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Study of Constrained Velocity Inversion of Seismic Data in North Sumatra Basin

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Highlight:

- Constrained velocity inversion is an interval velocity analysis using an exponential asymptotically bounded velocity model, which can show a delineation match between the model and the velocity inversion functions.
- The constrained velocity inversion results were adequate for use as seismic velocity input data and produced a better seismic section than the root mean square velocity input data, indicated by clear continuity of the reflectors and the fault.
- Constrained velocity inversion applied to seismic data in the North Sumatra Basin showed a more rigorous seismic section compared to that from the root mean square velocity analysis.

Abstract. Interval velocity analysis increases the precision of seismic velocity data with a complex structure and a high variation of velocity both laterally and vertically. In this study, interval velocity analysis was performed by applying the exponential asymptotically bounded function approach. An exponentially asymptotically bounded function was applied to calculate the interval velocity obtained from the root mean square velocity of seismic data using the Dix equation for conversion. To control this operation, a velocity constraint was applied in the interval velocity conversion. The velocity constraint used was the velocity trend gained from the root mean square velocity. This method is called constrained velocity inversion. In this study, interval velocity analysis using constrained velocity inversion was applied to seismic data from the North Sumatra Basin area. The seismic data interpretation resulted from the interval velocity analysis using constrained velocity inversion described the subsurface structure clearly. A corresponding anomaly at a time depth from 2000 ms to 2400 ms in the seismic time-domain data indicated a fault beneath an anticline. This result indicates that the interval velocity analysis of seismic data is more rigorous than the root mean square velocity analysis.

Keywords: *complex structure; constrained velocity inversion; interval velocity; North Sumatera Basin; seismic processing; velocity analysis.*

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1 Introduction

In seismic data processing, standard methods often fail when dealing with a complex structure that has high structural heterogeneity. In the research performed by Irawan, *et al.* [1] on a very complex structure, the inaccuracy of velocity increased with lateral velocity variations. The presence of lateral velocity variations caused bending waves at the layer boundaries. The bending waves caused the wave propagation to become more hyperbolic because of which the amplitude and travel time became incompatible.

Inhomogeneous subsurface structures produce complex seismic data. An example is the SBI field in the North Sumatra Basin. The North Sumatra Basin is a back-arc basin that is part of the Sunda Plate. Tectonic processes resulted in fold and fault formation in this area. These conditions resulted in a complex subsurface structure. Complex subsurface structures cause velocity variation both laterally and vertically. Heterogeneity of velocity variation requires advanced analysis of the seismic data to produce an adequate seismic data image.

When performing velocity analysis, it is necessary to select the most appropriate velocity values from the seismic data. The correct velocity value is obtained by analyzing each depth layer. Each reflector at a particular depth has a different velocity value, which according to Fagin [2] must be adapted to the circumstances of each layer. However, in practice, the velocity value used is the average velocity of all analyzed layers. For layered media with different densities, in general, the root mean square (RMS) velocity is used in the processing of seismic data.

Fagin [2] states that the RMS velocity ray path does not run into deflection (ray bending) conforming to Snell's law. Hence, the RMS velocity needs to be corrected. Correction can be performed by converting it to an interval velocity. Interval velocity analysis, as conducted by Fauzatun [3], thoroughly observes the value of the medium's velocity in each layer. Interval velocity calculation predicts the velocity changes in each layer. However, the analysis of the initial velocity (velocity picking) produces the RMS velocity and hence a calculation is required to obtain the interval velocity, as described by Yilmaz [4].

In this study, the method used to determine the velocity interval was constrained velocity inversion. Constrained velocity inversion is a method to analyze seismic data by estimating the velocity using the global velocity trend as proposed by Ravve, *et al*. [5]. This method uses the RMS velocity to find the global velocity trend. The global velocity trend is then used to control the interval velocity analysis obtained from the input data. The input data in this method are interval velocity data gained from Dix's equation. The interval velocity values are used to determine the velocity values in complex structures. The global velocity trend confines the velocity at irregular intervals (oscillation). The constraint avoids unnecessary sharpness of the interval velocity oscillation, laterally and vertically. The lateral and vertical velocity changes follow the global velocity trend.

