Prediction Model of Coal and Gas Outburst Based on Rough Set-Unascertained Measure Theory

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Abstract. This paper proposes a risk evaluation model based on rough sets (RS) and the unascertained measure theory (UMT) for solving the accuracy problem of coal and gas outburst prediction with the aim to reduce economic losses and casualties in coal mining. The coal and gas outburst prediction problem is constrained by the selection of the prediction indexes, the coupling of a single index, and the weight of each index. The proposed RS-UMT model applies two modified techniques. The first one is a method for index weight determination that was improved by rough set theory. The second one is a method for coupling a single index that was modified by the unascertained measure theory. The RS-UMT model not only well solves the problem of coupling a single index of coal and gas outbursts, but also solves the problem that the weight is susceptible to subjective factors and prior knowledge. The RS-UMT model was used to judge the risk degree of outburst of 10 mining faces in the Pingdingshan No. 8 Mine and No. 10 Mine. The predictive results of the model were basically identical to the actual measured results. The performance of the RS-UMT model was also compared to existing methods. Based on the case study it can be concluded that the RS-UMT model is an accurate and very promising method for solving the coal and gas outburst prediction problem.

Keywords: *coal and gas outburst; geodynamic; prediction; rough set; unascertained measure.*

1 Introduction

Coal and gas outburst is a complex geodynamic phenomenon in coal mines, which may lead to the projection of large volumes of fragmented coal and gas into the excavation space [1-3]. The occurrence of coal and gas outburst not only causes damage to equipment but can also cause a serious threat to the safety of miners. A primary task of safety management in coal mines is to control the risk of coal and gas outburst accidents. Predicting coal and gas outburst is an extremely effective method. Currently, the so-called 'comprehensive hypothesis' provides the mechanism of coal and gas outburst most widely accepted by researchers around the world. This hypothesis proposes ground stress, gas pressure and content, and coal physical properties as

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the most important elements that control the occurrence of coal and gas outburst [1-3]. Based on this theory, in recent years a considerable number of prediction methods have been proposed in the literature to predict a risk rating for such outbursts, including single-factor methods, complicated index methods, an artificial neural network (ANN) method, the fuzzy comprehensive evaluation (FCE) method, and the gray theory prediction (GTP) method. The single-factor methods [4,5] and the complicated index methods [4] are mainly based on the experience of field technicians and provide a simple and quick solution, but the accuracy of the results is too poor to fully meet the prediction requirements. Although the artificial neural network (ANN) method [6-7] can improve the prediction accuracy, its computational velocity is not high enough and its convergence is also poor. An improved ANN method [8] had been proposed by using fault tree theory. This method can enhance the computational velocity and convergence, but using fault tree to select prediction indexes may lead to missing some. The fuzzy comprehensive evaluation [9,10] may generate more accurate predictive results, but it requires an accurate weight of each evaluation index, which is susceptible to subjective factors and prior knowledge. The gray theory prediction [11] also has a weakness: it needs an equal-internal data set to initiate the computation and analysis. The BP neural network [12,13] has been proven to be effective for predicting outburst, but the studies provided few predictive samples. The extension theory [14] yielded accurate predictive results, but there was no result comparison in this study. A support vector machine was proposed to predict outburst by Chen, *et al.* [15], but this method needs large-scale sample data, which makes modeling more difficult. A genetic algorithm has been proposed by Wang, *et al.* [16], however, it has a strong dependence on the selection of the initial population.

The emergence and development of coal and gas outburst are affected and controlled by many factors, including geological structure, gas pressure, in-situ stress, and coal mechanical properties [17,18], where quantitative factors coexist with qualitative factors [8]. The prediction of coal and gas outburst is a multi-index decision-making process with great uncertainty and ambiguity, therefore a method that can handle a large amount of uncertain information needs to be applied. The unascertained measure theory (UMT) developed by Wang [19] and Liu [20] is a method that can reasonably integrate qualitative factors with quantitative factors [21,22] and improve the reliability and accuracy of risk prediction by a comprehensive quantitative analysis. Thus, this method can well meet the above requirements. Wang [22] applied the unascertained measure theory to evaluate highway traffic efficiency. It has also been applied to evaluate the bench blasting effect by Lei, *et al.* [23] and it was proven to be effective for this problem. Li, *et al.* [24] established an unascertained measure model to evaluate water and mud inrush risk; the evaluation results were in good agreement with the actual results. The unascertained measure theory has also been applied successfully in the evaluation of construction safety of high-rise buildings [25], mining resource environments [26], safety area delimiting of coal mining [27], and safety risk evaluation in mine construction sites [28].

However, using only the unascertained measure theory is not sufficient to identify the complex index systems of outburst. The degree of reliability of the index weight is critical to the accuracy of the evaluation results. At present, the methods for determining the index weight can be divided into subjective weighting methods and objective weighting methods. Expert scoring and analytic hierarchy process are examples of commonly used subjective weighting methods. Principal component analysis, gray relational analysis, and entropy weighting are examples of commonly used objective weighting methods. Both weighting methods have their own weaknesses: the results of subjective weighting are susceptible to subjective factors because the weight of the index is determined by an expert or an expert team, while objective weighting relies on prior knowledge of the decision maker in its application. Rough set theory was first introduced by Pawlak in 1982. It can obtain an objective weight without any prior knowledge of the decision maker and eliminate the impact of subjective factors. In less than four decades, rough set theory has rapidly established itself in many real-life applications, such as control algorithm acquisition and process control [29-30], malware analysis [31], and the automotive industry [32]. This paper proposes to ascertain the weight of the index system from the unascertained measure theory by rough set theory.

