Hollow Core Slabs on Winkler Foundation

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Abstract. This research dealt with the linear elastic behavior of hollow core slabs resting on a linear Winkler type foundation. A finite difference method was used to model the slabs as wide beams and the foundation as elastic springs. The finite element method was also used to model the problem using ABAQUS 6.10 software program. A comparison between the method proposed in this paper and methods from previous studies was made to check the accuracy of the solutions. Several important parameters were incorporated in the analysis, viz. the hollow core size and shape, subgrade reaction and slab depth, to trace their effects on deflections, bending moments and shear forces. A computer program coded in Fortran was developed for the analysis of hollow core slabs on an elastic foundation. It was found that the maximum difference in deflection between the present study and the exact solution was 3% for the finite difference and 7% for the finite element method.

Keywords: *finite differences; finite elements; hollow core; slabs; Winkler foundation.*

1 Introduction

The footing slabs of concrete buildings are supported directly by the soil medium. Modeling the soil medium is a very complicated problem. Therefore, structural engineers have tried to simplify soil behavior through using a simplified Winkler subgrade model. This model treats the soil as a linear elastic material that displaces independently at different points. The problem of reducing the dead load of a thick footing on soil is very important, especially for soils with low bearing capacity. The most efficient weight reduction for flexural members can be achieved through removing mass near the centroid of such members. One-way hollow core slabs are examples of this technique, which can be modeled using two-dimensional plate members or as wide beams. The latter approach approximates the problem through one-dimensional members or elements that support the applied loads through flexural and shearing impedances that develop in the slab cross-section. The famous formula (d4w/dx4 = q/EI) has been derived for the flexure of shallow or thin beams subjected to transverse distributed load (q) (per unit length). This classical equation discards the effects of deformation due to transverse shear [1,2].

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There are two basic theories that can be used to approximate the formulation of flexural beam models. In the case of a shallow or thin beam (beam depth to span h/L < 0.2), the model is approximated using the Euler-Bernoulli theory. This theory assumes that the beam cross-sections remain straight and normal to the neutral axis when loads are applied. It is also assumed that a small deformation develops in the beam [3,4]. Another theory of beam bending has been developed for deep beams (h/L > 0.2). This model was established through Timoshenko's theory. In addition to bending deformation, this theory includes shear deformation. Therefore, it assumed that after bending, the beam cross-section remains straight and not normal to the neutral axis.

Some previous studies have attempted to solve these problems. Zhan [5] used finite elements to solve the problem of supported beams on subgrade. The ABAQUS (6.10) software program was utilized to model the foundation and beam by using spring and plate elements. Bogdan [6] solved the problem of beams supported on an elastic foundation using finite differences in a Matlab-coded program. The beams have free ends and are subjected to three concentrated forces at different locations. Al-Azzawi [7] conducted a study on shear deformation in beams supported by a Winkler foundation. To proceed with the study, the author used a finite difference method for solving the problem. The findings of the study were compared with exact solutions and previous analytical studies to check the validity of the developed analysis.

2 Governing Differential Equation

The differential equations for bending of Timoshenko beams are based on small deformation theory and linear stress-strain relationships. The following assumptions are used:

1. The beam cross-section remains or stays plane after bending.
2. There will be an additional rotation in the beam cross section due to transverse shear.
3. The shear correction factor (r) is assumed to model a constant shear distribution.

Based on the above simplifying assumptions, the following differential equations are given [8]:

$$GrA\frac{d\theta}{dx}+q=K_z w-GrA\frac{d^2w}{dx^2} \tag{1}$$

$$EI\frac{d^2\theta}{dx^2}-GrA\,\theta=GrA\frac{dw}{dx} \tag{2}$$

where w = w (x) is the wide beam deflection, $\theta = \theta(x)$ is the wide beam rotation, G is the slab shear modulus, A is the area of the slab section, K_z is the subgrade modulus, EI is the slab bending rigidity, q is the distributed load, and r is the shear correction factor obtained from the equations mentioned in reference [9]. The slab cross section is shown in Figure 1.