2 Materials and Methods

2.1 Geological Setting

A case study was conducted in the North Sumatra Basin area. Pertamina-BEICIP [6] states that the North Sumatra Basin is a back-arc that is part of the Sunda Plate and includes a narrow path to the streak from Medan to Banda Aceh. Most of the North Sumatra Basin is characterized by non-marine depositions of black clay and micaceous sandstone. The North Sumatra Basin consists of various tectonic elements, as shown in Figure 1.

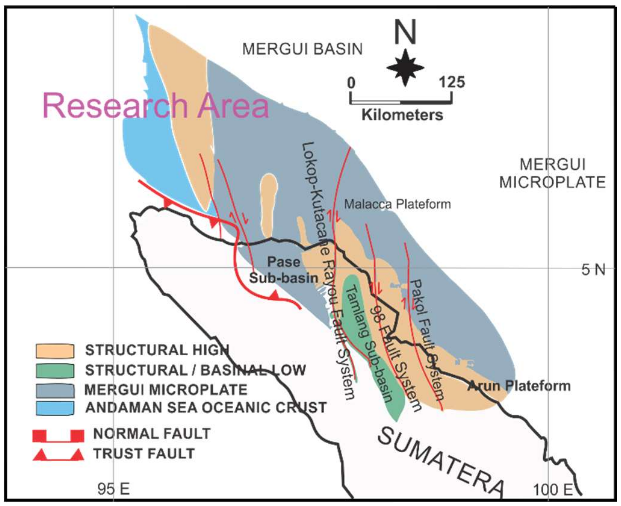

Figure 1 Tectonic elements of the North Sumatra Basin [7].

Wicaksono, *et al*. [7] describes the North Sumatera Basin as a deepening of the Paseh Basin that opens northwards to the offshore and southwards to the Tamiang Basin and the Medan Basin. This area is separated by a high area where the Peunulin/Telaga/Belumai formations directly cover the bedrock. The regional structure of the North Sumatran Basin is represented by a relatively well-defined range of folds stretching northwest-southeast, followed by a relatively higher western passage. According to this geological condition, the North Sumatera Basin has a complex subsurface structure. This makes the North Sumatera Basin suitable as a research area for the method investigated here. The field that was used as a research object is indicated in Figure 1.

2.2 Seismic Data

The seismic data used in this research for RMS velocity analysis were first subjected to data processing. The seismic data consisted of field data obtained from a drilling well and geological analysis data. Their interpretation was performed by geologists. The geologists interpreted some formations by indicating the top and bottom positions with a blue box along the vertical blue line. The vertical blue line in Figure 2 indicates the drilling well.

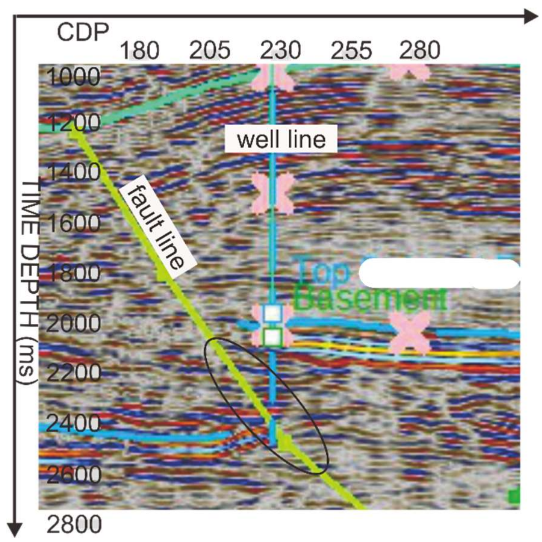

Figure 2 Seismic section using RMS velocity analysis [8].