In view of the objective of rough set theory and the accuracy of the unascertained measure method, a new prediction model based on a coupled application of rough set theory and the unascertained measure method is proposed in this paper. This model not only well solves the problem of coupling a single index of coal and gas outburst, but it also improves the prediction accuracy by solving the problem that the weight is susceptible to subjective factors and prior knowledge. The new model uses the unascertained measure method to construct an unascertained function of the optimal index and predicts the degree of coal and gas outburst risk according to the confidence criterion. This model was applied to the Pingdingshan No. 8 Mine and No. 10 Mine. The results showed that the model could accurately predict the risk of coal and gas outburst.

2 Establishment of the RS-UMT model

Set $\xi_1,\xi_2,\ldots,\xi_n$, as n evaluated mining faces, then $\xi = \{\xi_1,\xi_2,\ldots,\xi_n\}$ is the evaluation object collection. For each evaluation object $\xi_i \in \xi (i = 1,2,\ldots,n)$, there are m individual evaluation index spaces

$Y_1,Y_2,\ldots,Y_m$, and $Y = (Y_1,Y_2,\ldots,Y_m)$. Then ξ_i can be expressed as an *m* dimensional vector $\xi_i = (y_{i1}, y_{i2},\ldots,y_{im})$, where y_{ij} is the measured value of the coal and gas outburst evaluation index. As for each $y_{ij}(i = 1,2,\ldots,n; j = 1,2,\ldots,m)$, suppose there are *p* risk levels, the evaluation space can be recorded as *d*, $d = (d_1,d_2,\ldots,d_p)$. Set $d_k(k = 1,2\ldots,p)$ to the k^{th} risk level and the risk degree of $(k+1)^{th}$ is lower than k^{th}, which can be denoted as $d_k > d_{k+1}$. If it meets $d_1 > d_2 > \cdots > d_p$ or $d_1 < d_2 < \cdots < d_p$, $\{d_1,d_2,\ldots,d_p\}$ is an ordinal classification of evaluation space ξ.

2.1 Single Index Measure Evaluation Vector

If $\mu_{ijk} = \mu(x_{ij} \in d_k)$ indicates the level at which y_{ij} belongs to d_k, and μ must meet the following equations :

$$0 \le \mu(y_{ij} \in d_k) \le 1 \tag{1}$$

$$\mu(y_{ij} \in D) = 1 \tag{2}$$

$$\mu\left[y_{ij} \in \bigcup_{s=1}^{k} d_s \right] = \sum_{s=1}^{k} y_{ij} \in d_s\,, \quad (k = 1,2,\cdots,p) \tag{3}$$

Eqs. (1), (2) and (3) are respectively defined as non-negative boundedness, normalization and additivity. Satisfying Eqs. (1), (2) and (3) simultaneously can be defined as unascertained measurement. The following matrix is a single indicator measure evaluation matrix. The j^{th} row vector $\mu_{ij}(\mu_{j1},\mu_{j2},,\ldots,\mu_{jp})$ of the matrix is an evaluation vector of the single index measure of x_{ij}. Before building the matrix $(\mu_{ijk})_{m\times p}$, a single index measure function must be set up. At present, the straight line method, the parabolic curve method, the quadratic curve method, and the sinusoidal curve method are typical measurement function construction methods. Among them, the linear measure function is the most widely used unascertained measure function, so a linear unascertained measure function was adopted in this study. A graph of the linear unascertained measure function is shown in Figure 1. The expression of the linear unascertained measure function on interval $[a_i,a_{i+1}]$ is expressed as follows:

$$\begin{cases} \mu_i(x) = \begin{cases} \dfrac{-x}{a_{i+1}-a_i} + \dfrac{a_{i+1}}{a_{i+1}-a_i} & (a_i < x \le a_{i+1}) \\ 0 & (x > a_{i+1}) \end{cases} \\ \mu_{i+1}(x) = \begin{cases} 0 & (x \le a_i) \\ \dfrac{x}{a_{i+1}-a_i} - \dfrac{a_i}{a_{i+1}-a_i} & (a_i < x \le a_{i+1}) \end{cases} \end{cases} \tag{4}$$

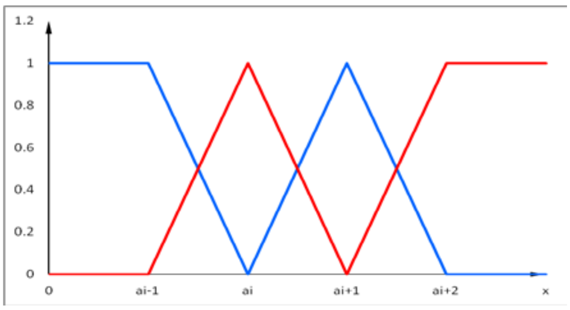

Figure 1 Graph of the linear unascertained measure function.

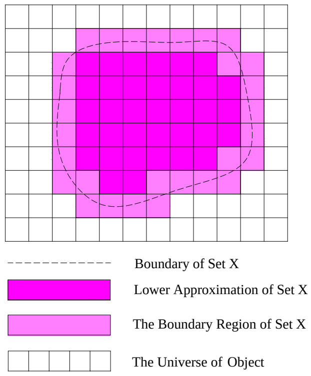

Figure 2 A rough set.