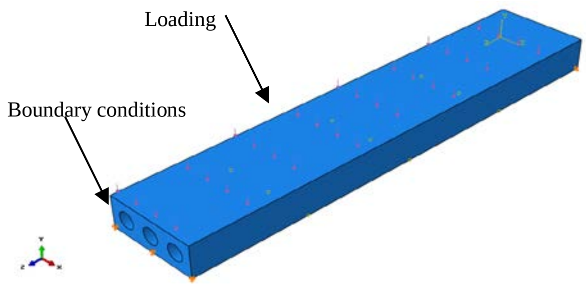


Figure 1 Hollow core slab footing.

3 Finite Difference Method

The finite difference method is the most preferred general numerical technique for modeling and solving complicated problems. The governing differential equation derivatives are transformed or converted into differences at different selected nodes. The term finite difference mesh refers to a grid that collects all these nodes or points. The differential equations for slabs on Winkler subgrade at each node are exchanged or replaced by difference equations to model the problem and forming simultaneous algebraic (SA) equations. The (SA) equations are solved numerically. The slab length is discretized or divided in the (x) direction into steps or intervals of length (Δx) (Figure 2).

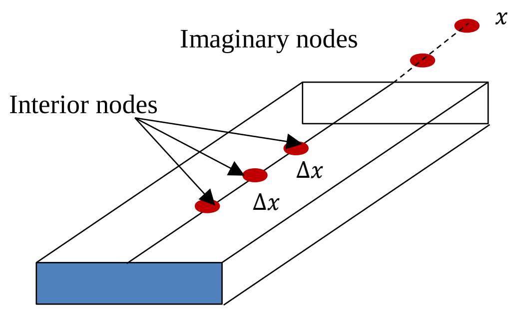


Figure 2 Finite difference grid or mesh.

In Figure 2, (n) is assumed to define the total number of slab nodes and (i) to represent the central node. In the finite difference method, the deflection profile is obtained approximately by a line connecting the slab nodes in finite differences for modeling the first (1st) derivatives. In the case of second (2nd) derivatives it is approximated by a parabolic curve.

4 Finite Element Method

The finite element method has been proven in previous studies to be suitable for application to slab problems. Three-dimensional (3D) finite element models can accurately solve most structural problems. In this study, the ABAQUS 6.10 software program was used to model hollow core slabs using 20 node brick elements and a linear spring element to model the Winkler subgrade. The finite element (FE) mesh of the hollow core slab is shown in Figure 3.

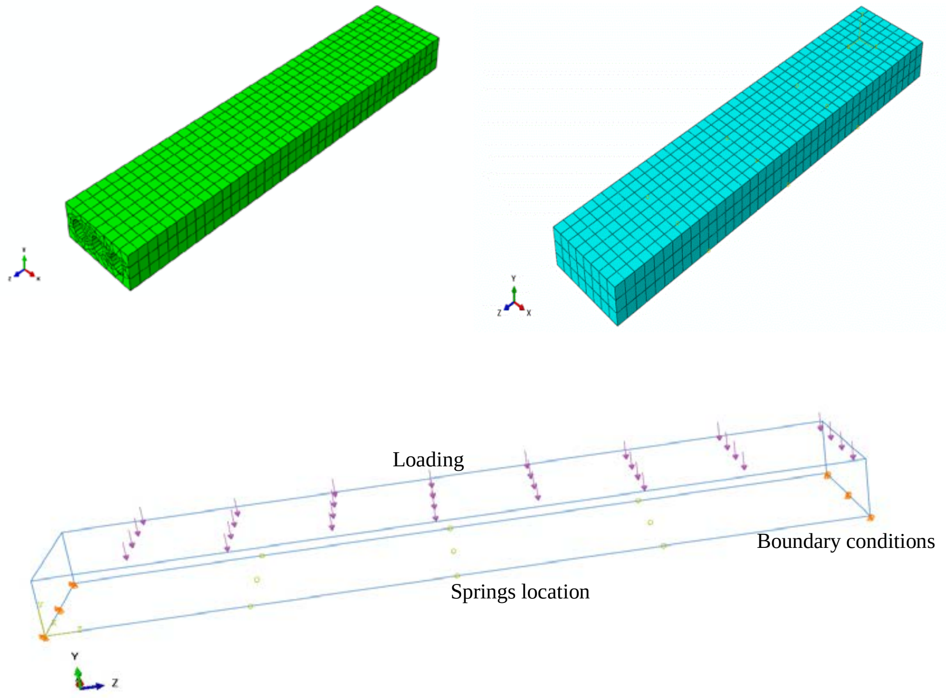


Figure 3 Finite element mesh, boundary conditions and loading.

