Simultaneous Optimization of Block Replacement and Spare Part Ordering Time for a Multi Component System with Separate Spare Part Ordering for Block and Failure Replacements

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Abstract. A block replacement schedule can be optimized simultaneously with a spare parts ordering schedule, since all items are replaced at a constant interval. The solution of joint optimization for spare parts ordering time and block replacement gives lower costs compared to separate optimization of ordering time and replacement time. The spare parts for replacement can be classified as stochastic demand for failure replacement and deterministic demand for block replacement. In this paper, we propose a simulation model for a separate spare parts ordering schedule. The solution was compared to the solution for a model with common spare parts for both failure and block replacement. The system has N identical components, each with a Weibull lifetime distribution. The costs of failure and block replacements, and also the costs of ordering, holding and shortage of spare parts are given. The proposed model was shown to perform better than the common order model. Also, compared to the age replacement model, the solution of the proposed model is relatively similar, yet the economies of scale would be an advantage for the block replacement over age replacement.

Keywords: *block replacement; deterministic demand; joint optimization; maintenance; spare parts inventory; stochastic demand.*

1 Introduction

One of the most important aspects affecting maintenance policies is spare part inventory control. Traditionally, optimization of maintenance does not take into account this aspect since it assumed that spares are always available upon request. In reality, this is not the case. To have spares always available, one incurs an inventory related cost (ordering and holding costs) and a shortage cost if the demand is higher than the stock on hand.

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According to the Aberdeen Benchmarking Study [1], spare parts and services account for 8% of the annual gross domestic product in the United States, with U.S. consumers and businesses spending more than $700 billion each year on spare parts and services for previously purchased assets, such as automobiles, aircraft, and industrial machinery. On a global basis, spending on such aftermarket parts and services totals more than $1.5 trillion annually. This implies that the overall magnitude of the problem is quite large. The dominant usage of spare parts is in the maintenance area, being either corrective or preventive.

To overcome the problem of making separate decisions on maintenance and spare parts, several papers have proposed joint optimization of inventory and maintenance policy. These papers show that such a policy is more cost effective than a separate optimization of maintenance and ordering policy, since there is an interaction effect between inventory and maintenance related costs. However, the coordination schemes developed so far are quite simple. The research question for this study was therefore focused on the question whether more advanced coordination mechanisms can lower costs.

Development of an analytical model for a joint optimization problem for a system consisting of multiple identical units that must be replaced under age based or block replacement policy is extremely difficult. For both systems it is difficult to calculate the renewal function, especially if block replacement can be delayed by a lack of spare parts. The easiest way to solve this problem is to use a simulation model.

The objective of this study was therefore to modify simulation models that have been developed in previously published articles and try out a number of new ideas. In the first place, by applying modified block replacement instead of block replacement. Smeitink and Dekker [2] showed that this could lower costs. Furthermore, we separate the ordering model into a model for stochastic failure replacement (stochastic demand) and a model for planned block replacement (deterministic demand). For the latter, separate replenishment orders can be made as the timing of preventive maintenance is, to some extent, known beforehand.

To gain general insights, we first define a base case and next make several extensions to obtain more general results.

2 Literature Review

In the past, maintenance and inventory optimizations were considered separately. In maintenance optimization theory, it is assumed that there is no cost associated with inventories to support maintenance activities. In reality, in order to support the maintenance activity, there must be many spare parts in stock to be used to replace the original units due to failure or block replacements.

There are many policies in maintenance practice, e.g. failure replacement (for a component with a constant failure rate), age-based block replacement (for a component with an increasing failure rate), block replacement, condition/inspection based replacement, and opportunity based replacement.

The first block replacement policy is age-based replacement for a single unit. This policy has been specified by Barlow and Proschan [3]. In their model, no consideration was given to spare part stocking. The model assumes that spares are always available upon request, without any cost.

In practice, this assumption is unrealistic. To keep stock available, one will incur an inventory cost. Also there is a delay between ordering and replenishment, known as lead time. Thus, the spares must be ordered and kept in such a way that the total inventory cost (ordering cost, holding cost and shortage cost as a consequence of lead time) is optimized. The model also assumes that the switch-over of spares is perfect and instantaneous.

Osaki and Kaio [4] developed a more realistic model than the Barlow and Proschan model, where there is a constant lead time. Park and Park [5] and Kalpakam and Hameed [6] developed models considering a random lead time. Their models assume that switch-over of spares is perfect and instantaneous. All these models optimize maintenance and inventory policy separately and sequentially. First comes optimizing the replacement interval, followed by optimizing the inventory stocking policy.

Acharya [7] developed a block replacement policy with periodic review of the spare part stock. This is the first model that tried to optimize maintenance and inventory policies simultaneously. This model assumes that the review period and block replacement interval coincide. This assumption is not applicable for a continuous review inventory policy since for a continuous review policy the cycle is in terms of inventory level, instead of in terms of time units. This model shows that joint optimization of maintenance and inventory policies gives a better solution than separate optimization.

All articles mentioned above only consider one type of lead time. In reality, if the failure of an original unit takes place before the regular ordering time (pre-specified time instant during the operation period of an original unit), it is possible to place an emergency order with a shorter lead time. Kaio and Osaki [8] developed a model to accommodate such a situation. Emergency cost is more expensive than regular ordering cost, but an emergency order can reduce downtime cost since it has a shorter delivery time. In this case, the trade-off is between downtime cost and emergency ordering cost.

It is often possible to do minimal repair instead of placing an emergency order. After minimal repair, the condition of the original unit will be as good as before failure. A regular order is placed at regular ordering time. When the unit is delivered, the original unit is replaced by the new unit. This situation has been modeled by Kaio and Osaki [9] and Sridharan [10]

For a single component system subject to random failure and with only one spare in stock or in order at any time, Armstrong and Atkins [11,12] give an analytical model. In their model, they used a deterministic lead time, L. The objective of their analytical model was to determine the optimum age replacement interval (T) and the optimum ordering time t_o in order to optimize the total maintenance and inventory costs. The optimum ordering time t_o is one lead time before T ($t_o = T- L$).

Kabir and Olayan [13] extended Amstrong and Atkins' [11,12] single component system to a multi-component system under age based replacement through the use of a simulation model. Their model is known as the (s,S,T) policy, with S is the maximum stock level, s is the reorder level (ROL), and T is the optimum BR interval. They concluded that for a multiple component system, as for a single component system [11,12], a joint optimization (s,S,T) policy gives a better solution than separate optimization (s,S) and (T).