2.2 Determination Weight of the Prediction Indexes

In this study, the importance of each index of coal and gas outburst was calculated by using rough set theory. Supposing that the knowledge sets can be represented with a four-group dataset $S=(U,R,V,f)$, where U is the whole domain, i.e. a finite set m of objects $\{X_1,X_2,L,X_m\}$, $R=C\bigcup D$ is the collection of attribute variables and the subsets C and D are respectively defined as conditions attributes and decision attributes, $V=\bigcup_{r\in R}V_r$ is the collection of the attribute values, where V_r indicates the attribute value of $r\in R$, $f:U\times R\to V$ is an information function such that $f(x,a)\in V_r$ for every $a\in R, x\in U$. The indistinguishable relation $ind(R)$ is the equivalence relation defined by attribute set R on U:

$$ind(R)=\{(x,y):x,y\in R\wedge f(x,R)=f(y,R)\} \tag{5}$$

$U/ind(R)$ represents the set of all equivalence classes divided by the equivalence relation $ind(R)$. For any concept $X\in U$ and attribute subset $A\in R$, X is approximated by the R-lower approximation and the R-upper approximation using the knowledge of A. The lower approximation of X is the collection of elements in U that surely belong to X, defined as:

$$\underline{R}(X)=\bigcup\{Y\in U/R\,|\,Y\subseteq X\} \tag{6}$$

The upper approximation of X is the collection of elements in U that possibly belong to X, defined as:

$$\overline{R}(X)=\bigcup\{Y\in U/R \mid Y\cap X\neq \Phi\} \tag{7}$$

The boundary region may be defined as: $BND_R(X)=\overline{R}(X)-\underline{R}(X)$. The elements in the boundary region may or may not belong to X on the basis of the available information, therefore the knowledge about the boundary region is vague. The number of elements in this region can serve as a measure of the uncertainty. Correspondingly, $pos_R=\underline{R(X)}$ is called the R positive domain of X. The above definitions are clearly depicted in Figure 2. Take $K=(U,S)$ as the knowledge base, and $P,Q\subseteq R$, if $k=\gamma_P(Q)=|pos_P(Q)|/|U|$ it can be guided as Q is dependent on knowledge P at degree k. Here, $|U|$ is the number of elements in U, $|pos_P(Q)|$ is the number of elements in $pos_P(Q)$. The importance of the attribute subset $C_i\subseteq C$ for D may be defined as:

$$Sig(C_i)=\gamma_C(D)-\gamma_{C-\{C_i\}}(D)=\left\| pos_C(D)\right| - \left| pos_{C-C_i}(D)\right\| / |U| \tag{8}$$

The greater the value of $Sig(C_i)$, the greater the importance of property C_i. According to the concept of attribute importance of rough set theory, the importance of each index can be obtained, the weight of each index ω_{C_i} can be obtained by normalizing the importance of each index, and finally ω_{C_i} can be calculated with Eq. (9):

$$\omega_{C_i}=Sig(C_i)/\sum_{i=1}^{n}Sig(C_i) \tag{9}$$

The weighting results of the rough set are taken as the index weights of the unascertained measure. According to Eq. (9), the weight of the index can be expressed as follows:

$$\omega_C=(\omega_{C_1},\omega_{C_2},\cdots,\omega_{C_m}) \tag{10}$$

Obviously, $0\le\omega_{C_j}\le 1(j=1,2,\cdots,m)$, $\sum_{j=1}^{m}\omega_{C_j}=1$.

2.3 Multi-Index Comprehensive Measure Evaluation Vector

Based on the single-index measure matrix and the weight of each index, the multi-index comprehensive measure evaluation vector and matrix respectively can be obtained as follows:

$$\mu_i=\left\{\mu_{i1},\mu_{i2},\cdots,\mu_{ip}\right\}$$

$$(\mu_{ik})_{n\times p}=\begin{bmatrix}\mu_{11} & \mu_{12} & \cdots & \mu_{1p}\\ \mu_{21} & \mu_{21} & \cdots & \mu_{2p}\\ \vdots & \vdots & \vdots & \vdots\\ \mu_{n1} & \mu_{n2} & \cdots & \mu_{np}\end{bmatrix}$$

$$\mu_{ik}=\sum_{j=1}^{m}\omega_j\mu_{ijk}(i=1,2,\cdots,n;k=1,2,\cdots,p) \quad (11)$$

μ_i expresses the unascertained risk grade. In order to gain the certainty risk grade, a risk evaluation is needed. Here $\mu_{ik}=\mu(R_i\in d_k)$ indicates the level at which ξ_i belongs to d_k, and d_k is the k^{th} risk grade.

2.4 The Confidence Recognition Criteria

If $d_1>d_2>\cdots>d_p$ or $d_1<d_2<\cdots<d_p$, the confidence recognition criteria are brought in. Set λ as the confidence, where $0.5\le\lambda<1$. Normally λ=0.6 or λ=0.7 ; this paper took λ=0.6.

If $$k_0=\min\left\{k:\sum_{s=1}^{k}\mu_{is}(d_i)\ge\lambda,1\le k\le p\right\} \quad (12)$$

then ξ_i belongs to grade d_{ki}.

