

Probabilistic Modeling of Seismic Risk Based Design for a Dual System Structure

Indra Djati Sidi

Department of Civil Engineering, Faculty of Civil and Environmental Engineering, Institut Teknologi Bandung, Jalan Ganesha 10, Bandung 40132, Indonesia E-mail: indradjati@gmail.com

Abstract. The dual system structure concept has gained popularity in the construction of high-rise buildings over the last decades. Meanwhile, earthquake engineering design provisions for buildings have moved from the uniform hazard concept to the uniform risk concept upon recognizing the uncertainties involved in the earthquake resistance of concrete structures. In this study, a probabilistic model for the evaluation of such risk is proposed for a dual system structure consisting of shear walls or core walls and a moment frame structure as earthquake resistant structure. Uncertainties in the earthquake resistance of the dual system structure due to record-to-record variability, limited amount of data, material variability and structure modeling are included in the formulation by means of the first-order second-moment method. The statistics of resistance against earthquake forces are estimated by making use of incremental nonlinear time history analysis using 10 recorded earthquake histories. Then, adopting the total probability theorem, the reliability of the structure is evaluated through a risk integral scheme by combining the earthquake resistance of the structure with the annual probability of exceedance for a given location where the building is being constructed.

Keywords: dual system structure; fragility function; probabilistic based design; tall buildings; uncertainty; uniform hazard; uniform risk.

1 Introduction

Currently, construction of Thamrin Nine Projects, with a 80-story (336 meters) high dual system structure, 6 basement levels, and around 90 meter deep 1.2 m-diameter bored pile foundations, has just started. The Signature Tower with a 111-story dual system of core and frame is under planning and design for immediate construction later this year. This development of super tall buildings in Jakarta, of course, raises several fundamental questions regarding design and construction principles, especially in view of the aspect of safety and reliability against earthquakes. Designed lifetime and relevant design earthquake loads, consideration of column shortening, creep and shrinkage analysis, the load factor adopted for the column design are among the questions that have to be addressed conceptually by the designers.

Meanwhile, Indonesia has made significant progress in regulating earthquake engineering design by formulating related provisions for buildings, moving from the uniform hazard concept to the uniform risk concept upon recognizing the uncertainties involved in the resistance of buildings subjected to earthquake loads. In contrast to the seismic design map from 2002, which was based on ground motion values with 10%-in-50-years exceedance probability, the probability portion of the targeted maximum considered earthquake risk (MCER) of the current code are equal to a 1% collapse probability of failure within 50 years. These values are different from 2%-in-50-years exceedance probability, or maximum considered earthquake (MCE). The associated increase in ground motion values is accompanied by a change in the performance objectives, from life safety (LS) to collapse prevention (CP), which has led to the introduction of a factor of 2/3 applied to the MCER ground motion [1]. As probability based design has gained acceptance in many design codes around the world, Indonesia under the coordination of Ministry of Public Works and other related institutions has also revised its provisions to adopt a reliability and probabilistic approach, especially for structural design.

The factor of safety and the load-and-resistance factor in structural design or nominal values used as design parameters, such as concrete compression strength or tension stress of steel, are consistently based on a certain targeted risk. All of the design values (nominal values) used in design practice should guarantee, conceptually speaking, that the structure has an acceptable performance in terms of reliability or risk during the lifetime of the building, e.g. 10⁻³ risk of failure for slab and beam under gravity loads. A design spectral acceleration with a certain value of probability of exceedance or expected return period does not mean anything for structural design. For designers, the most important is the consequence of earthquake loads for the respective buildings in terms of risk or probability of failure. This paper proposes a probabilistic methodology for modeling uncertainties systematically in designing a tall building subject to earthquake hazard. The model combines the result of the commonly used seismic hazard analysis of a certain area with the resistance of a structure obtained from incremental dynamic nonlinear analysis or time history analysis.