For a multi-component system under a block replacement policy, Sarker and Haque [14] have shown that joint optimization (s,S,T) gives a better solution than separate optimization. Similar to Kabir and Olayan [13], they also used a simulation model instead of an analytical model because it is extremely difficult to develop an analytical model for a joint optimization (s,S,T) policy.

In both models, they used a probabilistic lead time with two types of lead times, emergency lead time and regular lead time. In reality, it is common to use a deterministic lead time. Horenbeek, *et al*. [15] reviewed several articles related to joint optimization of maintenance and inventory systems. They listed the charasteristics of all reviewed articles. Among all the articles reviewed in their study, the model of Sarker and Hague [14] is the most closely related to our proposed model. The differences between Sarker and Hague [14] and the proposed model is in the spare parts ordering policy and the lead time. A separated spare part ordering schedule for planned replacement and for unplanned replacement is introduced and a deterministic lead time is used, as used by Amstrong and Atkins [12], instead of a probabilistic lead time, as used by Kabir Sarker and Haque [14].

Finally, we compared the performance of our modified policy to the (s,S,T) policy [14]. Also, we compared this modified block replacement policy to an age replacement policy (ARP). The details are explained in the next section.

3 Problem Formulation

Block replacement (BR) is a common practice in industrial maintenance. Block replacement is optimum for a system consisting of components with an increasing failure rate. For a system with many identical items, it is beneficial to replace all those identical items at the same time, regardless of the previous failure replacement of individual items. This policy is known as block replacement.

From a maintenance point of view, the determination of the optimum maintenance policy is only related to the long-run average costs, which include:

1. *Block replacement (BR) cost*: cost of spare part, cost of block replacement.
2. *Failure Replacement (FR) cost*: cost of spare part, cost of corrective replacement.
3. *Deterioration cost*: increasing operational cost due to component aging (wear-out).
4. *Downtime cost*: cost due to lost time/production loss due to preventive and corrective maintenance activity.

The effect of inventory costs (ordering cost, holding cost and shortage cost) of the spare parts needed for maintenance activities is not considered in this model. It is assumed that the spare parts are always available at any time without any costs. In reality, this assumption is only true if the stock of the spare part is high enough (overstock) and there is no holding cost to keep the stock available, no shortage cost, and no ordering cost. To optimize the block replacement (BR) interval, one therefore has to take into account not only maintenance cost, but also inventory cost.

If the BR interval is relatively long, then the stock of the spare will be relatively higher to cover the stochastic failure demands (failure replacement – FR) during that BR interval. The expected FR cost will also be relatively high. On the other hand, if the BR is relatively short, the BR cost will be relatively high but the stock of the spare will be relatively low since the expected demand for FR is lower.

The problem is to determine the optimum *BR interval* (T), the optimum *reorder point* (s), and the optimum *maximum stock level* (S) to minimize the total long-run average cost (cost related to maintenance activity and inventory cost). The demand in every BR and BR interval are constant. There are two types of demand, deterministic demand for BR and stochastic demand for FR.

If the lead time is relatively long, the ordering cost is relatively high and the holding cost is relatively low, it is more beneficial to place a common order for both BR and FR demands than to make a separate order for BR demand and FR demand. As an example, any time we have to place an order for BR demand it is beneficial to also include an order for FR demand. Conversely, any time we have to place an order for FR demand it is beneficial to include an order for the next BR demand.

As an extreme scenario, for a certain BR interval value one can order at one lead time before BR. The order quantity is equal to the BR demand plus the expected FR demand during one BR cycle. In this case, the ordering cost will be relatively low but the holding cost will be relatively high.

In a different scenario, we can place an order every one lead time before the BR time. The FR demand is ordered separately whenever there is a trigger to place an order (when the stock level is less than or equal to the reorder level). In this case, the ordering cost is relatively high and the holding cost relatively low.

It is possible to find the optimal BR interval (T), reorder point (s), and the maximum stock level (S) in order to achieve the optimum long-run average cost. In order to optimize the total cost for both inventory and maintenance related costs, there are two different models that can be used. The first is a separate optimization policy and the second is a joint optimization policy.

In the separate optimization policy, optimization of BR interval (T) and optimization of inventory stock (s,S) are done separately and sequentially. First, optimize the BR interval (T) and then, based on this optimum T, one optimizes the stocking policy (s,S). This policy is known as the (s,S), and (T) policy

In the joint optimization model, optimization of the BR interval and stocking policy (s,S) are done simultaneously, since there is an interaction effect among s,S, and T on the total average system cost. This policy is known as the (s,S,T) policy [13,14].

It is extremely difficult to solve this problem analytically. For a multi component system under an age replacement policy, a simulation model has been developed by Kabir and Olayan [13]. They concluded that joint optimization is more cost effective, in general, compared to separate optimization. For a multi component system under a block replacement policy, Sarker and Haque [14] also reached the same conclusion.

In this study, we modified a simulation model for the (s,S,T) policy as proposed by Sarker and Haque [14]. We propose (s,S,T) with separate part ordering for block and corrective replacements. For the (s,S,T) policy [14] we can give an illustration with a simple hypothetical system, as shown in Figure 1. We place an order only when the stock level is less than or equal to reorder level s. The stock is for both failure and block replacement demands. There will be a stock out possibility when we need a spare to replace a failure component or when we need to replace all components in every block replacement time. Otherwise, we have to stock more spares but this will imply a high inventory cost.

In a block replacement policy we know in advance that every block replacement time we have to replace all operating components (N). Therefore, every block replacement time we need N spares, hence the block replacement demand is deterministic. For deterministic lead time (L) we can place an order with a quantity of N units of a spare part one lead time before the block replacement time. This will guarantee that there will be no shortage every block replacement time and hence the shortage cost will decrease.

In this policy, there are two types of demand: stochastic demand for failure replacement and deterministic demand for block replacement. There are also 2 types of inventory evaluations. The first is based on (*s,S*) continuous review for failure replacement demand and the second is a periodic review one lead time (*L*) before every block replacement time. In the proposed (*s,S,T*) policy with separate part ordering for block and corrective replacements, every one regular lead time (*L*) before block replacement time (*T*) we place a regular order. The order quantity is as much as number of demand for next block replacement. Figure 2 shows a simple hypothetical system for this proposed policy.

The hypothetical system has N = 4, s = 1, S = 4, T = 12, a deterministic lead time *L* = 3, and the *threshold* is 4 time units before the block replacement time. Initial inventory level (IL_0) = 0 and initial inventory position (IP_0) = 0 (no outstanding order). Table 1 shows the description for the policies to be compared.

Table 1 Description of several policies.