3 Engineering Application of the RS-UMT Model

3.1 General Situation in the Research Region

In order to verify the accuracy and reliability of the Rough Set-Unascertained Measure Theory Model, the model was applied to evaluate the risk of coal and gas outburst in Pingdingshan No. 8 Mine and No. 10 Mine. The mines are located in the eastern part of the Pingdingshan mining area, which is located in the middle of Henan Province. There have occured 45 outburst accidents during the mining of these coal seams in the No. 8 Mine. Since the first coal and gas outburst happened in April 1988, 51 outburst accidents have occurred in the No. 10 Mine. After a statistical analysis of 96 coal and gas outburst cases in the two mines, the following common outburst features were obtained: (1) outburst accidents always took place in the tunneling working face; (2) most of the outburst accidents occurred in the geological tectonic zone, especially the effect of small faults on the outburst is obvious; (3) the phenomenon of abnormal gas emission always appeared before the occurrences of outburst, which indicates gas pressure and gas content were key factors influencing the coal and gas outburst; (4) tectonic coal is widely developed in the outburst accident regions.

Both the main mining coal and coal types are the same in the two mines. Furthermore, the geological conditions of the two mines are similar (medium complex). Therefore, The No. 8 Mine and No. 10 Mine have been taken as an integral whole in a number of published researches [35,36]. The more raw data the rough sets contain, the more accuratetely the weight can be determined. In order to ensure the accuracy of the weight obtained by rough set theory, sufficient information on coal and gas outbursts is needed, so in this study the No. 8 Mine and the No. 10 Mine were taken as a whole object of study.

3.2 Prediction Index System and Risk Classification Criteria of Coal and Gas Outburst

Based on the 'comprehensive hypothesis' and the tectonophysics of coal and gas outburst, combined with the outburst features of the two mines, a prediction index system was set up in this study, which included gas pressure (C_1), gas content (C_2), mining depth (C_3), geological structure (C_4), Protodyakonov's coefficient of coal (C_5), coal seam thickness (C_6) [17,18]. The prediction index of coal and gas outburst can be classified into two types of parameters. The first are quantitative and include the maximum gas pressure, the largest gas content, the maximum mining depth, the minimum firmness coefficient of coal, the average coal seam thickness, and the maximum amount of protruding coal. These parameters can be measured directly or indirectly by instruments. The second type of parameters are qualitative, e.g. the geological structure. The determination of such indicators is mainly based on the experience of mine geological engineers and in-situ research studies. According to the distribution of folds and faults, the fragmentation coal roof and floor, and the change of coal thickness in the forecasted region, the degree of geological structure complexity can be obtained by semi-quantifiable methods and may be classified into four grades: simple structure, little complex, medium complex, and very complex. According to the geological characteristics in the Pingdingshan No. 8 Mine and No. 10 Mine, referring to the researches by He and Zhang [7,8], the grading standard of the coal and gas outburst index was established as shown in Table 1. According to rough set theory, C_1, …, C_6 are condition attributes, and D is a decision attribute.

The risk level of coal and gas outburst is divided into four levels in this paper. The 'safe' level indicates that it is safe to carry out mining excavation. The 'low risk of outburst' level indicates that abnormal gas emission may appear frequently in the mining region; an outburst of throwing out less than 50 ton of coal may occur at this level. The 'medium risk of outburst' level indicates that an outburst throwing out 50-300 ton of coal may occur; the outburst is always accompanied by a large amount of methane at this level. The 'high risk of outburst' level indicates that an outburst throwing out more than 300 ton of coal may occur; the outburst is always accompanied by a large amount of methane at this level. The four risk levels of coal and gas outburst are denoted as 1-4 successively, as shown in Table 1.

Table 1 Prediction indexes and grading standards of coal and gas outburst in pingdingshan mining area.

Prediction indexes	**Safe level 1**	**Low risk of outburst level 2**	**Medium risk of outburst level 3**	**High risk of outburst level 4**
gas pressure （C_1/MP）	C_1<0.6	0.6 ≤C_1<1	1≤C_1<2	C_1≥2
gas content (C_2/m3/t ）	C_2<6	6 ≤C_2<10	10≤C_2<20	C_2≥20
mining depth (C_3/m)	C_3<300	300 ≤C_3<500	500 ≤C_3<800	C_3≥800
geological structure (C_4)	Simple structure	Little complex	Medium complex	Very complex
Protodyakonov's coefficient of coal (C_5)	C_5≥0.5	0.3≤C_5<0.5	0.15≤C_5<0.3	C_5<0.15
Coal seam thickness (C_6/m)	C_6<2	2≤C_6<3	3≤C_6<4	C_6≥4
Outburst strength (D/t)	D = 0	0<D<50	50≤D<300	D≥300

3.3 Determination of the Index Weight

Twelve outburst events of the No. 8 Mines and 15 outburst events of the No. 10 Mine were randomly selected as the original knowledge collection of the rough sets, respectively represented by $\xi_1, \xi_2,...,\xi_{12}$ and $\zeta_1, \zeta_2,..., \zeta_{15}$ (Table 2). According to the original outburst data, the objective weight of each outburst indicator was calculated by rough set theory. The knowledge system of coal and gas outburst is represented in tabular form in Table 2, where the columns of the table indicate 23 coal and gas outburst examples in Pingdingshan No. 8 Mine and No. 10 Mine, which were the object of the evaluation; the line indicates the coal and gas outburst prediction index system, which represents the attributes. As Table 2 shows, C_1, C_1, …, C_6 are the condition attributes of the information system, D is the decision attribute. On the basis of the discretization criterion in Table 1, the sample data of coal and gas outburst in Table 2 were discretized. The results are shown in Table 3.