2 Uncertainties in Dual System Structures

2.1 Moment Curvature and Section Properties

The moment curvature and section properties used in the analysis depend on the variability of the compressive strength of concrete and the tensile stress of steel. It is commonly assumed that the nominal values of compressive strength and steel are determined by the 10% lower tail values, which means there is 10%

probability that the real values are smaller than the nominal design values, hence, in terms of the nominal design, the mean value of the compressive strength f_c ' and the mean value of the yield stress of steel, f_y , may be given as following Eq. (1):

$$\overline{f_c} = \left[\frac{1}{1 - 1.28 \,\Omega_{f_c}}\right] f_c' \tag{1}$$

and the mean value of the yield strength of steel may be represented as in Eq. (2):

$$\overline{f_y} = \exp\left[\ln\left(\frac{f_y}{\sqrt{1 + \Omega_{f_y}^2}} + 1.28 \Omega_{f_y}\right)\right]$$
 (2)

where f_c' = the nominal value of the compressive strength of concrete, and $\overline{f_c}$ and Ω_{f_c} are the mean and coefficient of variation of f_c , f_y = the nominal yield stress of steel, and $\overline{f_y}$ and Ω_{f_y} are the mean and coefficient of variation of f_y . In this derivation, the compressive strength of concrete and the yield stress of steel are assumed to follow a normal and a lognormal probability density function respectively. In the following incremental nonlinear time history analysis the mean values are adopted for evaluating the capacity of the structure.

2.2 Structure Modeling for Dual System Structures

The structural concept for a super tall building using the dual system, as commonly adopted in Jakarta, uses a symmetric dual system consisting of shear walls and a frame on the outside perimeter, sometimes strengthened by a belt truss around the perimeter of the building connected to the shear wall by outrigger beams. The belt truss and outrigger beams are placed every other 1/3 and 2/3 of the height of the building to reduce shear lag and moment acting on the shear walls. Due to the random nature of the compressive strength of concrete, the yield stress of steel, simplification in the moment curvature of structural members, and structure idealization, including the condition of the base support, the response of a structure due to earthquake forces cannot be predicted with certainty. The random nature of the structural response is further magnified by the earthquake loads due to record-to-record variation, as shown in for example [2]. The influence of structure modeling and the random nature of the compressive strength of concrete and the yield stress of steel may be analyzed by using the first-order second-moment method [3] or applying Monte Carlo simulation [4] and [5]. In the proposed model it is assumed that the mean value of the model error is one, with a coefficient variation of 0.20.

2.3 Earthquake Loads and Its Effect, Probabilistic Seismic Hazard Analysis (PSHA) – Uniform Hazard Concept

The earthquake load is the biggest uncertainty involved in designing an earthquake resistant building due to our lack of understanding of earthquakes and their effect on buildings. However, engineers around the world have made significant progress in modeling earthquake loads deterministically or probabilistically. For the past decade, researchers have used a probabilistic approach to systematically incorporate all parameters involved in the determination of earthquake loads acting on the building, such as random occurrence, fault length and magnitude, into a prediction of annual probability of exceedance for a certain ground acceleration [6]. In this study, the annual probability of exceedance is incorporated into the formulation as the earthquake load effect on the structure (Figure 1) [7].

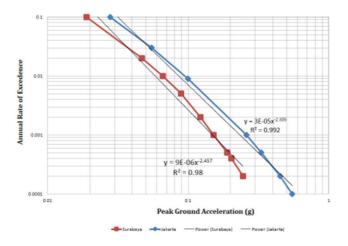


Figure 1 Annual seismic hazard for Jakarta and Surabaya [7].

The probabilistic seismic hazard analysis (PSHA) developed by Cornell [6] and McGuire [8] is the most commonly used method to assess seismic hazard by incorporating the occurrence frequency of earthquakes, the randomness of earthquake magnitude, the attenuation between earthquake magnitude and ground acceleration for a specific site, and the characteristic earthquake sources (e.g. line source, point source, or region sources) into a probabilistic framework to predict the annual probability of exceedance given by Eq. (3) [8]:

$$P(Y > y|y) = \sum_{i} v_{i} \iiint f_{M}(m) f_{R}(r) f_{\epsilon}(\epsilon) P(Y > y|m,r,\epsilon) dm dr d\epsilon$$
 (3)

where v_i is the activity rate for seismic source i; $f_M(m)$, $f_R(r)$, and $f_{\epsilon}(\epsilon)$ are the earthquake magnitude, source-to-site distance, and ground motion density

functions, respectively; ϵ is ground motion uncertainty; and $P(Y > y \mid m, r, \epsilon)$ is the probability that Y exceeds y for a given m and r, as shown in Figure 1, for a typical result. The solution of the triple integration of Eq. (3) requires a numerical algorithm. The results of Eq. (3) will depend on the quality of the input data, the attenuation law used in the model, and also the model error used in the formulation of PSHA. Nevertheless, PSHA is the best tool available to assess seismic hazard for a certain area.