Policy	Description
(s, S), T infinite	• No block replacement, replacement only at failure, there is only failure replacement (stochastic) demand • Place an order if inventory position is less than or equal to s
(s, S), (T) Barlow-Proschan	• Separate optimization. First, optimize block replacement interval (T), and then optimize stocking policy (s,S) based on T • Joint order for failure replacement (stochastic) demand and block replacement (deterministic) demand
(s,S,T) Haque-Sarker [14]	• Joint optimization. Optimize block replacement interval (T), and stocking policy (s,S) simultaneously • Joint order for failure replacement (stochastic) demand and block replacement (deterministic) demand
(s,S,T) separate spare part ordering for block and failure replacements	• Joint optimization. Optimize block replacement interval (T) and stocking policy (s,S) simultaneously • Separate order for failure replacement (stochastic) demand and block replacement (deterministic) demand • Block replacement order quantity is N
(s,S,T) age replacement [12]	• Replacement individual components upon failure or when age reaches T • Place an order if inventory position is less than or equal to s, order quantity is up to level S

4 Simulation Model

In this section the simulation model is described. This section consists of system description, cost formulation, algorithm of the simulation model, and verification of the simulation model.

4.1 System Description

The system comprises of N identical components. All components are statistically identical and independent. All components have the same failure distribution. For such a system, a block replacement policy is most suitable. All N components will be replaced after a certain interval of time (*block replacement interval – BRI*), regardless of the age of all components. This block replacement incurs a *block replacement cost* (*BRC*) per component.

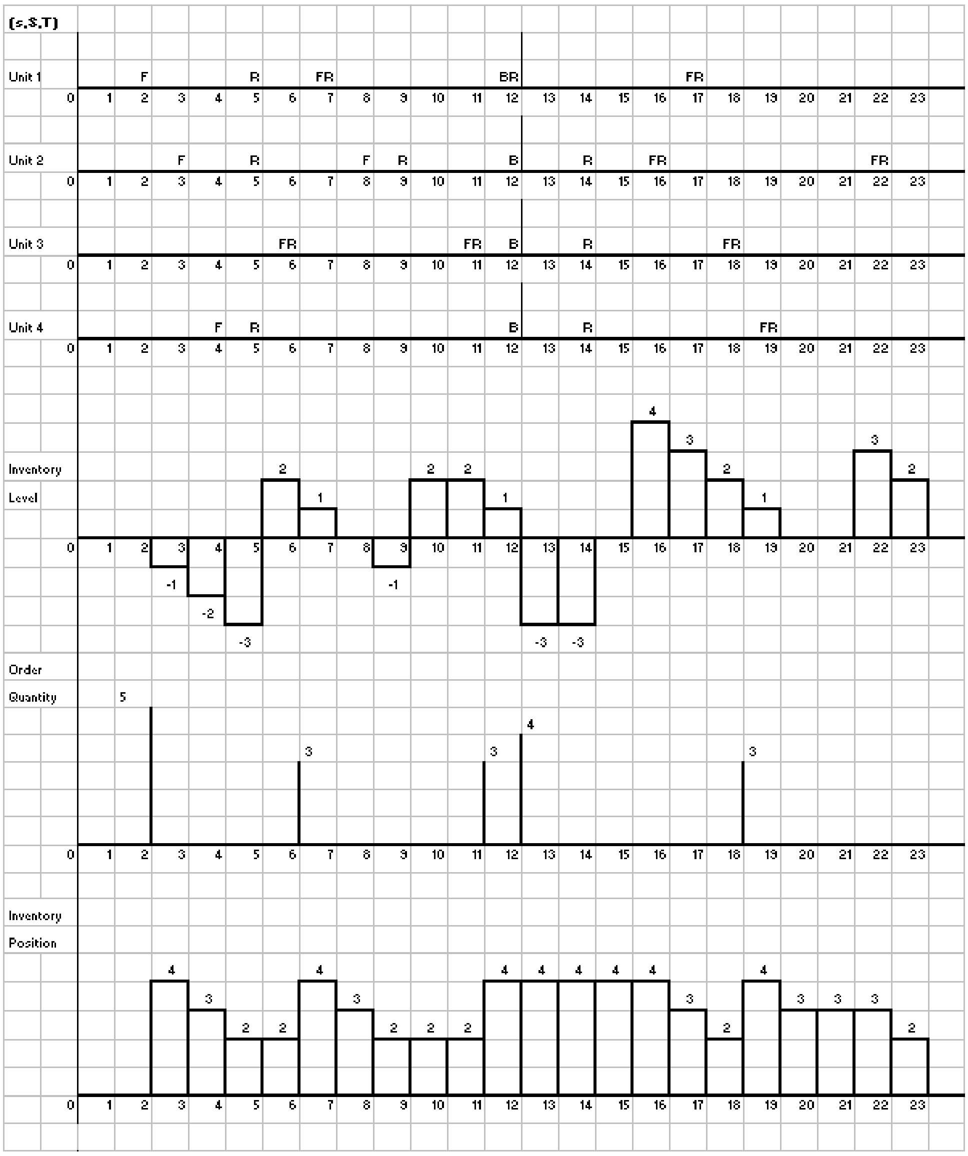
If the individual unit fails before the block replacement time, it will be replaced with a new component if there is stock available. This replacement incurs a *failure replacement cost (FRC*) per component. If the spare is not available, the unit will be replaced as soon as a spare is available, incurring a *shortage cost* (*SC*) per component per time unit. In general, *FRC* is higher than *BRC*.

The inventory policy is a (s,S) policy where s is the *reorder point* and S is the *maximum stock level*. In this policy, an order for (S-s) spares units is placed when the inventory position (*IP*) drops to s and this order will incur an ordering cost (*OC*). The order lead time is deterministic. There are two types of ordering: emergency ordering and regular ordering. Emergency ordering takes place when the inventory level is less than or equal to zero, while a regular order is placed when the inventory level is larger than zero. The emergency ordering cost is higher than the regular ordering cost, and the emergency lead time is shorter than the regular lead time. Spares kept in stock incur a holding cost (*HC*) per unit per time unit.

In this system, the total cost comprises of cost of block replacement (*BRC*) per unit component, cost of individual failure replacement (*FRC*), shortage cost (*SC*), ordering cost (*OC*), and holding cost (*HC*). This total cost can be minimized by determining the optimal values of block replacement interval *BRI* maximum stock level S, and reorder level s.