Table 2 Data set of coal and gas outburst example.

Data set no.	C_1/MPa	C_2/m^3/t	C_3/m	C_4	C_5	C_6/m	D/t
ξ_1	1.0	11	420	Medium complex	0.12	4.3	48
ξ_2	1.2	14	515	Very complex	0.24	4.5	15
ξ_3	2	18	568	Very complex	0.19	4.5	76
ξ_4	1.6	13	730	Medium complex	0.16	4.5	28
ξ_5	0.55	8	510	Little complex	0.26	4.2	0
ξ_6	1.12	9	687	Little complex	0.15	4.6	19
ξ_7	1.5	20	735	Very complex	0.14	3.5	326
ξ_8	0.8	9	520	Little complex	0.28	3.5	0
ξ_9	2.4	20	913	Medium complex	0.11	3.5	200
ξ_{10}	1.3	20.5	767	Medium complex	0.13	3.5	683
ξ_{11}	2.4	18	896	Very complex	0.13	3.5	159
ξ_{12}	2.7	21	937	Very complex	0.11	3.5	2000
ζ_1	0.9	14	487	Very complex	0.13	3.95	20
ζ_2	0.8	12	529	Medium complex	0.35	4	5
ζ_3	1.2	15	537	Very complex	0.2	3.9	48
ζ_4	1.9	21	840	Medium complex	0.13	4.2	551
ζ_5	0.9	5.8	415	Medium complex	0.25	4.4	0
ζ_6	0.9	13	520	Very complex	0.25	4.3	45.5
ζ_7	1.0	16	614	Little complex	0.3	4.5	7
ζ_8	1.8	20	697	Little complex	0.35	4.2	14
ζ_9	1.2	16	622	Little complex	0.2	4.3	64
ζ_{10}	1.8	11	424	Medium complex	0.42	3.5	6
ζ_{11}	1.9	21	566	Medium complex	0.17	3.9	240
ζ_{12}	0.5	8.3	350	Little complex	0.46	4.2	0
ζ_{13}	1.9	12	485	Very complex	0.12	3.8	450
ζ_{14}	0.95	12	490	Very complex	0.14	4.4	478
ζ_{15}	2.1	20	589	Very complex	0.09	3.9	215

According to Table 3, the following knowledge expression can be obtained:

$$pos_C(D)=\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27\}$$

$$pos_{C-\{C_1\}}(D)=\{1,4,8,9,10,11,12,14,15,16,17,19,20,22,23,24,26,27\}$$

$$pos_{C-\{C_2\}}(D)=\{1,2,3,4,5,7,8,9,10,13,14,15,16,17,18,19,20,22,23,24,25,26,27\}$$

$$pos_{C-\{C_3\}}(D)=\{1,2,3,4,5,6,7,8,9,10,11,13,14,15,16,17,18,19,20,21,22,23,24,25,26\}$$

$$pos_{C-C_4}(D)=\{1,3,5,6,7,8,10,11,13,14,15,16,17,18,19,20,23,24,26,27\}$$

$$pos_{C-C_5}(D)=\{1,2,3,4,5,6,7,8,9,11,12,13,14,15,16,17,18,20,22,24,25,26,27\}$$

$$pos_{C-C_6}(D)=\{1,2,3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,25,27\}$$

$$Sig(C_1)=\frac{\left\|pos_C(D)\right|-\left|pos_{C-C_1}(D)\right\|}{|U|}=\frac{27\text{-}18}{27}=\frac{9}{27}$$

Similarly, $Sig(C_2)=\frac{4}{27}$, $Sig(C_3)=\frac{2}{27}$, $Sig(C_4)=\frac{7}{27}$, $Sig(C_5)=\frac{4}{27}$, $Sig(C_6)=\frac{2}{27}$.

Furthermore, the degree of importance of each conditional attribute is normalized according to Eq.(9). The weights of the indicators of coal and gas outburst respectively are calculated as follows:

$$\omega_{C_1} = Sig(C_1) / \sum_{i=1}^{6} Sig(C_i) = 0.3214$$

Similarly, $\omega_{C_2} = 0.1429$, $\omega_{C_3} = 0.0714$, $\omega_{C_4} = 0.25$, $\omega_{C_5} = 0.1429$, $\omega_{C_6} = 0.0714$.

Table 3 Discretization of coal and gas outburst data.

	C_1/MPa	C_2/m^3/t	C_3/m	C_4	C_5	C_6/m	D/t
ξ_1	3	3	2	3	4	4	2
ξ_2	3	3	3	4	3	4	2
ξ_3	4	3	3	4	3	4	3
ξ_4	3	3	3	3	3	4	2
ξ_5	1	2	3	2	3	4	1
ξ_6	3	2	3	2	3	4	2
ξ_7	3	4	3	4	4	3	4
ξ_8	2	2	3	2	3	3	1
ξ_9	4	4	4	3	4	3	3
ξ_{10}	3	4	3	3	4	3	4
ξ_{11}	4	3	4	4	4	3	3
ξ_{12}	4	4	4	4	4	3	4
ζ_1	2	3	2	4	4	3	2
ζ_2	2	3	3	3	2	2	2
ζ_3	3	3	3	4	3	3	2
ζ_4	3	4	4	3	4	4	4
ζ_5	2	1	2	3	3	4	1
ζ_6	2	3	3	4	3	4	2
ζ_7	3	3	3	2	2	4	2
ζ_8	3	4	3	2	2	4	2
ζ_9	3	3	3	2	3	4	3
ζ_{10}	3	3	2	3	4	3	2
ζ_{11}	3	4	3	3	3	3	3
ζ_{12}	1	2	2	2	2	4	1
ζ_{13}	3	3	2	4	4	3	4
ζ_{14}	2	3	2	4	4	4	4
ζ_{15}	4	4	3	4	4	3	3