The finite element mesh was selected based on the solution results. When the results was found to be constant, the suitable number of elements was selected. In this study, elements with a size of 0.1 m x 0.1 m were selected.

5 Applications

The results from the adopted finite difference and finite element methods were compared with closed form or exact solution results to check the accuracy of the obtained numerical solution. The following case studies, which include the linear elastic behavior of solid and hollow core slabs resting on a one-parameter Winker foundation, were considered.

5.1 Simply Supported Solid Slab Resting on Winkler Subgrade

A solid slab of (E = 24*106 kN/m2, $\nu = 0.15$) with a total length of (4.0 m), width (B = 0.8 m), thickness (H = 0. 3 m), subjected to a uniform stress or load (q = 40 kN/m2), is considered. The slab shown in Figure 4 is resting on Winkler subgrade with coefficient (Kz = 10000 kN/m^3). In the present study, the effect of mesh size in the finite element method on the maximum deflection result was studied as shown in Figure 5.

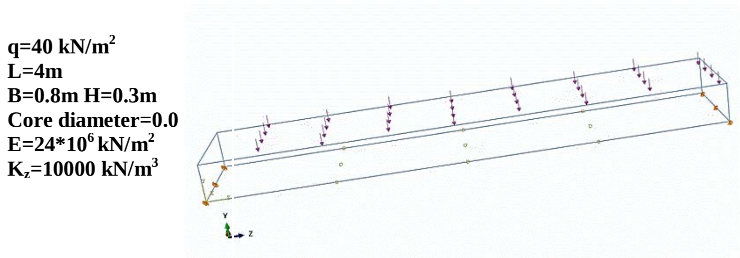


Figure 4 Simply supported solid slab resting on Winkler subgrade loaded with uniform load.

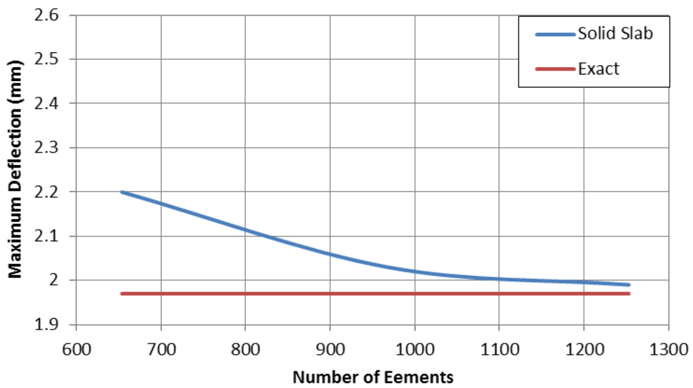


Figure 5 Mesh size effect on deflection results for simply supported solid slab resting on Winkler subgrade loaded with uniform load.

The optimum number of elements was found to be 986. Previously, this case was solved analytically by Hetenyi [10], assuming the slab to be a wide beam. The same case was tackled here by using the two adopted methods: the finite difference and finite element methods. The slab deflections, bending moments and shear force were plotted simultaneously with the exact solution results, as shown in Figures 6 to 8. The figures and the obtained results demonstrate that the solutions were in acceptable agreement. The variation percentages between the results of maximum deflections, bending moments and shear force obtained from the exact solution and the finite difference method were (1%), (3%) and (2%) and the finite element method were (4%), (7%) and (6%) respectively.

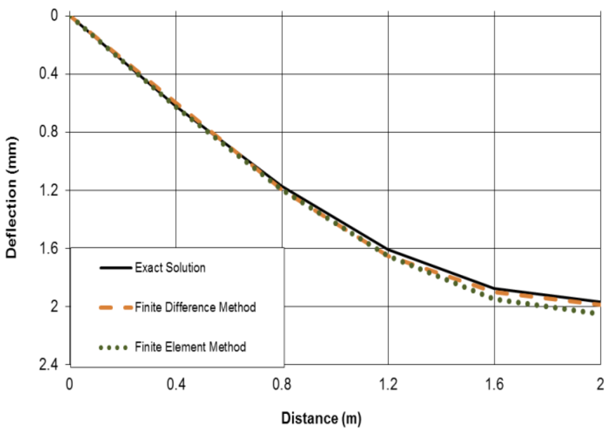


Figure 6 Deflection curves for a half span of simply supported solid slab on Winkler subgrade.