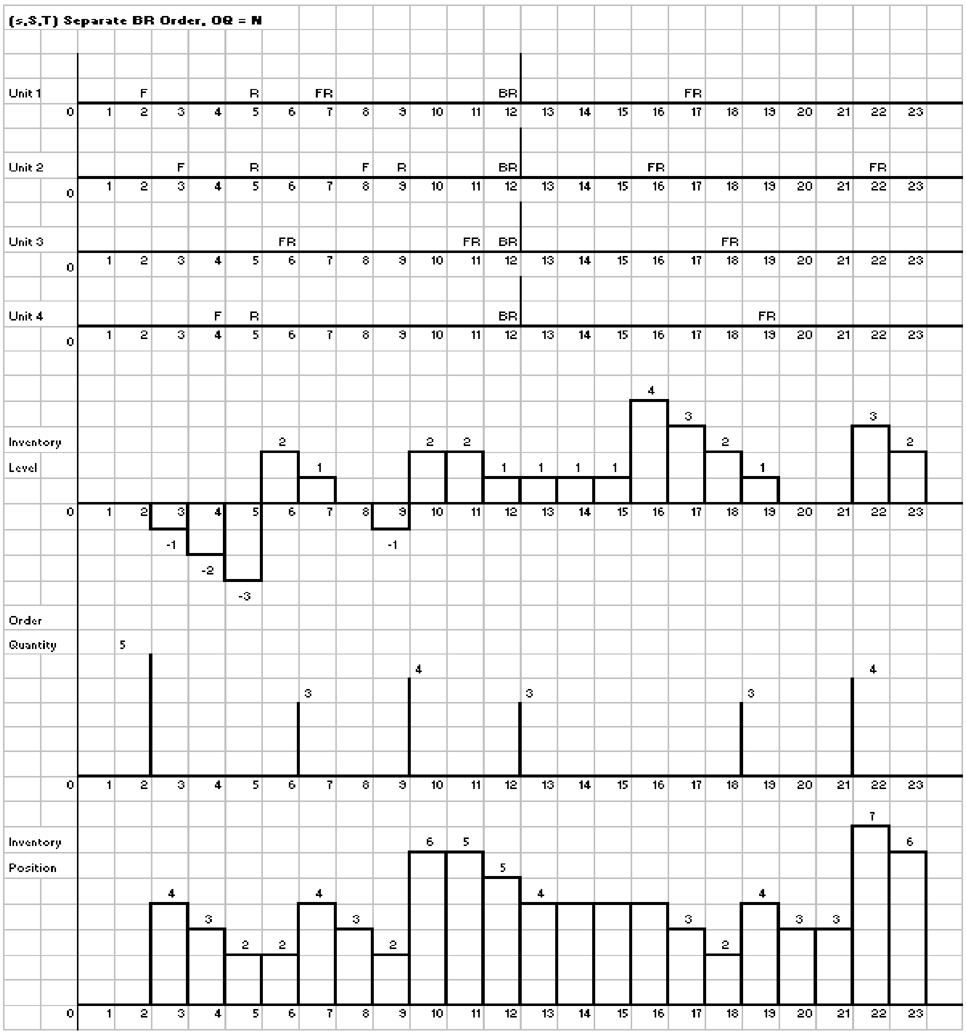
4.2 Cost Formulation

1. *Ordering costs* (*OC*) are incurred if at time *t* the *inventory position* (*IP*) drops to or below reorder level s. There are two types of ordering costs: emergency ordering cost, if inventory level < 0, and regular ordering cost, if IL > 0. The emergency delivery time is shorter than the regular delivery time.
   $TCu = TCp + OCo$ if $IP_t < s$ and $IL_t > 0$
   $TCu = TCp + OCe$ if $IP_t < 0$ and $IL_t <= 0$
   OCo: Regular ordering cost
   OCe: Emergency ordering cost
   Order quantity (OQ) is $S\text{-}IL_t$, S = maximum stock level
2. Holding costs (*HC*) are computed for the inventory level between any two events (both failure time and block replacement time) for $IL > 0$
   $TCu = TCp + HC\ (t - t_p).\ IL_p$
3. *Shortage cost* (*SC*) is accrued for each spare unit remaining not available for each time unit ($IL < 0$)
   $TCu = TCp + SC\ (t - t_p).\ IL_p$
4. *Failure replacement cost* (*FRC*) is incurred if an individual unit fails before the *block replacement time (BRT)* and the spare required is available.
   $TCu = TCp + FRC$ gives the spares available.
   Otherwise, the replacement will be delayed until spares are available; shortage cost will be incurred.
5. Block replacement cost (*BRC)* is incurred every *block replacement time (BRT)* interval, regardless of the previous failure replacement time for each unit. If the available number of spares is larger than or equal to the number of units to be replaced, *N*, the block replacement cost is
   $TCu = TCp + BRC\ x\ N$
   If $0 < IL < N,$
   $TCu = TCp + BRC\ x\ IL$ If $0 < IL < N,$ and the rest of the components will remain inoperative until the spares required are available; shortage cost and block replacement cost are incurred.
   If $IL <= 0$, all units will remain inoperative until the spares required are available; shortage cost and block replacement cost are incurred.
   TCu: Total updated cost
   TCp: Total previous updated cost



F : Failure time
R : Replacement time
FR : Failure and replacement time
BR : Block replacement time

Figure 1 Hypothetical system with four operational units with *(s,S,T)* policy.



F : Failure time
R : Replacement time
FR : Failure and replacement time
BR : Block replacement time

Figure 2 Hypothetical system with four operational units with (s,S,T) separate part ordering for block and corrective replacements (block replacement order quantity = N).