In addition, the geological condition shown in Figure 2 indicates the presence of folds and faults in the subsurface. The faults can be interpreted by analyzing the information on the sequence boundaries and the time structure map. The result of seismic data processing shown in the seismic section does not indicate a sequence boundary conformity as a fault, as can be seen in Figure 2. The fault interpreted by geologists is indicated by a yellow cross line in Figure 2. In this research, residual RMS velocity analysis was performed to obtain the optimal result and an appropriate comparison between RMS velocity and interval velocity analysis.

2.3 Stacking Velocity Inversion

Stacking velocity inversion was first presented by Gerritsma [9] and later by Van der Made [10], Toldi [11], and Zijlstra [12]. This method utilizes the stacking velocity to generate the interval velocity values. The interval velocity is the velocity that produces the moveout curve that best matches the stacking velocity curve. In Figure 2, the best moveout curves were constructed by varying the velocity intervals in the target layer. In the curve constructed with matching velocity there is a slight difference between the stacking velocity and the interval velocity of the target layer.

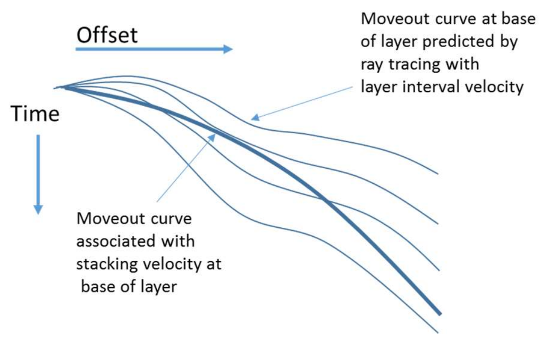

Figure 3 Stacking velocity inversion using a model-based approach [2].

The parameters used to build the velocity model are the moveout, effects of the subsurface structure, and the velocity. Dix's method [13] has the disadvantage of the assumption of flat layers and small raypath deflection. In a complex structure, the model-based method is the most appropriate method to use according to Fagin [2].

2.4 Velocity Inversion

Harlan [14] explained that the Dix equation [13] produces the interval velocity in a single trace or a local area. Calculations based on the Dix equation have a poor velocity estimation because it does not consider the effect of the structure. Interval velocity calculations using the Dix equation provide a solution that is non-realistic. The best method to use is a model that can predict the effects of the structure. The result of the calculation compensates for the effect of (irregular) velocity oscillations. The oscillations occur both laterally and vertically, even small variations shown by the root mean square velocity analysis.

Ravve, *et al*. [5] proposed a method to make interval velocity models. The interval velocity is obtained from the RMS velocity or the stacking velocity. The RMS velocity values are obtained from a data set of vertical functions from velocity picking. Calculations are used to predict the interval velocity values of the seismic data. The method used by Ravve *et al.* is called constrained velocity inversion.

2.5 Initial Trend Function

Ravve, *et al.* [15] used the equations of Slotnick [16], Huston [17], Faust [18], and Al Chalabi [19] as the basis for the asymptotic velocity equation. This basic velocity equation is used to create the exponential asymptotic velocity equation. The function of the linear velocity in the depth domain was first expressed by Slotnick [16] and further developed by Ravve, *et al.* [15].

The velocity model with asymptotic function uses the velocity trend. The velocity trend describes the vertical variation of instantaneous velocity. Instantaneous velocity is an exponential function with the asymptotic approach from Ravve *et al.* [15], as shown in Eq. (5):

$$v_{sesaat}^{exp}(z) = v_a + \Delta v \left[1 - \exp\left(-\frac{k_a z}{\Delta v}\right)\right],\ v_a + \Delta v = v_\infty \qquad (5)$$

where v_{sesaat}^{exp} is the instantaneous velocity at the surface, k_a is the vertical gradient in position a; v_∞ is the asymptotic velocity at an infinite depth; and Δv is the difference in instantaneous velocity between the two points.