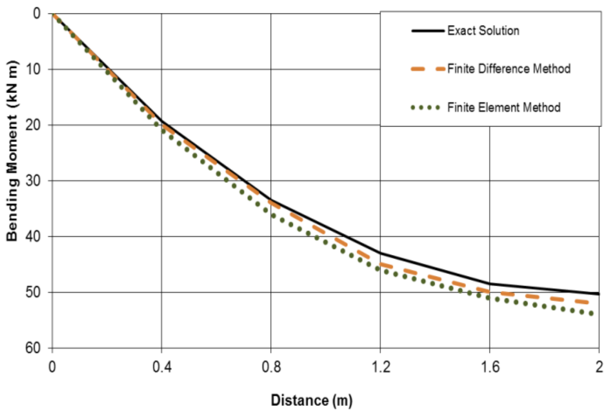


Figure 7 Bending moment curves or diagrams for a half span of simply supported solid slab on Winkler subgrade.

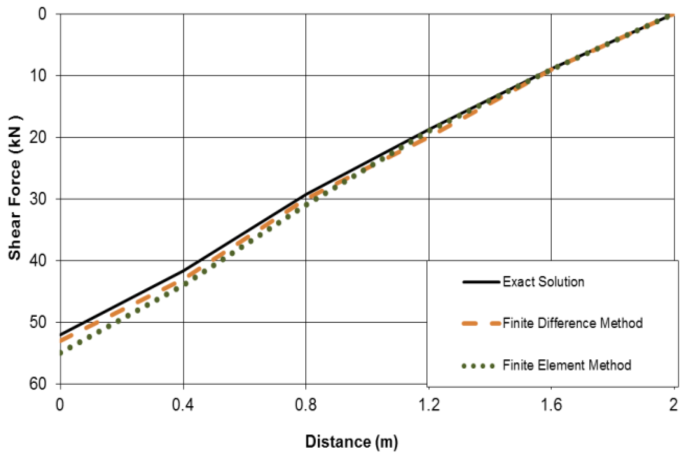


Figure 8 Shear force curves or diagrams for a half span of simply supported solid slab on Winkler subgrade.

5.2 Simply Supported Hollow Core Slab resting on Winkler Subgrade

A hollow core slab of (E = 24*10^6 kN/m^2, $\nu = 0.15$) with a total length of (4.0 m), width (B = 0.8 m), thickness (H = 0.3 m), loaded with uniform stress or load (q = 40 kN/m^2), is considered. The slab has 3 cores with a core diameter of (0.1 m) giving a void percentage of 10% of cross-sectional area. The slab shown in Figure 9 is resting on Winkler subgrade with coefficient (K_z = 10000 kN/m^3). In the present study the effect of mesh size in the finite element method on the maximum deflection result was studied, as shown in Figure 10.

The optimum number of elements was found to be 5687, which is higher than for the solid slab, which is attributed to the existence of a hollow core. Previously, this case was solved by Bowles [11] using a closed form solution. This case was solved here using the two adopted methods: finite difference and finite element. The slab deflections, bending moments and shear force were plotted simultaneously with the exact or closed form solution results, as shown in Figures 11 to 13.

The figures and the obtained results demonstrate that the solutions were in acceptable agreement. The variation percentages between the results of the maximum deflections, bending moments and shear force obtained from exact solution and the finite difference method were (1.5%), (3.5%) and (1.8%) and the finite element method were (3.6%), (6%) and (3.7%) respectively.