4.3 Simulation Program Algorithm

1. Set the number of components (*N*), unit life time distribution *f(t)*, unit holding cost (*HC*), unit shortage cost (*SC*), emergency ordering cost (*EOC*), regular ordering cost (*ROC*), failure replacement cost (*FRC*),block replacement cost (*BRC*), block replacement interval (*BRI*), regular lead time (*RLT*), emergency lead time (*ELT*), simulation length, number of replication, set initial inventory level IL_0 and initial inventory position IP_0.
2. Generate unit life time. If unit life time ≥ block replacement time then the component will be replaced preventively (block replacement) at the block replacement time. Else, go to step 14.
3. Check inventory level. If inventory level ≥ N, go to step 4. Else go to step 5
4. Replace all components.All components will start as good as new. Update inventory position. If inventory position ≤ reorder level, and inventory position ≤ 0, place an emergency order. If inventory position ≤ reorder level, and inventory position ≥ 0, place a regular order. If inventory position ≤ reorder level, and inventory position < 0, place a regular order. Set the next block replacement time. Go to step 2.
5. If 0 < inventory level < N, go to step 6. Else go to step 8
6. Replace component as much as the inventory level. Go to step 2. The rest of the components will be replaced as soon as the order arrives. Current inventory position = previous inventory position - N. Current inventory level = previous inventory level - N. If inventory position ≤ reorder level, go to step 7. Else go to step 8.
7. Place an emergency order. The order quantity is the maximum stock level (*S*) - inventory position (*IP*). If the spares arrive before the unit fails, do a block replacement (block replacement cost) at order arrival time. If the unit fails before thespare arrives then do a failure replacement and failure replacement cost is incurred. Since there is a shortage between failure time and order arrival time, shortage cost is incurred.
8. Update total block replacement cost, total failure replacement cost, total shortage cost, total holding cost, total emergency ordering cost, inventory position, and inventory level. Set the next block replacement time. If inventory position ≤ reorder level, place an order. Go to step 2.
9. If shortage cost due to waiting for outstanding order ≥ emergency ordering cost plus shortage cost due to waiting for emergency order, place an emergency order. Update total emergency ordering cost. Else, go to step 9.
10. Wait until the order arrives and replace all components. If the spare arrives before the unit fails do a block replacement when the order arrives, block replacement cost is incurred. If the unit fails before thespare arrives do a

failure replacement, failure replacement cost is incurred. Since there is a shortage between failure time and order arrival time (spares available), shortage cost is incurred.

11. Update total block replacement cost, total failure replacement cost, total shortage cost, total holding cost, inventory position, and inventory level. If inventory position ≤ reorder level, place an order. Set the next block replacement time. Go to step 2.
12. If inventory position ≥ N, go to step 11. Else go to 13.
13. If shortage cost due to waiting for outstanding order > emergency ordering cost plus shortage cost due to waiting for emergency order, place an emergency order. Update the total emergency ordering cost. Else go to step 9.
14. Wait until the order arrives and replace all components. If the spare arrives before the unit fails do a block replacement when the order arrives, block replacement cost is incurred. If the unit fails before thespare arrives do a failure replacement, failure replacement cost is incurred. Since there is a shortage between failure time and order arrival time (spares available), shortage cost is incurred.
15. Update total block replacement cost, total failure replacement cost, total shortage cost, total holding cost, total emergency ordering cost, inventory position, and inventory level. If inventory position ≤ reorder level place an order. Set the next block replacement time. Go to step 2.
16. If inventory position < N, place an emergency order. The order quantity is N - inventory position. Wait until the order arrives and replace all components. If the spare arrives before the unit fails do a block replacement (block replacement cost) at order arrival time. If the unit fails before the spare arrives then do a failure replacement, failure replacement cost is incurred. Since there is a difference between failure time and order arrival time, shortage cost is incurred.
17. Update total block replacement cost, total failure replacement cost, total shortage cost, total holding cost, total emergency ordering cost, inventory position, and inventory level. If inventory position ≤ reorder level, place an order. Set the next block replacement time. Go to step 2.
18. Check inventory level. If inventory level > 0 go to step 15. Else go to step 16.
19. Replace the component. The component will start as good as new and only individual failure replacement cost is incurred. Set the next block replacement time. Current inventory position = inventory position - 1. Current inventory level = inventory level - 1. If current inventory position ≤

reorder level and current inventory level ≥ 0 place a regular order. If current inventory position ≤ reorder level and current inventory level < 0 place an emergency order. Go to step 2.
20. If inventory position ≥ reorder level go to step 17. Else go to step 19.
21. If shortage cost due to waiting for outstanding order > emergency ordering cost plus shortage cost due to waiting for emergency order place an emergency order. Update total emergency ordering cost. Else go to step 18.
22. Replace the component as soon as the order arrives. Shortage cost and individual failure replacement cost are incurred. Update total failure replacement cost, total shortage cost, total holding cost, inventory position, and inventory level. Set the next block replacement time. Go to step 2.
23. Place an emergency order. Replace the component as soon as the order arrives. Shortage cost and individual failure replacement cost are incurred. Update total emergency ordering cost, total failure replacement cost, total shortage cost, inventory position, and inventory level. Set the next block replacement time. Go to step 2.
24. End.

This algorithm is for the *(s,S,T)* policy. For the *(s,S,T) separate order* policy, every one regular lead time before the block replacement time, we order N units of spare parts.

4.4 Simulation Program

The simulation program was developed using simulation package ARENA. We used ARENA since it combines the ease of use found in high-level simulators with the flexibility of a simulation language, all the way down to the general-purpose procedural languages [16]. The modules in ARENA are composed of simulation language SIMAN components so we could create our own module.

ARENA also has an OpQuest package that is designed to find the decision variables (input-control) that give the best solution to an objective function. OptQuest automates or controls ARENA to set variable values, start and continue simulation runs, and retrieve the simulation results.

When an optimization runs, OptQuest starts the simulation by issuing a start over command. It then changes the values of the control variables and resource capacities to those identified by OptQuest for the simulation scenario. Next, OptQuest instructs ARENA to perform the first replication.

OptQuest combine the metaheuristics procedures of tabu search, neural networks and scatter search into a single composite method [17]. The exact procedure, however, is kept secret. For more information we refer to Glover, *et al.* [18].

5 Numerical Experiments and Discussion

In this section, we describe our numerical experiments and analyze the differences between the policies. First, a base case is used and then various parameters will be tested in order to gain some insight into the effects of changing the parameters values.

Also, the saving cost from our proposed policy (policy 4) and policy 5 [13] to policy 3 [14] were compared.

5.1 Numerical Experiments with Base Case Parameters

The simulation was run for all policies with base case parameters. The parameters for this base case numerical experiment were:

1. A Weibull distribution was used to represent the unit failure time. The probability density function *f(t)* of this distribution is given by
2. $f(t) = \alpha\beta^{-\alpha}t^{\alpha-1}\exp\left\{-\left(\frac{t}{\beta}\right)^{\alpha}\right\}$ for $t > 0$, otherwise and *f(t)* = 0
3. Where α is the shape parameter and β is the scale parameter.
4. In this base case simulation, the values of β = 50 and α = 3.
5. Number of identical components (N) = 5
6. Regular lead time: 5
7. Emergency lead time: 1
8. Regular ordering cost: 5 per order
9. Emergency ordering cost: 30 per order
10. Shortage cost: 20 /unit/time unit
11. Holding cost: 1 /unit/time unit
12. Block replacement cost: 20 per unit/replacement
13. Failure replacement cost: 100 per unit/replacement
14. Threshold = T/2

We ran the simulation to find the optimum value for the (*s,S,T*) combination. The range of *block replacement interval* (*T*) was set from 10 to 50 (scale parameter of failure distribution) with a step increment of 1. We chose this range since it is economical to replace the components before its average life time (in this case, scale parameter, which is 50). *Reorder level* (*s*) was set from 0 to 5 with step increment 1. Maximum stock level (*S*) was set from 1 to 7 with a step increment of 1. We chose this value since there are 5 identical components in the system. With this setting there are 41 x 6 x 7 combinations of (*s,S,T*).