The vertical function used in the study was the time domain. The trend function in Eq. (5) is the function of velocity in the depth domain. Eq. (5) is rewritten as a function of time (one way traveltime) in Eq. (6) as follows:

$$v_{sesaat}^{\text{exp}}(t) = \frac{v_a\, v_\infty}{v_a + \Delta v \exp\left(-\frac{k_a t\, v_\infty}{\Delta v}\right)} \qquad (6)$$

where v_{sesaat}^{exp} is an instantaneous velocity time domain function.

The inversion operation for the trend parameter at each grid point is solved using the least square solution. The final result expected is an exponential function of RMS velocity close to the RMS velocity values resulted from picking. RMS velocity is expressed generally in Eq. (7) from Yilmaz [20]:

$$v_{RMS}(t) = \sqrt{\frac{1}{t}\int_0^t v_{sesaat}^2(\tau)d\tau} = \sqrt{\frac{W(t)}{t}} \qquad (7)$$

where $\text{W}=\int_0^t V_{sesaat}^2(\tau)d\tau$ is a hyperbolic parameter. Thus, the distribution exponential can be written in Eq. (8) as follows:

$$W^{exp}(t) = \frac{\Delta V\, V_\infty}{k_a} \cdot \ln\frac{\text{S}}{V_\infty} - \frac{V_a\, \Delta V^2}{k_a} \cdot \frac{\lambda - 1}{S} \qquad (8)$$

where $\lambda = \exp\left(\frac{k_a t\, V_\infty}{\Delta V}\right)$ dan $S = V_a\, \lambda + \Delta V$

2.6 Constrained Velocity Inversion

Inversion is performed on every single set of the vertical function. However, if the function calculates the global trend, then the inversion calculation is done globally. The picking point that produces the vertical function is selected randomly, both laterally and for the time variable. Irregular conditions necessitate regularization of the time function. Regularization is performed over the interval $\Delta t_n = \Delta t_n - t_{n-1}$ between two nodes. The RMS velocity measured at layer n can be calculated with Eq. (9):

$$U_n^{data} = \sqrt{\frac{v_{RMS,n}^2 . t_n - v_{RMS,n-1}^2 . t_{n-1}}{\Delta t_n}} \tag{9}$$

U_n^{data} is the velocity interval from Dix's conversion. If the instantaneous velocity is a linear function of depth, linearization is necessary.

Linearization of the velocity function is performed on the interval between two points of instantaneous velocity as a function of time $v_{a,n}^{Lin}$ and $v_{b,n}^{Lin}$. Sripanich, *et al.* [21] describe linearization estimation to a simple system appropriate for handling weak lateral variations. Linear distribution in the depth domain of RMS velocity is formulated in Eq. (10):

$$U_n^{Lin}(v_{a,n}, v_{b,n}) = \sqrt{\frac{v_{b,n}^2 - v_{a,n}^2}{2 \ln[v_{b,n}/v_{a,n}]}} \tag{10}$$

RMS velocity inversion should be close to the velocity of the RMS data. Eq. (10) is expected to match well with the velocity of the RMS data, which can be written in Eq. (11) as follows:

$$U_n^{Lin}(v_{RMS,n-1}, v_{RMS,n}) = U_n^{data} \tag{11}$$

The velocity resulting from the inversion should be close to the velocity of the trend function. The velocity function model approaching the trend is given by Eq. (12):

$$f_{tren} \equiv \frac{1}{2} \sum_{n=1}^{N} \int_0^t \left[v_{RMS,n}^{Lin}(\tau) - v_{RMS}^{Tren}(t_{n-1} + \tau) \right]^2 d\tau \tag{12}$$

Rocha, *et al*. [22] found that velocity conversion of Dix's equation produces irregular velocity intervals. A constraint is used to control irregular interval velocity variation. The interval velocity is controlled in order to approach the velocity of the global trend according to Eq. (12). The picking function is used as a velocity constraint of interval velocity inversion to approach the RMS velocity data in Eq. (13):

$$f_{pick} \equiv \frac{1}{2} \sum_{n=1}^{N} \Delta t_n \, . w_n \left[U_n^{Lin}(v_{RMS,\ n-1}, v_{RMS,\ n}) - U_n^{data} \right]^2 \tag{13}$$

where w_n is weighted to n intervals of the vertical function.