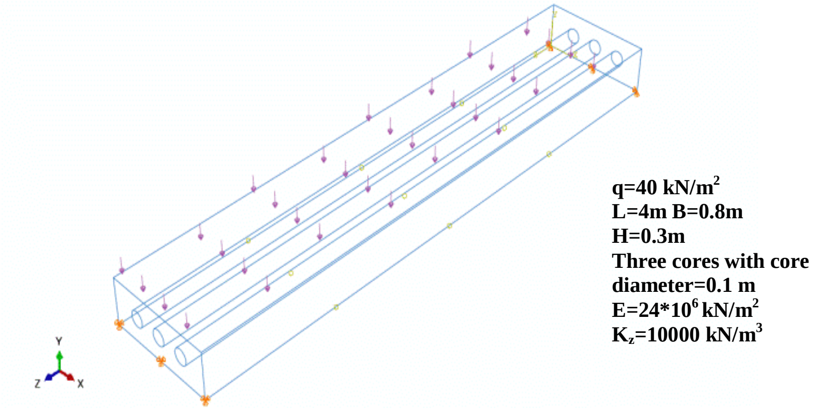


Figure 9 Simply supported hollow core slab resting on Winkler subgrade loaded with uniform load.

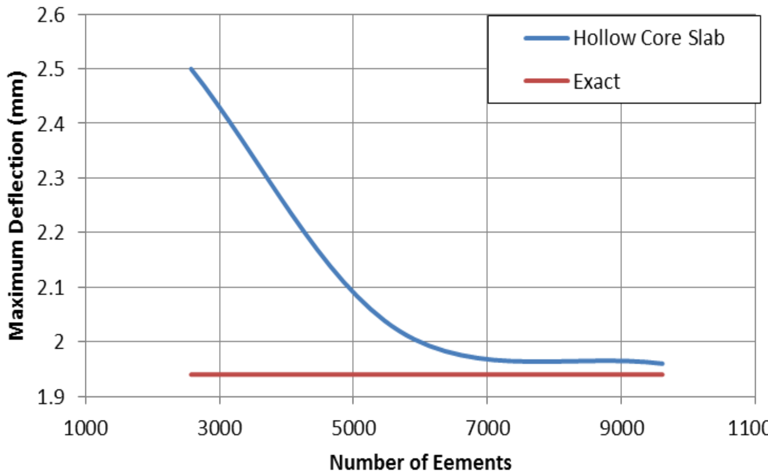


Figure 10 Mesh size effect on deflection results for simply supported hollow core slab resting on Winkler subgrade loaded with uniform load.

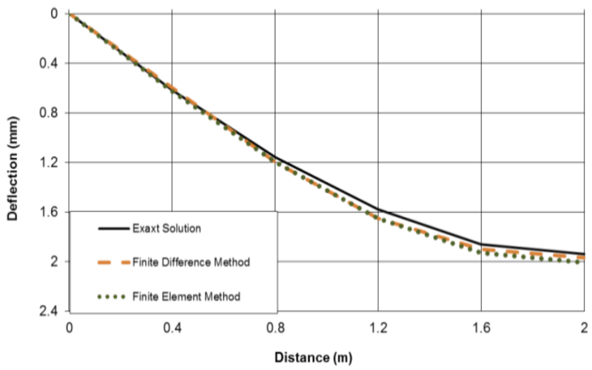


Figure 11 Deflection curves for a half span of simply supported hollow core slab resting on Winkler subgrade.

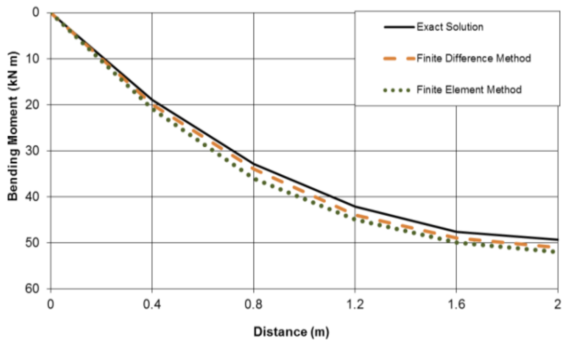


Figure 12 Bending moment curves for a half span of simply supported hollow core slab resting on Winkler subgrade

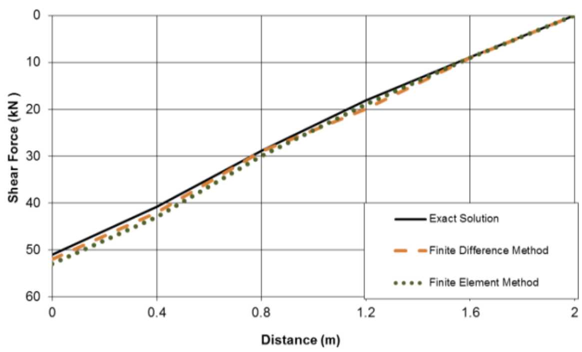


Figure 13 Shear force curves or diagrams for a half span of simply supported hollow core slab resting on Winkler subgrade.