Every simulation ran for 100 replications with a replication length of 10.000 time units (200 time units is the life time scale parameter). A 95% confidence interval was constructed for the results of each experiment. ARENA gives the mean of each result over the replication with the half width of a (nominal) 95% confidence interval on the expected value of the output result. Table 2 shows the output of the simulation. The differences between policy 3 [14] and our proposed policy (policy 4) are significant.

The results for the holding cost can be verified using the following reasoning. The upper bound for the holding cost per time unit is equal to 1 since the holding cost is equal to 1 and in general we keep the maximum stock level equal to 1. However, sometimes the inventory level is equal to zero or below zero, therefore the average holding cost per time unit is less than 1. However, for policy 1 the maximum stock level is equal to 2, so the upper bound for the holding cost per time unit is equal to 2 and the lower bound for the holding cost per time unit is equal to 1, since the reorder point is equal to 1. Next, we will discuss the results of each policy in detail.

1. (s,S), T Infinite Policy (Policy 1)

Since there is only failure demand, only one demand at one time, and there is not a routine demand for block replacement, the optimum stock policy is (0,2). This policy gives the highest average total cost. This result is in line with theory, which states that for a system consisting of components with an increasing failure rate, the failure replacement policy is costly. It is more economical to do block replacement.

2. *(s,S),(T)* Policy (Policy 2)

In this case the maximum stock level is 1, so that in any instance the maximum inventory position is equal to 1 and the maximum inventory level is equal to 1. This stock is only for failure demand between two successive block replacement times. In every block replacement time the stock is always less than the demand (N = 5). Hence, in every block replacement time we place an emergency order.

The lower bound for emergency ordering cost per time unit is equal to the emergency ordering cost (30) divided by the block replacement interval (24), which is equal to 1.25. It is also possible that an emergency order is triggered by failure demand. This explains why the average emergency ordering cost is 1.309 per time unit (higher than 1.25).

Regular orders are only triggered by failure demand. On average, one failure occurs in about every 46 time units and will trigger a regular order or an emergency order. The upper bound for the average regular ordering cost per time unit is equal to the regular ordering cost (5) divided by 45 = 0.110 per time unit. The average regular ordering cost is 0.089, which is less than 0.110 because sometimes failure of a component triggers an emergency order instead of a regular order.

This result is in line with theory, for a system consisting of components with an increasing failure rate, a block replacement policy gives lower costs compared to a failure replacement policy (Policy 1).

3. *(s,S,T)* Policy (Policy 3)

In this case the maximum stock level is 1, so that in any instance the maximum inventory position is equal to 1 and the maximum inventory level is equal to 1. This stock is only for failure demand between two successive block replacement times. In every block replacement time, the stock is always less than the demand (N = 5). Hence, in every block replacement time, we place an emergency order.

The lower bound for the emergency ordering cost per time unit is equal to the emergency ordering cost (30) divided by the block replacement interval (25), which is equal to 1.20. It is also possible that an emergency order is triggered by failure demand.This explains why the average emergency ordering cost is 1.274 per time unit (higher than 1.20).

Regular orders are only triggered by failure demand. On average, one failure occurs in about every 43 time units and will trigger a regular order or an emergency order. The upper bound for the average regular ordering cost per time unit is equal to the regular ordering cost (5) divided by 43 = 0.118 per time unit. The average regular ordering cost is 0.095, which is less than 0.118 because sometimes failure demand triggers an emergency order instead of a regular order.

As concluded in [13] and [14], a joint optimization (s,S,T) policy (Policy 3) has better performance than a separate optimization (s,S),(T) policy (Policy 2) for both age replacement and block replacement policies. The total average cost is lower.

4. *(s,S,T)* Separate Part Ordering for Block and Corrective Replacement Policy (Policy 4)

In this policy, since we place regular orders for block replacement demand in advance with regular ordering cost, which is cheaper, the total ordering cost becomes cheaper. There are no emergency order in every block replacement time.

On average, the lower bound for the average regular ordering cost per time unit is equal to the regular ordering cost (5) divided by the block replacement interval (24), which is 0.208. Regular orders are also triggered by failure demand, hence the average regular ordering cost is more than 0.208, which is lower than 0.304. The difference comes from the regular ordering cost triggered by failure demand.

Emergency orders are only triggered by failure demand and occur less frequently (0.070 per time unit). The regular ordering cost triggered by failure demand is 0.304 - 0.028, which is equal to 0.096. This shows that the regular ordering cost triggered by block replacement demand (0.208) is higher than the regular cost triggered by failure demand (0.096).

It also shows that, in case of failure demand the average regular ordering cost (0.096) is higher than the average emergency ordering cost (0.070). The average holding cost in this policy is higher since stock on hand will not be used in every block replacement time. In every block replacement time, we only use N spares, which we order separately. If there is stock on hand, this stock is only used to fulfill failure replacement demand. If there is no failure between the replacement time and the block replacement time (T - L), this stock will be left over after the block replacement time and will be used for the next failure replacement.

As a comparison between the block replacement policy and the age replacement policy, we simulate the same case for *(s,S,T)* age replacement policy (Policy 5).

Table 2 Optimum solution for base case parameters.

Policy	s	S	T	Average Maintenance Cost: Failure	Preventive	Total Cost	Average Inventory Cost: Emergency ordering cost	Regular ordering cost	Holding cost	Shortage cost	Total cost	Average Total cost
1	0	2	∞	11.110	0.000	11.110	0.365	0.245	1.079	0.503	2.192	13,302 ±0.028
2	0	1	24	2.184	4.112	6.296	1.309	0.089	0.876	0.057	2.331	8.627 ±0.031
3	0	1	25	2.354	3.939	6.293	1.274	0.095	0.872	0.065	2.303	8.596 ±0.034
4	0	1	24	2.182	4.151	6.333	0.070	0.304	0.925	0.040	1.338	7.671 ±0.034
5	0	2	24	2.337	3.666	6.003	0.299	0.477	0.615	0.255	1.646	7.650 ±0.034

These results are in line with theory. The proposed policy (Policy 4) gives a better solution compared to Policy 1, Policy 2, and Policy 3. The separate spare part order schedule gives a better result due to inventory cost reduction. The cost reduction is gained from the emergency ordering cost and the shortage cost. Since the block spare parts are ordered at one lead time before the planned schedule of block replacement, the shortage probability for block replacement is very low and hence emergency order probability is very low as well.