For the case of Indonesia, based on PSHA, the maximum considered earthquake (MCE) ground motion map in the proposed new code is derived by including the latest earthquake data (e.g. Aceh 2004, West Sumatera 2009, Aceh 2012, etc.) with the criteria of 2%-in-50-years probability of exceedance. An example of the resulting map is represented in Figure 2 [9]. This recommendation is in line with NEHRP Provisions 2003 and ASCE Standard 7-05. The associated increase in the ground motion values is accompanied by a change in the performance objective, from life safety (LS) to collapse prevention (CP), which has led to the introduction of a factor of 2/3 applied to the MCE ground motion [1].

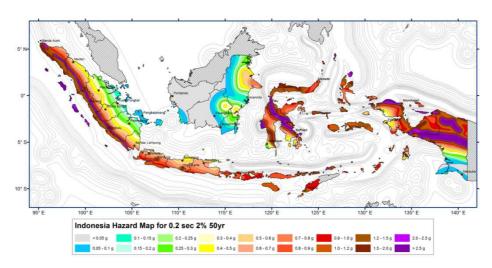


Figure 2 Indonesia Hazard Map for Site Class B (Rock) and Ss (T = 0.2 second), x = 5% [9].

3 Probabilistic Based Design – General Approach

Uncertainties are always part of the design process and are a fact of life in engineering design. Engineers cannot avoid uncertainties, but they can minimize the effect of uncertainties on the performance of the buildings they design to a certain acceptable level. In this case, probabilistic methods offer a

systematic and conceptual scheme for modeling uncertainties in design practice to seek a safe and yet economic structure that can resist future earthquakes. Many current design codes are based on a probabilistic approach [1], including the determination of future earthquake design loads and the related resistance to be provided by a building. However, the classic questions that always come up in the mind of an engineer are: what design acceleration should be used, how are we going to design the structure system, how we are going to reinforce the beams, the columns and the shear walls, and is there any guarantee that, if we use sufficiently severe earthquake loads (high spectral acceleration), the building will withstand future earthquakes during the lifetime of the building without jeopardizing life safety.

Nevertheless, several facts regarding earthquakes remain uncertain, namely: when and where they strike, their magnitude, the ground acceleration and the impact on a specific building, and, finally, the behavior of the resistance of the building against earthquake loads shows a nondeterministic performance. In short, earthquake loads and earthquake resistance are random variables in nature and the attached risk is unavoidable. Failure or collapse during the lifetime of a building can be represented by following Eq. (4):

$$F = R < L \tag{4}$$

where F = failure, R = resistance, and L = maximum earthquake load effect on a building within the lifetime (T) of the building, and the corresponding probability of failure or risk can be formulated as:

$$Risk = P\left[ln\left(\frac{R}{L}\right) \le 0\right] \tag{5}$$

A formidable task for researchers and engineers in the evaluation of Eq. (5) is the determination of the probability distribution function of L with the respective parameters obtained from field measurement or inferred from the statistics of extremes. Fortunately, however, the model of probability seismic hazard analysis (PSHA) of Cornell [6] and McGuire [8] delivers a quantitative value for the annual probability of exceedance, $P(Y > y \mid y)$ for a given value of y, which may be used in the risk or reliability analysis. In the past, the annual probability of exceedance was assumed to be identical to the probability of failure of the structure, which implies that the resistance is a deterministic variable. That is not the case, however, the resistance of a tall building structure is a random variable.

