepsilon B} \text{ and } C_b = \frac{C_{b0}}{1 + \eta R}, \tag{3}$$

with C_{r0} and C_{b0} are the carrying capacity of Rhinos in the absence of Bulls, and the carrying capacity of Bulls in the absence of Rhinos, respectively, while ε and η are nonnegative constants. This carrying capacity function (3) is a decreasing function with respect to Bulls' (or Rhinos') density with positive concavity. The parameter ε and η denotes the decreasing rate of the carrying capacity of Rhinos and Bulls. In the absence of Rhinos (Bulls) the system becomes a usual logistic system for Bulls (Rhinos). Notice also that at $\varepsilon = 0$ and $\eta = 0$, the system becomes System (2).

3.2.1. Normalization of The Modified Model

Now, we want to normalize the system with Rhinos' carrying capacity. The normalized Rhinos' and Bulls' density with respect to Rhinos' carrying capacity are $\overline{R} = \frac{R}{C_{r0}}$ and $\overline{B} = \frac{B}{C_{r0}}$. Here, we introduce another parameter that measure the proportion of bull's and rhino's carrying capacity, i.e. $\rho = \frac{C_{b0}}{C_{r0}}$. Thus, after substitution and getting rid the overline, the Modified System is given by:

$$\begin{aligned} \dot{R} &= \mu_R R\left(1 - R(1 + \varepsilon C_{r0} B)\right), \\ \dot{B} &= \mu_B B\left(1 - \frac{B}{\rho}(1 + \eta C_{r0} R)\right). \end{aligned} \tag{4}$$

Next, if we let $a = \varepsilon\, C_{r0}$ and $b = \eta\, C_{r0}$, and $c = \frac{1}{\rho}$, The System (4) becomes:

$$\begin{aligned} \dot{R} &= \mu_R R\left(1 - R(1 + aB)\right), \\ \dot{B} &= \mu_B B\left(1 - cB(1 + bR)\right). \end{aligned} \tag{5}$$

3.2.2. The Equilibria of The Modified System (5)

The System (5) have at most 5 equilibria. Three equilibria are given by

1) $(0, 0)$, which is purely unstable,
2) $(1, 0)$ and $\left(0, \frac{1}{c}\right)$, which are unstable saddle.

The other 2 equilibria must satisfy

$$0 = (bc)R^2 + (a + c - bc)R - c, \tag{6}$$

$$B = \frac{1 - R}{aR}. \tag{7}$$

There are two others equilibria at most that satisfies (6) and (7). From (6), the zeros of R have different sign, and by discriminant of it,

$$(a + c - bc)^2 + 4bc^2 = 0$$

if and only if

$$bc = 0.$$

Because the proportion between Rhinos' and Bulls' carrying capacity, $c > 0$, then $b = 0$ ($\eta = 0$). If $b = 0$, then $a = -c$, but that's not posible. Consequently, the two equilibria never collide and it is not possible to have fold bifurcation in this system. This can only mean there are no changes in numbers of equilibria.

Lemma 3.1. *Fold bifurcation does not occur at The System* (5) .

The positive zeros of R is

$$R^* = \frac{-(a + c - bc) + \sqrt{(a + c - bc)^2 + 4bc^2}}{2bc}.$$

Hence, the fourth equilibrium is given by

$$E^* = \left(R^*, \frac{1 - R^*}{aR^*}\right)$$

Let's have a look at equilibria E^*. We would like to know whether B^* is also positive, for the equilibrium E^* to have biological meaning. To show this, we will prove that $R^* < 1$. Because

$$(a + c + bc)^2 - (a + c - bc)^2 - 4bc^2 = 4abc > 0,$$

then

$$(a + c + bc)^2 > (a + c - bc)^2 + 4bc^2. \tag{8}$$

Both the left and right-hand side of (8) are positive, so

$$\begin{aligned} (a+c+bc) &> \sqrt{(a+c-bc)^2+4bc^2} \\ -(a+c-bc)+\sqrt{(a+c-bc)^2+4bc^2} &< 2bc. \end{aligned}$$

Thus, we have shown that $R^* < 1$. As a consequence, in order to have biological significance, we can only have one of the two equilibria.
Let $\mathcal{D} = \{(R,B) \in \mathbb{R}^2 | R \geq 0, B \geq 0\}$, then this system have three equilibria in the boundary of D, ∂D, and one equilibria in $Int(\mathcal{D})$.

Corollary 3.1.1. *There are no changes on the number of equilibria in the System* (5)*, at Closure*(D).

This means

Theorem 3.2. *The System* (5) *have three equilibria in* ∂D *and one equilibrium in* $Int(\mathcal{D})$.