6 Parametric Study

In the present study, the influence or effect of selected important parameters or factors on the response of slabs resting on Winkler subgrade were investigated. The study parameters were as follows:

1. Core diameter (void percentage)
2. Core shape
3. Subgrade reaction (K_z)
4. Slab thickness (h)
5. Boundary conditions

To study the effects of these parameters, the previous problem of slabs was considered under different loading types and ends or boundary conditions.

6.1 Effect of Core Diameter

Different core diameter values were considered in the parametric study. The assumed core diameter values were (0.0, 0.1m and 0.15m) with three cores for each analyzed slab. Figure 14 shows the variation of core diameter with slab mid-span deflection. It is obvious from the figure that the mid-span vertical displacement or deflection increased at an increasing rate as the core diameter was increased. For increasing the core diameter or void percentage from (0.0 to 22%), the mid-span deflection showed increased values by 2%, 4% for the finite difference and the finite element method respectively.

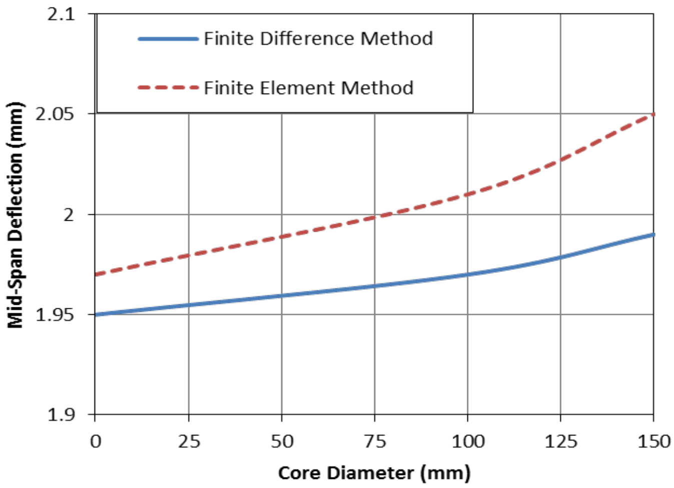


Figure 14 Effect of core diameter on mid-span deflection.

Figure 15 shows the variation of core diameter with slab mid-span moment. From this figure it is obvious that the mid-span moment decreased with increasing core diameter or void percentage. If the void percentage increased from (0.0 to 22%), the mid-span moment decreased by 10% and 11% for the finite difference and the finite element method respectively. Figure 16 shows the variation of core diameter with slab maximum shear force. The maximum shear force decreased at an increasing rate as the void percentage was increased. If the void percentage was increased from (0.0 to 22%), the maximum shear force decreased by 7% and 9% for the finite difference and the finite element method respectively.

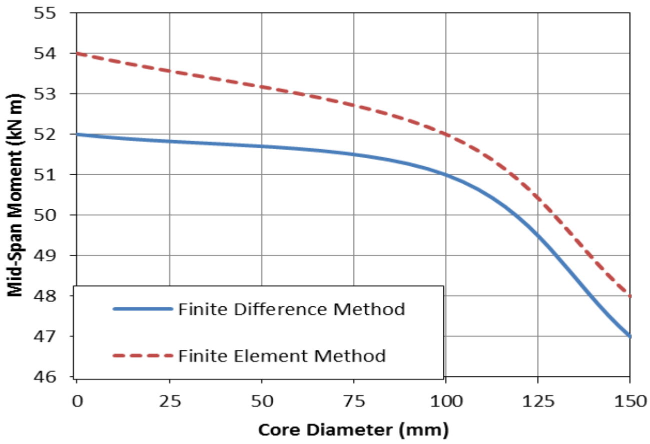


Figure 15 Effect of core diameter on mid-span moment.