The evaluation statistics of R, nevertheless, are relatively easy and straightforward. The statistics of R may be obtained from model tests or performing incremental nonlinear time history dynamic analysis [10] and

combining with the first-order second-moment method proposed by Ang and Tang [3]. Designing a building with a specific value of ground acceleration with a certain probability of exceedance y, namely 2% within 50 years or annual probability of exceedance of 10^{-4} , doesn't mean that the building has an annual risk of 10^{-4} , because, as mentioned before, the building capacity or earthquake resistance is not a deterministic variable, it is in fact a random variable. If the earthquake resistance of a tall building is equal to y and the capacity is a deterministic variable, then the annual risk, P_{an} , can be defined as in Eq. (6):

$$Pan = P (Y > y \mid Resistance = y)$$
 (6)

Figure 1 represents the annual seismic risk of Jakarta and Surabaya [7]. Since the earthquake resistance of a tall building is also a random variable and by adopting the total probability theorem the final annual risk can be represented by the risk integral in following Eq. (7).

$$Pan = \int P((Y > y \mid r) f_R(r) dr$$
 (7)

where $f_R(r)$ is the probability density function of the capacity of a tall building in terms of inter-story drift or rotational capacity of structural members (e.g. columns, beams or shear walls), known as the fragility function.

4 Seismic Risk Analysis for Dual System Structures

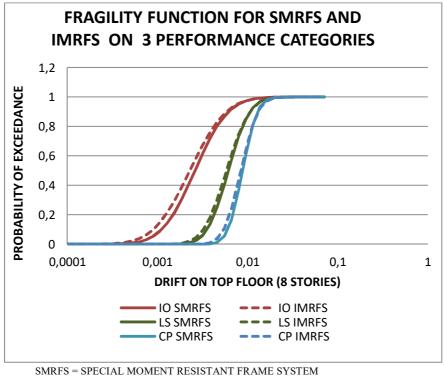
4.1 Fragility Function

A dual system is a structural system in which an essentially complete frame provides support for gravity loads and resistance to lateral loads is provided by a specially detailed moment-resisting frame and shear walls or braced frames. Both shear walls and the frame participate in resisting the lateral loads resulting from earthquakes and the portion of the forces resisted by each depends on its stiffness, modulus of elasticity and ductility, and the possibility to develop plastic hinges in its parts. The moment-resisting frame may be either made from steel or concrete but concrete intermediate frames cannot be used in seismic zones 3 or 4. The moment-resisting frame must be capable of resisting at least 25 percent of the base shear [9] and both systems must be designed to resist the total lateral load in proportion to their relative stiffness [11].

The probability density function of R, also known as the fragility function, follows a lognormal distribution characterized by its logarithmic mean and standard deviation [12,13]. In other words, the probability density function may be represented by a lognormal distribution as in Eq. (8):

$$f_{R}(r) = \frac{1}{\sqrt{2\pi}\xi r} \exp\left\{-\frac{1}{2} \left[\frac{\ln r - \lambda}{\xi}\right]^{2}\right\}$$
 (8)

where λ and ξ are the logarithmic mean and the standard deviation of fragility function f_R (r), or the resistance of the structure against earthquake forces. The typical form of the fragility function is shown in Figure 3 [12].



SMRFS = SPECIAL MOMENT RESISTANT FRAME SYSTEM
IMRFS = INTERMEDIATE MOMENT RESISTANT FRAME SYSTEM

Figure 3 Fragility function for SMRFS and IMRFS [12].

The statistics of resistance are measured by applying an incremental nonlinear time history dynamic analysis from a set of recorded earthquakes to a structure until the structure collapses at a certain value of PGA of the corresponding earthquake. If p_i is the corresponding PGA of the earthquake when the structure collapses, then the mean value of p_i is given by Eq. (9):

$$\mu = \frac{1}{n} \sum_{i}^{n} p_{i} \tag{9}$$

And the variance of p is given by Eq. (10):

Var (p) =
$$\frac{1}{n-1} \sum (p_i - \mu) 2$$
 (10)