Irregularities in the conversion velocity resulting from Dix's equation can be muted (restricted) by damping. The damping function constrains oscillations in the vertical data. Eq. (14) provides damping in interval velocity conversion:

$$f_{dump} \equiv \frac{v^2.\Delta t}{2} \sum_{n=1}^{N-1} w_n^{dump} \Delta t_n \,.\, \Delta t_{n-1} (k_{n+1} - k_n) \tag{14}$$

where k is the vertical gradient, and n is the node.

2.7 Application of Constrained Velocity Inversion Method

The initial model built from the RMS velocity analysis is matched or adapted to the geological data structure. The velocity data should correctly follow the condition of the existing structure. Each layer requires proper analysis. The analysis begins from the top layer and moves down to the bottom layer. An inappropriate analysis results in data errors. The error amount must be minimized. If an error occurs in the top layer, then the error in the layer below is greater. Therefore, consideration of possible errors should be done at the beginning of the picking process so the solution is more easily reached.

The calculation of the interval velocity by constrained velocity inversion using a basic equation has been proposed by Dix in [13]. The constrained velocity inversion is developed based on the estimation from the interval velocity calculation using Dix's equation. The improvement concerns the control of the input data. The conversion interval velocity is controlled by the global velocity trend. The calculations are made by considering the interval velocity as the input data and the velocity trend as the controller data. Dominant input and controller data can be adjusted according to the expected result. The input data are based on existing data. If the data describe the structure insufficiently appropriately, the velocity trend can be increased.

3 Results and Discussion

3.1 Interval Velocity Model Analysis

The constraint value is adjusted to the existing data. If the existing data are reasonably appropriate to describe the geological condition, less constraint is needed. The condition of appropriate data is characterized by the value of interval velocity change not being significant (irregular oscillation) both laterally and vertically. An input constraint trend value that is too large causes the velocity interval to have a pattern that is similar to the global velocity trend. The global velocity trend is used as the velocity constraint. This allows the inversion result to follow the pattern of the existing global trend. If the input data are not good enough and the constraint used is too small, then the velocity interval looks rough.

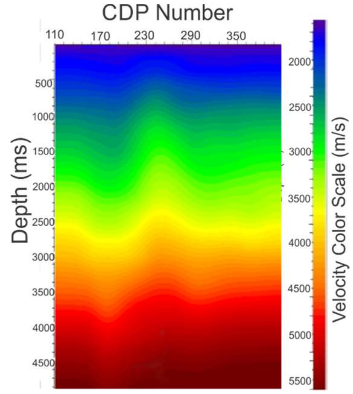

Figure 4 Global velocity trend.

Consideration of the input of data values and the velocity trend produces different models. If the input value of the velocity trend is increased, the result has a tendency to follow the pattern of the velocity trend. An interval velocity model (section) with a high velocity trend value has a pattern similar to the pattern of the velocity trend, as shown in Figure 4. A similar trend in interval velocity is shown in Figure 5(a). If the value is increased, as shown in Figure 5(c), it increasingly follows the data. The data used in this case are interval velocity values.

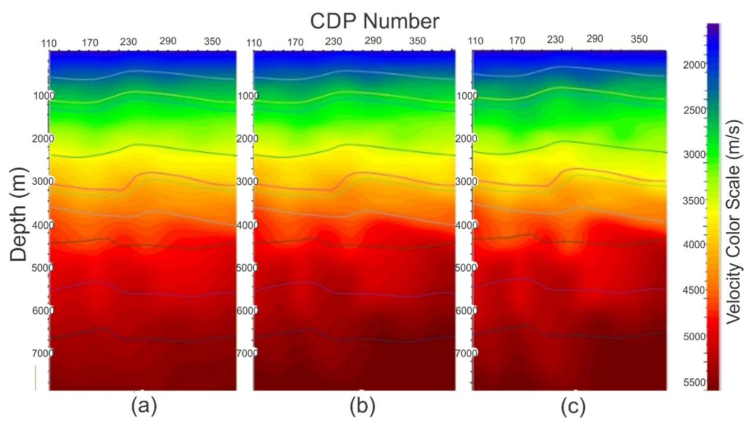

Figure 5 Velocity constrained interval velocity inversion results with the percentage ratio (in %) value compared with the velocity trend: (a) 30:60, (b) 45:45, (c) 60:30 and damping 10%.