3.4 Construction of Single Index Measure Evaluation Vector and Matrix

In order to verify the accuracy of the RS-UMT prediction model, the geological and gas data of the 10 sampling locations were collected from the previously mined area in Pingdingshan No. 8 Mine and No. 10 Mine, where ξ_{13}, ξ_{14}, ..., ξ_{17} are from the No. 8 Mine and ζ_{16}, ζ_{15}, ..., ζ_{20} are from the No. 10 Mine. These areas have detailed gas, geological and coal seam data (Table 4). Furthermore, these sample points do not contain the 23 sampling points that were used previously to determine the weight of the index by the rough set theory. The measured gas and geologic data sets of these areas were inputted into the RS-UMT prediction model. The predicted result was obtained according to the prediction step of the model.

Table 4 Basis data of mining face to be evaluated.

Mining face	C_1/MPa	C_2/m^3/t	C_3/m	C_4	C_5	C_6/m
ξ_{13}	1.0	10	583	Little complex	0.35	4.3
ξ_{14}	0.73	14	652	Simple	0.33	3.6
ξ_{15}	1.8	20	636	Very complex	0.11	4.2
ξ_{16}	1.3	16	622	Medium	0.17	3.6
ξ_{17}	0.5	6.5	405	Simple	0.5	4.2
ζ_{16}	1.1	11.5	535	Little complex	0.35	2.2
ζ_{17}	1.5	13	485	Medium	0.17	4.5
ζ_{18}	1.2	15	481	Medium	0.25	4.6
ζ_{19}	1.6	15	536	Very complex	0.21	4.3
ζ_{20}	0.67	5.8	380	Simple	0.45	3.8

In accordance with Table 1, the risk of coal and gas outburst was divided into four levels. The evaluation level can be expressed as $D = \{d_1,d_2, d_3,d_4\}$. d_1,d_2, d_3,d_4 respectively represent the safe, low outburst risk, medium outburst risk, and high outburst risk. The grading standards used for each indicator are shown in Table 1. With the maximum gas pressure (C_1) as an example, the membership function was established based on the construction method of the linear unascertained measure function, as follows:

$$\mu_1(x\in d_1)=\begin{cases} 1 & x<0.6 \\ \dfrac{\frac{0.6+1}{2}-x}{\frac{0.6+1}{2}-0.6} & 0.6\le x<\frac{0.6+1}{2} \\ 0 & \frac{0.6+1}{2}\le x \end{cases} \qquad \mu_1(x\in d_2)=\begin{cases} \dfrac{x-0.6}{\frac{0.6+1}{2}-0.6} & 0.6\le x<\frac{0.6+1}{2} \\ \dfrac{\frac{1+2}{2}-x}{\frac{1+2}{2}-\frac{0.6+1}{2}} & \frac{0.6+1}{2}\le x<\frac{1+2}{2} \\ 0 & others \end{cases}$$

$$\mu_1(x\in d_3)=\begin{cases} \dfrac{x-\frac{0.6+1}{2}}{\frac{1+2}{2}-\frac{0.6+1}{2}} & \frac{0.6+1}{2}\le x<\frac{1+2}{2} \\ \dfrac{2-x}{2-\frac{1+2}{2}} & \frac{1+2}{2}\le x<2 \\ 0 & others \end{cases} \qquad \mu_1(x\in d_4)=\begin{cases} 0 & x<\frac{1+2}{2} \\ \dfrac{x-\frac{1+2}{2}}{2-\frac{1+2}{2}} & \frac{1+2}{2}\le x<2 \\ 1 & 2\le x \end{cases}$$

The membership function of other predictive indexes was established in the same way (Figures 3-7), and take sampling point ξ_{13} as an example, the single index measure vector was calculated, as shown in Table 5.

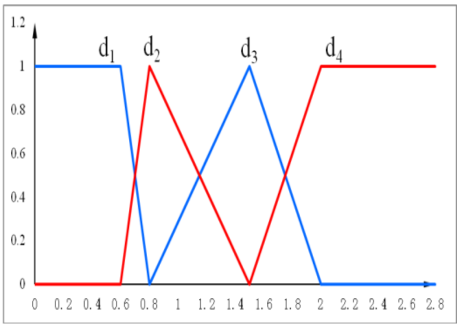

Figure 3 Single index measure function chart of gas content.

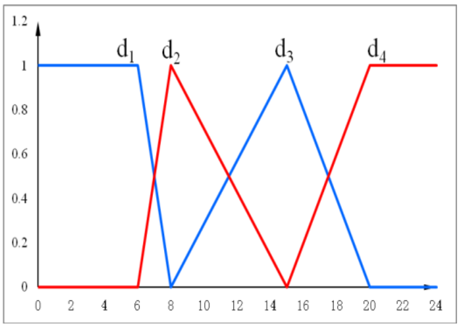

Figure 4 Single index measure function chart of gas pressure.