And the corresponding standard deviation and coefficient of variation of p are given as in Eq. (11):

$$\sigma = \sqrt{\text{Var}(p)} \tag{11}$$

and Eq. (12):

$$\Omega_{\rm p} = \frac{\sigma}{\mu} \tag{12}$$

Hence, the logarithmic mean value can be written as in Eq. (13):

$$\lambda = \ln \mu - 0.5 \ln(1 + \Omega_{\rm p}^2) \tag{13}$$

and the probability density function of R may be written as in Eq. (14):

$$f_{R}(r) = \frac{1}{\sqrt{2\pi}\xi r} \exp\left\{-\frac{1}{2} \left[\frac{\ln r - \ln \mu + 0.5 \ln(1 + \Omega_{p}^{2})}{\xi} \right]^{2} \right\}$$
 (14)

And the final form of annual risk integral may be represented by:

Pan =
$$\int P(Y > y \mid r) \frac{1}{\sqrt{2\pi}\xi r} \exp \left\{ -\frac{1}{2} \left[\frac{\ln r - \ln \mu + 0.5 \ln(1 + \Omega_p^2)}{\xi} \right]^2 \right\}$$
 (15)

Evaluation of Eq. (15) depends on the hazard analysis for a specific area and the statistics of resistance in terms of logarithmic mean λ and logarithmic standard deviation ξ . The values of ξ depend on record-to-record variation, the amount of data used in the analysis, material variability, and the structure model error given by [13].

$$\Omega_R^2 = \Omega_D^2 + \Omega_D^2 + \Omega_S^2 + \Omega_M^2 \tag{16}$$

Since R follows a logarithmic distribution, ξ may be calculated as:

$$\xi = \sqrt{\ln(1 + \Omega_{\rm R}^2)} \tag{17}$$

where in Eqs. (16) and (17), Ω_R = coefficient variation of R, Ω_p = correction due to record-to-record variability, Ω_D = correction due to limited amount of data, Ω_S = correction due to structure model error, and Ω_M = correction due to material variability. Luco, *et al.* [2] proposed a ξ equal to 0.8, although they also used ξ = 0.6 without showing significantly different results with ξ = 0.8. Some of the students' final projects at the Department of Civil Engineering, Institut Teknologi Bandung, e.g. Yogi [12], evaluated ξ for Special Moment Resisting Frame System and Intermediate Moment Resisting Frame System for buildings designed according to the Indonesian Code, and found ξ = 0.6 and ξ = 0.62,

respectively for the city of Jakarta (Figure 1). Haselton and Deierlein [14] proposed a ξ value of 0.65, which already included the structure modeling error. Liel, *et al.* [10] calculated a ξ value of 0.624 for a 4-story reinforced concrete frame. The Indonesian Team for new provisions in [15] suggests ξ values between 0.65 and 0.70.

Clearly, a uniform hazard concept with an expected return period of 2500 years does not produce structures with a uniform risk or the same probability of failure due to uncertainties in the building's capacity. Consequently, the expected return period of each site is not 2500 years, it varies depending on the seismic characteristics of a site and the level of uncertainty involved in resistance against earthquakes.

4.2 Life Time Risk and Annual Risk

Assuming the occurrences of consecutive earthquakes are independent random variables, the relationship between lifetime risk and annual risk for lifetime T may be expressed as one minus probability of no collapse within time T, given as:

P (collapse in T) =
$$1 - [1 - P_{an}]^T$$
 (18)

For small values of P_{an}, Eq. (18) can be written as:

$$P \text{ (collapse in T)} \approx T \times Pan$$
 (19)

Eqs. (15) to (19) provide a systematic way for calculating the risk or probability of failure of a structure subjected to earthquake loads for a certain lifetime in order to design for a targeted risk. To account for the importance of a structure, the SNI and ASCE code introduced the use of the importance factor to lower the earthquake risk. However, the risk level remains unknown. By applying the proposed method one can evaluate the risk and make proper adjustment to the geometry and dimensions until a certain risk is achieved, e.g. 10^{-3} for a 50-year lifetime or a longer design lifetime.