In Figure 5(a), the input trend value used was 0.3 and 0.6 with input data damping at 0.1. This means that the constraint value used was 60% of the value of the data. The inversion velocity resulted in a model that had the same trend as the control. The values of the input data interval velocity and velocity trend in Figure 5(b) are equal. Comparison of the velocity feedback in Figure 5(b) shows 0.45 and 0.45 for the input data and the velocity trend with damping at 0.1. In Figure 4(c), the value of the input data for the velocity interval was 0.6 and the velocity trend as control was 0.3 with damping at 0.1.

In this study, the data input used was 0.6 or 60% of the total input data and the velocity trend as control was 0.3 or 30%. The input in this study was determined based on consideration of the interval velocity model. If the interval velocity model has a result that is adequate to describe the structure seen in the data stack time domain, then the data used are good enough. The trend velocity constraint is only used as a control in order to make the interval velocity value more smooth, which limits the occurrence of irregular reflector nodes.

Damping is used to control the presence of significant changes in velocity gradient vertically. In this method, the number of data that have irregular or less predictable changes can be reduced, so that the value does not go up or down significantly. The calculation of velocity value in this study used a damping of 0.1, i.e. 10% of the overall number of data used.

3.2 Constrain Velocity Inversion Model Validation

Figure 6 shows the analysis of the RMS velocity model generated from RMS velocity picking. RMS velocity values were picked from RMS velocity observation data. The RMS velocity data observation function is indicated by the black dash line. The black line shows the interval velocity from picking (smoothing). The blue line is the velocity model function (prediction).

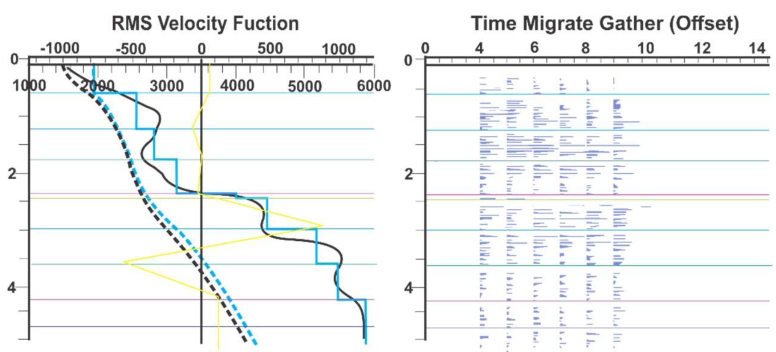

Figure 6 Velocity analysis validation time domain.

The analysis showed that the RMS velocity prediction almost exactly coincided with the velocity from the RMS data. Errors occurred mainly in deep layers. The error amount increases with depth because errors in the determination of the initial velocity will affect the velocity of the underlying layers. However, overall, the data sample used for the prediction result showed fair accuracy. This velocity function is the velocity model function used to obtain the interval velocity inversion.

Figure 7 shows the analysis interval velocity inversion obtained from the constrained velocity inversion. The analysis performed on the CDP gather appropriately represents the velocity applied. The indicator of the appropriate velocity is shown in the velocity analysis validation and the CDP gather. In the CDP gather, an appropriate velocity value is indicated by normal moveout correction. An appropriate velocity value applied in the CDP gather flattens the reflector events. In seismic data processing as described by Yilmaz [20], a flat reflector event means that the normal moveout has been corrected. The QC depth gate shows that a flat reflector event was obtained. In the velocity analysis validation, the model of the velocity function is indicated by the red line. The blue line indicates the velocity inversion results. The appropriate velocity function is shown by two dash lines (blue and red), which follow each other closely. The blue dash line is the velocity function that is used to generate the interval velocity in depth.