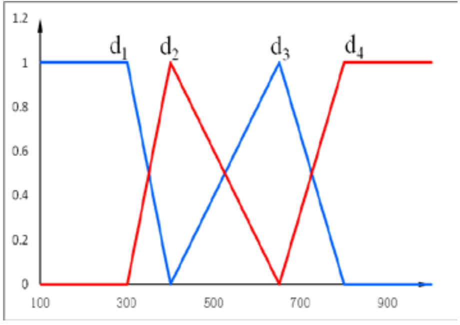

Figure 5 Single index measure function chart of mining depth.

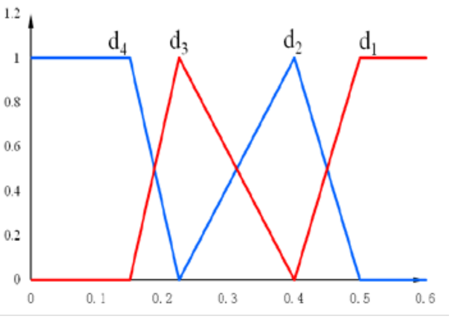

Figure 6 Single index measure function chart of Protodyakonov's coefficient of coal.

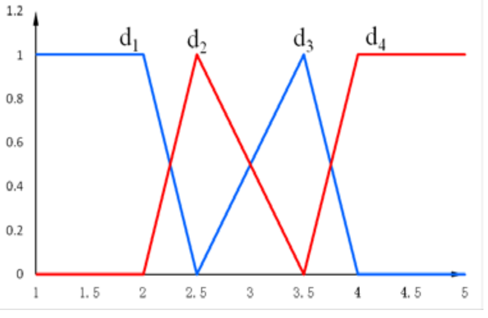

Figure 7 Single index measure function Chart of coal-seam thickness.

Table 5 Single index measure evaluation vectors of the index system.

Index system	Prediction index	Single index measure evaluation vector
Influence factors of coal and gas outburst C	gas pressure (C_1/MPa)	(0 0.7143 0.2857 0)
	gas content (C_2/m3/t)	(0 0.7143 0.2857 0)
	mining depth (C_3/m)	(0 0.268 0.732 0)
	geological structure (C_4)	(0 1 0 0)
	Protodyakonov's coefficient (C_5)	(0 0.2857 0.7143 0)
	coal seam thickness (C_6/m)	(0 0 0 1)

According to the above measure vector, the individual index measure matrix of ξ_{13} was established as follows:

$$(\mu_{1jk})_{6\times4}=\begin{pmatrix} 0 & 0.7143 & 0.2857 & 0 \\ 0 & 0.7143 & 0.2857 & 0 \\ 0 & 0.268 & 0.732 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.2857 & 0.7143 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Similarly, the individual index measure matrixes of the 9 other sampling points were established.

3.5 Construction of Multi-Index Comprehensive Measure Evaluation Matrix

The weight of each index determined by the rough set theory was taken as the input weight of the unascertained measure. The multi-index comprehensive measure matrix of each evaluation object was obtained by Eq.(11), as follows:

$$(\mu_{ik})_{10\times4}=\begin{pmatrix} 0 & 0.6416 & 0.287 & 0.0714 \\ 0.3625 & 0.3151 & 0.3072 & 0.0152 \\ 0 & 0.004 & 0.196 & 0.8 \\ 0 & 0.0998 & 0.7525 & 0.1477 \\ 0.8215 & 0.1057 & 0.0014 & 0.0714 \\ 0.0428 & 0.6686 & 0.2886 & 0 \\ 0 & 0.088 & 0.7359 & 0.1761 \\ 0 & 0.2065 & 0.7222 & 0.0714 \\ 0 & 0.0326 & 0.5532 & 0.4143 \\ 0.6875 & 0.2411 & 0.0286 & 0.0428 \end{pmatrix}$$

According to Eq. (12), the outburst risk levels for 10 sampling areas were identified. Here λ was set to 0.6. Take ξ_{17} for example $k_0=1$, when, $\mu_{71}(d_1)=0<\lambda=0.6$,

when $k_0=2$, $\sum_{s=1}^{k}\mu_{is}(d_i)=\mu_{71}(d_1)+\mu_{72}(d_2)=0+0.088=0.088<\lambda=0.6$,

when $k_0=3$, $\sum_{s=1}^{k}\mu_{is}(d_i)=\mu_{71}(d_1)+\mu_{72}(d_2)+\mu_{73}(d_3)=0+0.088+0.7359=0.8239>\lambda=0.6$.

Therefore, for $\zeta_{17}, k_0=3$, and ζ_{17} belongs to the medium level risk of outburst. Similarly, ζ_{17}, ζ_{20} belong to the safe level, $\xi_{13}, \xi_{14}, \zeta_{16}$ belong to the low level risk of outburst, $\xi_{16}, \xi_{17}, \zeta_{18}$ belong to the medium level risk of outburst, ξ_{15}, ζ_{19} belong to the high level risk of outburst.

Table 6 Comparative analysis of prediction results applied to the Pingdingshan No. 8 Mine and No. 10 Mine.