4.3 Return Period

The return period of a certain earthquake with probability of exceedance P_{an} can be modeled as a Bernoulli sequence with a geometric distribution, given by Eq. (20):

$$(T=t) = P_{an}(1 - P_{an})^{t-1}$$
(20)

in which the first occurrence of a certain earthquake is realized on the t^{th} trial. In other words, there must be no occurrence of this particular earthquake in any of the prior (t-1) trials.

The mean value of Eq. (20) is given by Eq. (21):

$$\overline{T} = E(T) = \frac{1}{P_{an}}$$
 (21)

Strictly speaking, we do not know the exact return period for a certain earthquake since the return period is a random variable, but we do know the expected value of the return period or the mean value of t.

5 Risk Evaluation of a 25-story Building with a Dual System Structure

To demonstrate the usefulness of the method, a dual system structure consisting of four shear walls and a frame with a height of 100 m was evaluated (Figure 4). The size of the floor plan was 32.5 m width by 35 m length. The fundamental periods of the building were 2.16 seconds in the strong direction and 5.2 seconds in the weak direction respectively. Moreover, 60% of the base shear was resisted by the shear walls and the other 40% by the moment frame system. Then, the capacity of the structure was determined by observing the PGA that causes the collapse prevention state at a particular member of the structure subjected to historical records by using nonlinear time history analysis. The observed capacity was obtained by performing incremental time history analysis.

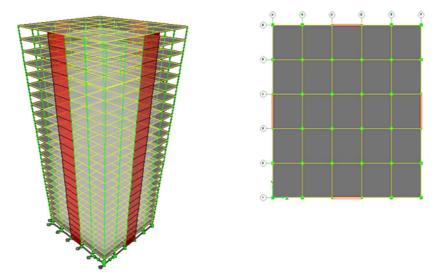


Figure 4 25-story building with a dual system structure.

Table 1 shows the results of the capacity in terms of PGA for 10 historical records. Observe that the ξ value is 0.68, which is rather close to 0.70, the value

assumed in the Indonesian Code. The statistics of the fragility function are summarized in Table 2; they were assumed to follow a lognormal distribution.

Year	Earthquake	PGA (g)	Scale	Xi	$(x_i-x_{bar})^2$
1995	Mexico	0.0998	1.8	0.17964	0.591361
1968	Hyuganda	0.3698	0.5	0.1849	0.583298
1978	Miyagi	0.3251	0.7	0.22757	0.519942
1971	San Fernando	0.2547	2.2	0.56034	0.150777
1940	El Centro	0.3569	2.6	0.92794	0.000428
1989	Loma Prieta	0.2755	3.7	1.01935	0.004999
1994	Northridge	0.6038	1.7	1.02646	0.006056
1979	James RD El Centro	0.5952	2.3	1.36898	0.176669
1995	Kobe	0.8211	1.8	1.47798	0.280201
1952	Taft Lincoln	0.2371	10.6	2.51326	2.448036

Table 1 Nonlinear time history results for 10 historical records.

Table 2 Statistics of fragility function or capacity in terms of PGA.

n	10
μ_{x}	0.9486
Var[x]	0.529
σ_{x}	0.7274
Ω_{x}	0.767
$\zeta_{\rm x}$	0.68
λ_{x}	-0.284

The annual risk was then evaluated using Eq. (15) and the lifetime risk was evaluated using Eq. (18), accordingly. The lifetime risk is 0.017×10^{-2} and 3.71×10^{-3} for buildings located in Jakarta and Surabaya respectively. If in the analysis the limited amount of data and the model errors are included in the calculation, then the risk would be 0.022×10^{-2} and 5.3×10^{-3} , an increase between 30% and 43%. It is observed that plastic hinges occur only at the beams, obviously a consequence of the strong-column weak-beam concept, which means that the capacity of the structure is determined by the ability of the beams to form plastic hinges until it reaches the collapse prevention state.

6 Conclusions

There are many uncertainties involved in the collapse capacity of a structure, which cannot be avoided due to record-to-record variability, structure idealization, material variability, and limited data used in the analysis. Consequently, the previous code of 2002 did not produce structures with equal

risk subjected to earthquake loads, even though they are subject to the same seismic hazard level.