3.2.3. Local Stability of Equilibrium E^*

Next, we will consider the local stability of equilibrium E^* in Int($\mathcal{D}$). The characteristics polynomial of the Jacobian matrix of The System (5) evaluated at E^* is $A_2\lambda^2 + A_1\lambda + A_0$ dengan

$$\begin{aligned} A_2 &= aR^*, \\ A_1 &= -2bc\mu_B {R^*}^2 + (2bc\mu_B - a\mu_B + a\mu_R - 2c\mu_B)R^* + 2c\mu_B, \\ A_0 &= -\mu_B\mu_R(bc(R^*)^3 + (a-bc+2c)R^* - 2c). \end{aligned}$$

It is sufficient to show that A_1 and A_0 are positive.

$$\begin{aligned} A_1 &= A_1 + 2\mu_B(bc{R^*}^2 + (a+c-bc)R^* - c) \\ &= aR^*(\mu_B+\mu_R) \\ &> 0 \end{aligned}$$

Because $R^* < 1$, similarly, we can show that $A_0 < 0$. Thus, the fourth equilibrium is a stable equilibrium.

Another way to see the stability is via the isocline of $\dot{R} = 0$ and $\dot{B} = 0$. The isocline $\dot{B} = 0$ are hyperbolic function that intersects at $(R,B) = (0, \frac{1}{c})$ with horizontal asymptote at $B = 0$ and vertical asymptote at $R = -\frac{1}{b}$ in $B - R$ plane. On the top of this isocline, the trajectory of the orbit is heading downward, while under the isocline the trajectory is heading upward (Figure 3).

On the other hand, the isocline $\dot{R} = 0$ are hyperbolic function that intersects at $(R,B) = (1,0)$ with horizontal asymptote $B = -\frac{1}{a}$ and vertical asymptote $R = 0$ in the $B - R$ plane. On the top of this isocline, the trajectory of the orbit is facing right, while under the isocline the trajectory is facing left. Thus, the equilibrium E^* is a stable equilibrium. Following the same argument, we conclude that the orbit is always bounded.

Theorem 3.3. *The System* (2) *with carrying capacity function* (3) *have three unstable equilibria (two unstable saddles and one purely unstable) in* $\partial\mathcal{D}$ *and one stable equilibrium in* $Int(\mathcal{D})$*. Moreover, the orbit is bounded.*

3.2.4. Sensitivity Analysis of The Modified System

To analyze the connection between parameters, we show two graphs in $a-b$ plane and $c-P$ plane, where P is the proportion between Rhinos' density and Bulls' density at the fourth equilibrium, i.e. $P = \frac{R^*}{B^*}$. We found that at a fixed decreasing rate of carrying capacity of Rhinos (i.e. ε parameter in a), we can make the the density of Rhinos larger by increasing the decreasing rate of carrying capacity of Bulls. For example, if the decreasing rate of Rhinos, $\varepsilon = 0.0029565$, and carrying capacity of Rhinos and Bulls are 115 and 500, respectively, we have $a = 0.34$. Here, we can increase the density of the the Rhinos from 0.1 of the density's of Bulls to 0.15 times the density of Bulls, by increasing the decreasing rate of the Bulls (η) from 0.001 to 0.0065 (from the dash-dotted line to the dashed line or from $b = 0.115798$ to $b = 0.7525515$ at Figure 4).

The same phenomena can be observed from Figure 5. Here, for a fixed porportion of the carrying capacity of Rhinos' and Bulls', the density of Rhinos' can be increased by increasing the decreasing rate of carrying

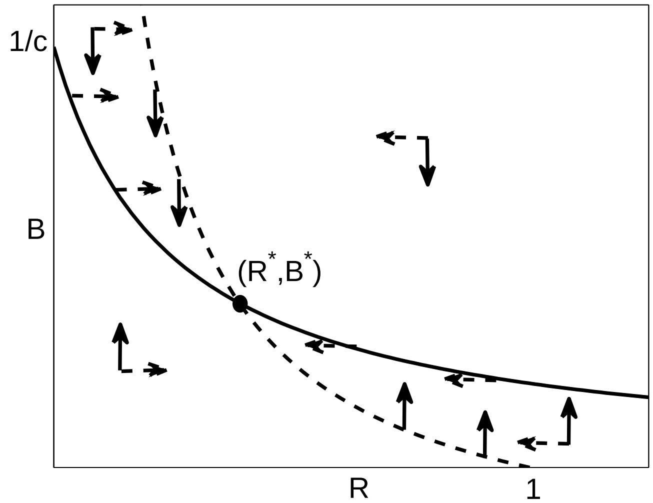

Fig. 3: The isocline of $\dot{R} = 0$ (dashed), $\dot{B} = 0$ (solid) and their trajectory. The equilibrium (R^*, B^*) is a stable one.