Mining face	Comprehensive unascertained measure				The complex index method	Fuzzy comprehensive evaluation model	The RS-UMT Model	Risk level from in situ records
	d_1	d_2	d_3	d_4				
ξ_{13}	0	0.6416	0.287	0.0714	d_2	d_2	d_2	d_2
ξ_{14}	0.3625	0.3151	0.3072	0.0152	d_2	d_1	d_2	d_2
ξ_{15}	0	0.004	0.196	0.8	d_2	d_4	d_4	d_4
ξ_{16}	0	0.0998	0.7525	0.1477	d_2	d_3	d_3	d_3
ξ_{17}	0.8215	0.1057	0.0014	0.0714	d_1	d_1	d_1	d_1
ζ_{16}	0.0428	0.6686	0.2886	0	d_2	d_2	d_2	d_2
ζ_{17}	0	0.088	0.7359	0.1761	d_2	d_3	d_3	d_3
ζ_{18}	0	0.2065	0.7222	0.0714	d_2	d_3	d_3	d_3
ζ_{19}	0	0.0326	0.5532	0.4143	d_2	d_3	d_4	d_4
ζ_{20}	0.6875	0.2411	0.0286	0.0428	d_1	d_1	d_1	d_1

3.6 Results and Discussion

A total of 96 prominent accidents have occurred in Pingdingshan No. 8 Mine and No. 10 Mine, which were selected as the research object in this study. Several feasible methods to conduct outburst risk prediction have been explored in this paper. Rough set theory was used to analyze the original outburst accident data and the objective weight of each evaluation index was calculated, which was then used as the input weight of the unascertained measure model. The unascertained measure function of each evaluation index was established according to the measured data and the confidence criterion was used to judge the risk degree of outburst of 10 mining faces in the No. 8 Mine and No. 10 Mine. The geological and gas data associated with 27 basic events and 10 other additional parameter sets were collected from an adjacent previously mined zone at the two mines. To analyze the weights of different predictive indicators 27 basic events were used and the remaining 10 additional incidents were used to verify the accuracy of the predicted results. The geological and gas data sets were then applied to RS-UMT, without knowledge of the records of the actual outburst events that have been recorded within this mining zone. To verify the accuracy of the RS-UMT prediction model, the predictive results of the model were compared with the actual measured results and other evaluation methods.

A comparative analysis of the RS-UMT prediction model, the fuzzy comprehensive evaluation (FCE), the complex index method and the record of actual outburst levels are shown in Table 6. Nine out of ten of the predicted results by the RS-UMT model were accurately predicted, while 7 predicted results of the FCE model and 5 predicted results of the comprehensive index method were consistent with the records of the actual outburst level. The prediction accuracy of the RS-UMT prediction model was 90%, whereas the FCE model achieved 70% and the comprehensive index method achieved only 40%. Furthermore, the predicted SAFE levels by the RS-UMT model were completely consistent with the risk level from the in situ records.

It can be concluded that the RS-UMT prediction model developed in this study may provide a reliable way of evaluating the risk level of coal and gas outburst. Compared with other methods, the RS-UMT model has two main advantages in predicting coal and gas outburst: (1) the data for calculating the weights of the prediction indicators by rough set theory are all from real outburst accidents so the calculation results are more realistic and objective, overcoming the shortcomings of other methods in determining the weights, such as the impact of subjective factors, which requires the decision maker to have some prior knowledge. (2) The unascertained measure theory (UMT) can deal better with the coupling problem of outburst indicators. Furthermore, the unascertained measure evaluation method uses a more reasonable confidence criterion to replace the maximum membership principle. Based on the above two advantages, the success rate of the RS-UMT model in predicting outburst is much higher than that of the complex index method and the FCE model, which provide alternatives for coal and gas outburst prediction. Moreover, the accuracy of the predicted model can still be improved by determining a more reasonable index system and index weight, which requires a more in-depth understanding of the outburst mechanism and more outburst data.

4 Conclusions

A new prediction model for coal and gas outburst based on rough set theory and unascertained measure theory was established. This paper presented an analysis of the results of a case study investigation that considered the application of the proposed Rough Set-Unascertained Measure Theory model to predict the risk of coal and gas outburst events at the No. 8 Mine and No. 10 Mine located within the Pingdingshan coal mining region. The prediction results of the model were compared with actual measured results and other evaluation methods. Nine out of ten of the prediction results by the Rough Set-Unascertained Measure Theory model were accurately predicted, while 7 prediction results of the fuzzy comprehensive evaluation model and 5 prediction results of the comprehensive index method were consistent with the records of actual outburst levels. It can be concluded that the success rate of the developed model in predicting outburst was much higher than that of the complex index method and the fuzzy comprehensive evaluation model. The higher prediction accuracy of the Rough Set-Unascertained Measure Theory model is mainly attributed to its two major advantages: (1) the determination of index weight by rough set theory overcomes the shortcomings of the other methods so that it can obtain more objective and reliable index weights without any prior knowledge of the decision maker, and eliminate the impact of subjective factors; (2) the Rough Set-Unascertained Measure Theory model can better deal with the coupling problem of outburst indicators, making the evaluation results more clear and accurate. The developed model not only provides a more effective method for determining the weight of outburst indicators but also provides a new and accurate problem-solving approach for coal and gas outburst prediction.

The Rough Set-Unascertained Measure Theory model may be applicable to other coal mines with similar geological and mining conditions as the No. 8 Mine and No. 10 Mine in Pingdingshan. Moreover, if the prediction index and the weights are updated according to the outburst data of other coal fields, the proposed model may properly be applied to the mines of these coal fields. The rough set theory can be combined with other methods to determine the weight of outburst indicators, which will be a direction for the improvement of the prediction model in the future. Consequently, it can be concluded that the developed Rough Set-Unascertained Measure Theory model is a reliable and very promising method for predicting the risk of coal and gas outburst.

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