For tall buildings the reliability of a structure against earthquake loads should be explicitly evaluated based on available earthquake data, designed lifetime, and a certain targeted risk that reflects the importance of the specific super tall building. Much work is needed until we can come up with a consensus on designed lifetime and design reliability or acceptable risk for tall buildings.

In this paper, a procedure for evaluating the uncertainties involved in designing a dual system structure was introduced using the first-order of second-moment method. By combining the annual exceedance from PSHA and the fragility function obtained from incremental nonlinear time history dynamic analysis, the reliability of a structure subjected to earthquake loads may be evaluated. The method may be repeated until a certain predetermined target is achieved. It was demonstrated that a dual system structure designed using the new Indonesian Code reaches a target of 1% risk for a 50-year lifetime.

References

- [1] ASCE, ASCE Standard Minimum Design Loads for Buildings and Other Structures, ASCE/SEI 7-10, American Society of Civil Engineers, Reston, Virginia, 2010.
- [2] Luco, N., Ellingwood, B.R., Hamburger, R.O., Hoope, J.D., Kimball, J. K. & Kircher, C.A., *Risk-Targeted versus Current Seismic Design Maps for the Conterminous United States*, SEAOC 2007 Convention Proceedings, pp. 1-13, 2007.
- [3] Ang, A.H-S. & Tang, W.H., Probability Concepts in Engineering Planning and Design Volume I Basic Principles, 2nd ed., John Wiley, pp. 183-190, 2007.
- [4] Rubinstein, R.Y., *Simulation and the Monte Carlo Method*, John Wiley, pp. 86-91, 1981.
- [5] Ang, A.H-S. & Tang, W.H., Probability Concepts in Engineering Planning and Design volume II Decision, Risk, and Reliability. John Wiley, 274-296, 1990.
- [6] Cornell, C.A., *Engineering Seismic Risk Analysis*, Bulletin of the Seismological Society of America, **58**(5), pp. 1583-1606, 1968.
- [7] Irsyam, M., Dangkua, D.T., Hendriyawan, H., Boediono, B., Kusumastuti, D. & Kertapati, E.K., *Proposed site-specific response spectra for Surabaya-Madura Bridge*, Civil Engineering Dimension 10, **2**, pp. 65-70, 2008.
- [8] McGuire, R.K., *Seismic Hazard and Risk Analysis*, Earthquake Engineering Research Institute, pp. 163-171, 2004.

- [9] National Standard of Indonesia, *SNI 1726:2012* Recommendation for Earthquake Resistant Design for Buildings and Civil Structures, 2012.
- [10] Liel, A., Haselton, C., Deierlein, G. & Baker, J., Assessing the Collapse Risk of RC Frame Structures, Including the effects of Modeling Uncertainties, Special Workshop on Risk Acceptance and Risk Communication, Stanford University, 2007.
- [11] Govindan, N.D., Behavior of Reinforced Concrete Dual Structural System: Strength, Deformation Characteristics, and Failure Mechanism, IACSIT International Journal of Engineering and Technology, 5(1), p. 14, February 2013.
- [12] Yogi, M.R.A., Comparison Study of Fragility Curve for Special Resisting Moment Frame for Jakarta Region Subjected to Earthquake with 2500 Years Return Period, Master Thesis, Department of Civil Engineering, ITB, Bandung, 2012.
- [13] Porter, K., Hamburger, R. & Kennedy, R., *Practical Development and Application of Fragility Function*, Structural Engineering Research Frontiers, http://dx.doi.org/10.1061/40944/249/23, June 20, 2012.
- [14] Haselton, C.B. & Deierlein, G.G., Assessing Seismic Collapse Safety for Modern RC Moment Frame Buildings, Pacific Earthquake Engineering Research Center, University of California, Berkeley, February 2008.
- [15] Sengara, I.W., Irsyam, M., Sidi, I.D., Merati, W., Pribadi, K.S., Suarjana, M. & Edwards, M., Some Recent Efforts in earthquake Hazard and Risk Analyses for Disaster Risk Reduction in Indonesia, Proceedings, 2nd International Conference on Earthquake Engineering and Disaster Mitigation for Advancing Earthquake Safety of Structures, Tavio, 2011.