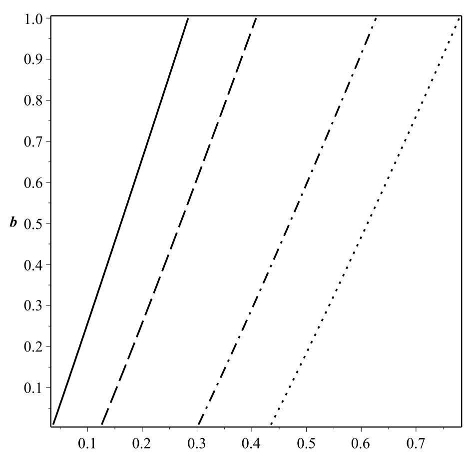

Fig. 4: Sensitivity analysis in $a - b$ plane for $c = \frac{115}{500}$. The dotted line, the dash-dotted line, the dashed line, and the solid line correspond to $P = 0.08$, $P = 0.1$, $P = 0.15$, and $P = 0.2$ respectively, where $P = \frac{R^*}{B^*}$.

capacity of Bulls'. But, in this graph, we see that an increase in the rate of decreasing of Bulls' carrying capacity, increase Rhinos' carrying capacity not significantly. Thus, the increase in Rhinos' density is slow compared to the increase of the decreasing rate of carrying capacity of Bulls'.

In Figure 6, The System (4) with $a = 0.34$ and $b = 0.115798$ have equilibrium $(R, B) = (0.4150, 4.1484)$, with the value $P = \frac{0.4150}{4.1484} = 0.1$ that matches the value P in Figure 4. By varying the b to $b = 0.7525515$, the value of P becomes $P = \frac{0.4796}{3.1944} = 0.15$ that matches the value P in Figure 4.

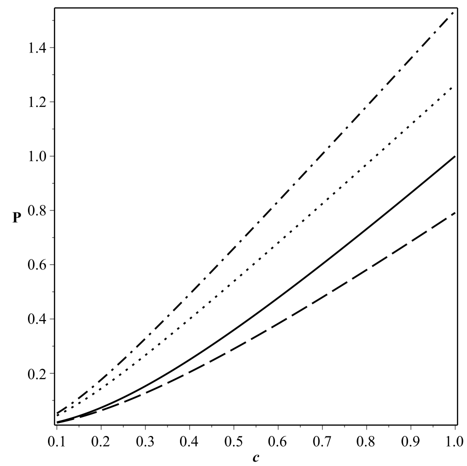

Fig. 5: Sensitivity analysis in $c - P$ plane. The dotted line is for $a = 0.2$ and $b = 0.5$, while the dash-dotted line is for $a = 0.2$ and $b = 0.8$. The dashed line is for $a = 0.5$ and $b = 0.2$, while the solid one is for $a = 0.5$ and $b = 0.5$.

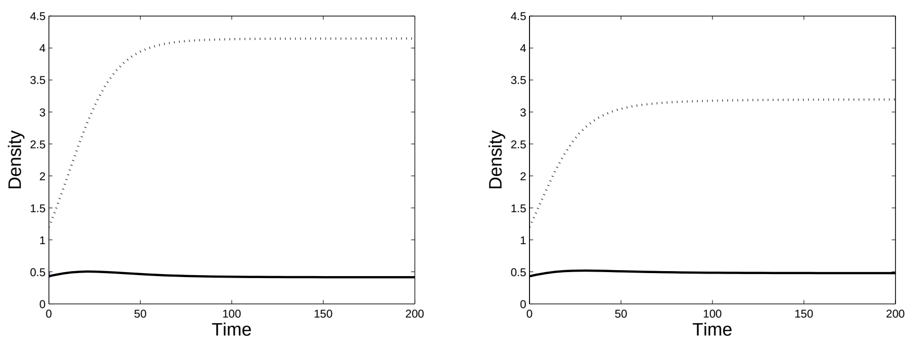

Fig. 6: For the same $c = \frac{115}{500}$, $a = 0.34$ of System (4). Left: The Rhinos' density (solid line) for $b = 0.115798$ and $P = 0.1$, right: The Bulls' density (dotted line) for $b = 0.7525515$ and $P = 0.15$

4. Conclusion

By modifying the constant carrying capacity of Rhinos to a variable that depends on the carrying capacity of Bulls, we can model the territorial competition between Rhinos and Bulls. The density of Rhinos can be increased by increasing the decreasing rate of Bulls' carrying capacity (Figure 4). Unfortunately, the increased on Rhinos' carrying capacity have little effect on the increased of Rhinos' density (Figure 5).

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