The regular ordering cost for Policy 4 is relatively high compared to Policy no 3 since there are two regular-order components, one for failure replacement and one for block replacement. However, the increase in the regular ordering cost is very low compared to the decrease in the emergency ordering cost.

The holding cost for Policy 4 is slightly higher compared to Policy 3, since the expected amount of spare part inventory in Policy 4 is higher due to the possibility of excess inventory for failure replacement at the time of block replacement. Overall, in terms of total cost, Policy 4 outperforms Policy 3.

The age replacement policy (Policy 5) performs slightly better compared to our proposed policy. This result is in line with Archibald and Dekker [19].

However, block replacement has an economic of scale advantage compared to age replacement, especially when the number of identical items is relatively high.

5.2 Numerical Experiments with Various Parameters

In order to gain some insight into the effects of various parameters on all policies, a number of case problems were constructed. It was proved that for a system consisting of components with an increasing failure rate, block replacement (Policy 2) is always better than failure replacement (replacement only at failure, Policy 1).

For a system under age based replacement, Kabir and Olayan [13] proved that Policy 3 yields better results than Policy 2. Meanwhile, Sarker and Haque [14] indicate the same result for a system under a block replacement policy. Therefore, in our numerical experiments we only compared between our proposed policy (Policy 4) and Policy 3 [13]. As a benchmark, we compared block replacement policy (Policy 3) to age replacement (Policy 5).

The parameter values selected for these numerical experiments were:

1. Unit life time distribution follows a Weibull distribution with scale parameter $\beta = 50$ and shape parameter $\alpha = 1.5, 2$ and 3
2. Number of components (N) = 5
3. Regular lead time: 5, 10 and 15
4. Emergency lead time: 0, 1 and 2
5. Regular ordering cost: 1,5 and 10
6. Emergency ordering cost: 10, 30 and 100
7. Unit shortage cost: 10, 20 and 100
8. Unit holding cost: 0.1, 0.5, and 1
9. Block replacement cost /unit: 10, 20 and 50
10. Threshold time = T/2

In these numerical experiments, the value of one of the parameters was changed while the values of the other parameters were kept the same as the base case values. The number of replications was 100 and the simulation length was 10000 time units. A 95% confidence interval was constructed for the results for each experiment.

5.2.1 Effect of Unit Life Time Distribution Shape Parameter

The unit life time distribution shape parameter affects the optimum block replacement interval. The higher this parameter, the shorter the block replacement interval. These results are in line with Kabir and Olayan [13] and Dekker and Dijkstra [20]. It can also be seen from the results that the higher the distribution shape parameter, the higher the cost saving from these policies. For Policy 4, the cost saving ranges from 5.70% to 10.76%. Table 3 shows the effect of the distribution shape parameter on the total average cost for the different policies.

Table 3 Effect of unit life time distribution shape parameter on average total cost, *s, S, & T*.

Shape Parameter	Average Total Cost			Comparison (%)	
	Policy 3	Policy 4	Policy 5	Policy 4 to Policy 3	Policy 5 to Policy 3
1.5	13.116±0.059	12.369±0.062	12.011±0.055	94.30	91.58
2	11.106±0.044	10.202±0.042	10.077±0.045	91.86	90.73
3	8.596±0.034	7.671±0.034	7.650±0.034	89.24	88.99

5.2.2 Effect of Regular Lead Time

The regular lead time does not significantly affect the optimum s, S, and T for Policy 4 but significantly affects Policy 5.The cost saving for Policy 4 ranges from 10.63% to 10.76 %, while the cost saving for Policy 5 ranges from 1.88 to 11.01 %. Table 4 shows the effect of the regular lead time on the total average cost for the different policies.

Table 4 Effect of regular lead time on average total cost, s, *S*, & *T*.

Regular Lead Time	Average Total Cost			Comparison (%)	
	Policy 3	Policy 4	Policy 5	Policy 4 to Policy 3	Policy 5 to Policy 3
5	8.596±0.034	7.671±0.034	7.650±0.034	89.24	88.99
10	8.568±0.034	7.648±0.034	8.0364±0.033	89.26	93.79
15	8.525±0.034	7.619±0.034	8.365±0.044	89.37	98.12

5.2.3 Effect of Emergency Lead Time

Emergency lead time significantly affects optimum *T* for Policy 3, since in this policy we have to place an emergency order in every block replacement time. In order to decrease the emergency ordering cost, *T* must increase. For all policies, as the emergency lead time increases, the cost saving decreases. For Policy 4, the cost saving ranges from 10.28% to 11.68%. For Policy 5, the cost saving ranges from 1.60% to 2.90%, and for Policy 5, the cost saving ranges from 10.05% to 12.61%. Table 5 shows the effect of the emergency lead time on the total average cost for the different policies.

Table 5 Effect of emergency lead time on average total cost, *s, S,* & *T.*

Emergency Lead Time	Average Total Cost			Comparison (%)	
	Policy 3	Policy 4	Policy 5	Policy 4 to Policy 3	Policy 5 to Policy 3
0	8.629±0.033	7.621±0.031	7.541±0.032	88.32	87.39
1	8.596±0.034	7.671±0.034	7.650±0.034	89.24	88.99
2	8.588±0.034	7.705±0.033	7.725±0.036	89.72	89.95

5.2.4 Effect of Regular Ordering Cost

The regular ordering cost affects cost saving significantly for Policy 4. As the regular ordering cost increases, the cost saving decreases. For Policy 4, the cost saving ranges from 8.25% to 12.82 %, while for Policy 7, the cost saving ranges from 7.86% to 14.71%. Table 6 shows the effect of the regular ordering cost on the total average cost for the different policies. The expected number of regular orders for Policy 4 is higher than for Policy 3 since it separates the order schedules for failure replacement and block replacement. Therefore, if the regular ordering cost increases the cost saving of Policy 4 decreases.

Table 6 Effect of regular ordering cost on average total cost, *s*, *S*, & *T*.