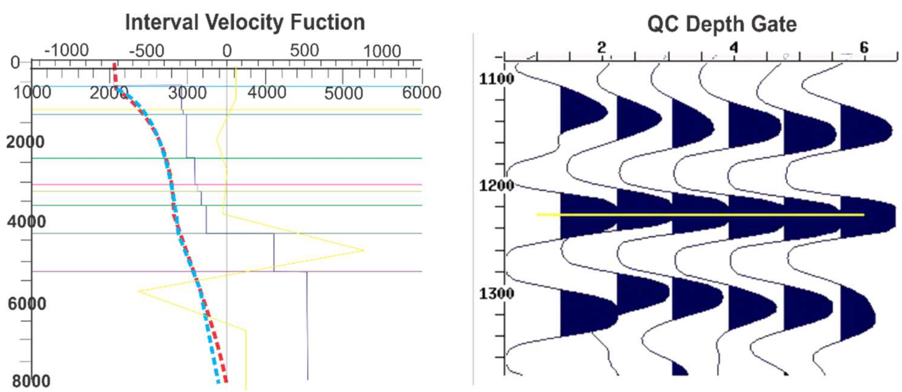

Figure 7 Velocity analysis validation and QC CDP gather in the depth domain.

This result was compared with the drilling well data obtained from a field report in Figure 8. The well field data were also used as velocity control when velocity picking was performed. In Figure 8, the velocity data are presented in analog or non-digital data shown as interval velocity versus depth and average velocity versus depth. In these data, the interval velocity is presented with a 50-m increment from the surface to 2391 m of depth. It is better if the velocity data can be presented throughout the data analyzed and the interval velocity analysis is performed with an increment of the same value. However, velocity picking in the seismic data processing is performed by analyzing clear wiggle traces, which indicate a reflector.

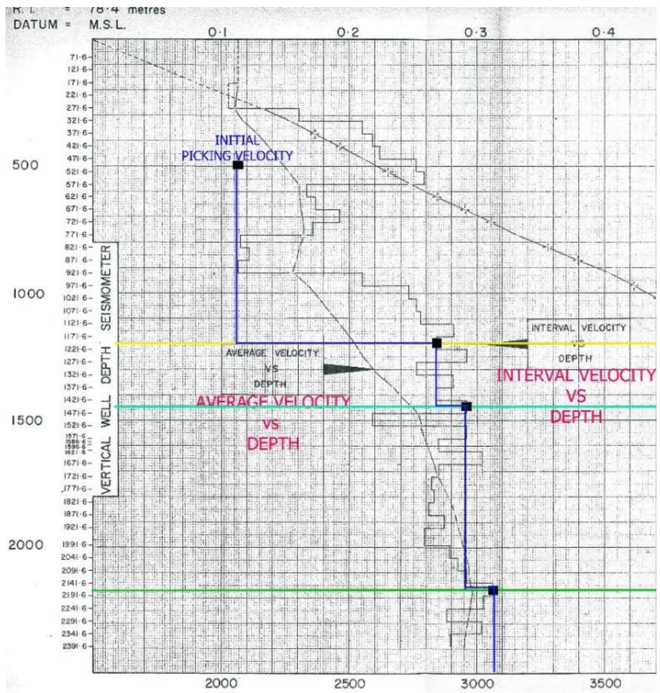

Figure 8 Well field report 0 to 2391 m in vertical depth [6].

In the seismic data processing, velocity picking is performed using the coherence obtained from clear wiggle traces in the seismic data, see Yilmaz [20]. The black dots in Figure 8 are the velocity picking points. The yellow, sea-green, and green lines are marker lines to show reflectors up to 2150 m depth. The reflectors show the boundaries of the layers. Each layer has its own velocity value. The blue line indicates the interval velocity function for each layer. The control velocity function of the well field report was matched with the velocity function obtained from the inversion in Figure 7.

3.3 Seismic Data Interpretation Result

The interval velocity analysis calculates the velocity value in every medium that has an actual velocity. The interval velocity analysis considers wave refraction (ray tracing) based on any change in the medium that has been passed through at different velocity. Interval velocity is sensitive to reflector prediction with complex variations. Any velocity change in the medium is calculated allowing a ray path that conforms to Snell's law as described by Fagin [2].

Interpretation is justified if the seismic data are clearly depicted and the seismic section is realistic. As can be seen in Figure 7, seismic interval velocity using constrained velocity inversion has an appropriate delineation match. The indicator is a velocity match function between model and inversion. The other indicator is the normal moveout corrected in the CDP gather. These results indicate that the interval velocity provides appropriate velocity input values. Appropriate velocity values input make the imaging of the subsurface in the seismic section more realistic. Continuity of reflectors and other geological conditions such as faults can be seen more clearly in the analysis based on the interval velocity.

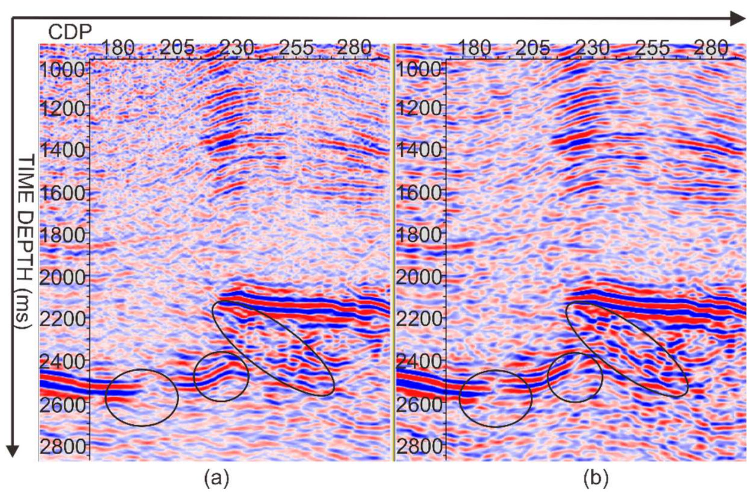

Figure 9 Time-domain seismic section with input velocity: (a) RMS velocity, and (b) constrained velocity inversion.

The seismic section along with the interval velocity input data is shown in Figure 9(b). The seismic data image shows clearly visible reflectors. This can be compared with the seismic section from the RMS velocity analysis in Figure 9(a). In the seismic section, clearly continuous reflectors are visible at a time depth of 2000 ms to 2600 ms. Corresponding anomalies can be seen at a time depth of 2200 ms to 2600 ms. The existence of a fault can be seen from 2200 ms to 2500 ms. The comparison is indicated by the black circled areas. The corresponding anomalies are clearer in the seismic section based on constrained velocity inversion than in the seismic section based on RMS velocity. The reflectors as well as the fault display continuity. This result confirms the interpretation of the geologists, indicating a fault that was not visible in Figure 2. This image suggests a conclusion that the reflectors are continuous and reasonable. This result can seen as a validation of the analysis of seismic data using constrained velocity inversion.

4 Conclusion

The constrained velocity inversion result obtained in this study was appropriate for use as velocity input of seismic data. This was indicated by the validation performed on the velocity function and the data gathered. The velocity functions showed an appropriate delineation match between the model and the inversion. The flat CPD gather indicated that the velocity inputs were appropriate values.

The constrained velocity inversion input produced a better seismic section than the RMS velocity input. This was indicated by a clear continuity of the reflectors and the fault. The seismic interpretation indicated a fault in the target layer. The fault of the seismic section was visible at a time depth from 2200 ms to 2500 ms beneath an anticline structure.

The proposed method is appropriate to analyze complex subsurface structures. In the case study, the regional structure of the North Sumatra Basin was represented by a relatively well-defined range of folds. The North Sumatera Basin consists of various tectonic elements, such as faults and folds, which could be seen clearly in the seismic section.

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