Regular Ordering Cost	Average Total Cost			Comparison (%)	
	Policy 3	Policy 4	Policy 5	Policy 4 to Policy 3	Policy 5 to Policy 3
1	8.520±0.033	7.428±0.033	7.267±0.034	87.18	85.29
5	8.596±0.034	7.671±0.034	7.650±0.034	89.24	88.99
10	8.691±0.035	7.974±0.035	8.008±0.031	91.75	92.14

5.2.5 Effect of Emergency Ordering Cost

For Policy 3, the higher the emergency ordering cost, the higher the maximum stock level (*S*). Since the emergency ordering cost is higher, one avoids placing an emergency order in every block replacement time. Therefore, *S* must be high enough so that it can fulfill all block replacement demand (shortage will not trigger an emergency order). The emergency ordering cost affects the cost saving significantly for Policy 4. As the emergency ordering cost increases, the cost saving also increases.

The cost saving ranges from 1.68% to 29.45% for policy, while for Policy 5 the cost saving ranges from 4.13% to 30.26%. Table 7 shows the effect of the emergency ordering cost on the total average cost for the different policies. The expected number of emergency orders for Policy 4 is lower than for Policy 3 since the expected shortage for block replacements for Policy 4 is lower than for Policy 3. Therefore, if the emergency ordering cost increases, the cost saving of Policy 4 increases.

Table 7 Effect of emergency ordering cost on average total cost, *s*, *S*, & *T*.

Emergency Ordering Cost	Average Total Cost			Comparison (%)	
	Policy 3	Policy 4	Policy 5	Policy 4 to Policy 3	Policy 5 to Policy 3
10	7.754±0.030	7.624±0.032	7.434±0.031	98.32	95.87
30	8.596±0.034	7.671±0.034	7.650±0.034	89.24	88.99
100	11.104±0.040	7.834±0.039	7.744±0.034	70.55	69.74

5.2.6 Effect of Unit Shortage Cost

As unit shortage cost increases, optimum T decreases for Policy 3. But for Policies 4 and 5, varying unit shortage cost does not affect optimum T. Varying this parameter does not significantly affect cost saving for all policies except for Policy 5. Cost saving ranges from 5.40% to 12.27% for Policy 5. Table 8 shows the effect of the unit shortage cost on the total average cost for the different policies. The expected block replacement shortage for Policy 4 is lower than that of Policy 3. Therefore, if the unit shortage cost increases, the cost saving of Policy 4 decreases.

Table 8 Effect of unit shortage cost on average total cost, *s*, *S*, & *T*.

Unit Shortage Cost	Average Total Cost			Comparison (%)	
	Policy 3	Policy 4	Policy 5	Policy 4 to Policy 3	Policy 5 to Policy 3
10	8.560±0.033	7.651±0.033	7.510±0.032	89.38	87.73
20	8.596±0.034	7.671±0.034	7.650±0.034	89.24	88.99
100	8.856±0.036	7.829±0.039	8.378±0.046	88.40	94.60

5.2.7 Effect of Unit Holding Cost

The lower the unit holding cost, the higher maximum stock level S and the longer optimum *T* for policy 3. On the other hand, for proposed Policy 4 and Policy 5, varying the unit holding cost does not affect optimum *T*.

In Policies 3, 4 and 5, the lower the unit holding cost, the higher the maximum stock level (*S*). With a higher stock level, the emergency ordering cost decreases, since we don't have to place an emergency ordering in every block replacement time (for block replacement demand). There is a trade-off between holding cost and emergency order cost. The higher the emergency ordering cost and the lower the unit holding cost, the higher the maximum stock level (*S*).

For Policy 4, the cost saving ranges from 4.81% to 11.67 %, while for Policy 5 the cost saving ranges from 8.05% to 12.23 %. Table 9 shows the effect of the unit holding cost on the total average cost for the different policies.

If the unit holding cost is relatively low it is more beneficial to reduce the number of orders. For Policy 4, the expected number of orders is higher compared to Policy 3 since it separates the block replacement and the failure replacement. Hence, the lower the unit holding cost, the lower the cost saving of Policy 4.

Table 9 Effect of unit holding cost on average total cost, *s*, *S*, & *T*

Unit Holding Cost	Average Total Cost			Comparison (%)	
	Policy 3	Policy 4	Policy 5	Policy 4 to Policy 3	Policy 5 to Policy 3
0.1	7.132±0.032	6.789±0.031	6.558±0.027	95.19	91.95
0.5	8.160±0.035	7.208±0.034	7.162±0.031	88.33	87.77
1	8.596±0.034	7.671±0.034	7.650±0.034	89.24	88.99

5.2.8 Effect of The Block Replacement Cost

The block replacement cost affects the optimum *T* significantly. If the block replacement cost increases, the block replacement interval *T* increases in order to reduce the average block replacement cost.

Table 10 Effect of block replacement cost on average total cost, *s*, *S*, & *T*.

Block Replacement Cost	Average Total Cost			Comparison (%)	
	Policy 3	Policy 4	Policy 5	Policy 4 to Policy 3	Policy 5 to Policy 3
10	6.514±0.030	5.368±0.028	5.493±0.030	82.41	84.33
20	8.596±0.034	7.671±0.034	7.650±0.034	89.24	88.99
50	13.664±0.036	12.978±0.038	12.038±0.033	94.98	88.10

For Policy 4, as the block replacement cost increases, the cost saving decreases ranging from 17.59% to 5.02%. Cost saving for Policy 5 ranges from 11.01% to

15.67%. Table 10 shows the effect of the block replacement cost on the total average cost for different policies.

6 Conclusion and Future Research

Our experiments showed, that for a system consisting of components with an increasing failure rate, a block replacement policy gives lower costs compared to a failure replacement policy (replacement only at failure), as is already well-accepted in maintenance theory. Our experiment also confirmed the result from previous experiments [13,14] that joint optimization of maintenance and inventory policy produces better results than the combination of separate and sequential optimization policies.

The most important result of our experiments, for a system under block replacement, is that our proposed policy (Policy 4) yielded better and more cost effective solutions. In Policy 4, ordering cost was optimized with a separate order policy. This policy produced the lowest inventory cost, while the maintenance cost was about the same as for Policy 3 [14].

For the age replacement policy (ARP) we also tried to separate the ordering for block replacement demand and failure replacement demand. However, in ARP the effect of separate ordering on the average total cost is insignificant. In contrast, in the block replacement policy (BRP), the effect of separate ordering on the average total cost is significant.

In this study, we used a simulation model in our experiment. The direction of future research is to develop an analytical model for all our three proposed policies, in order to find the exact solution for these problems. The main problem for a multiple component system is to calculate the cycle time. For a single component system, the time between two successive block replacements can be treated as a cycle. Unlike in a one-unit system, in a multiple-unit system it is difficult to define the time cycle since there is an overlap in the replacement time units.

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