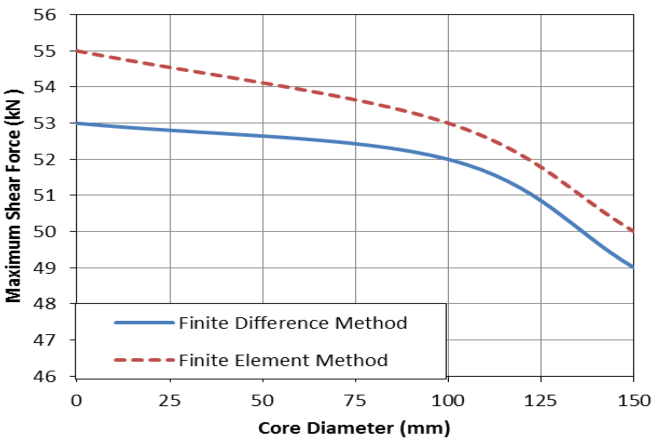
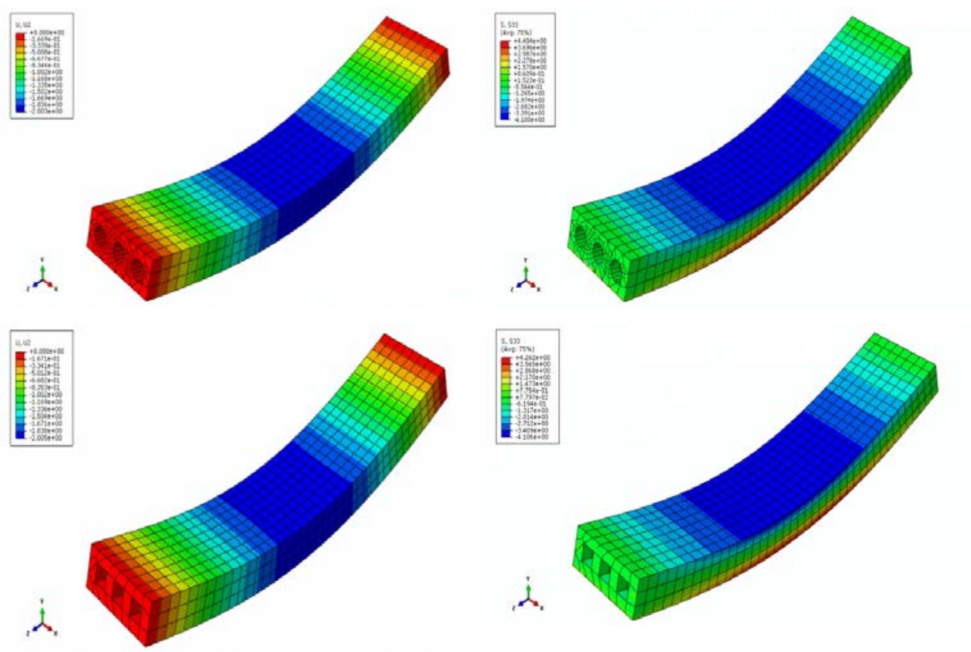


Figure 16 Effect of core diameter on maximum shear.

6.2 Effect of Core Shape

To show the effect of core shape, circular and square shapes of the core with equal areas were considered, as shown in Figure 17. The slabs were solid and hollow core with three core sizes (0.15 m diameter for the circular core and 0.13 side lengths for the square core) were assumed for each slab. Figure 18 shows the relationship between hollow core slab mid-span deflection and core shape.

The maximum or mid-span deflection increased as the core diameter increased and the core shape became square. It was found that by changing the core shape from circular to square, the mid-span deflection was increased by 0.15%. Figure 19 shows the variation of mid-span moment of the slab with core shape. The mid-span moment decreased by 2% as the core shape was changed from circular to square. Figure 20 shows the variation of maximum shear force of the slab with core shape. The maximum shear force decreased by 2.5% as the core shape was changed from circular to square.



Deflection (mm) Normal stress (z) (MPa)

Figure 17 Deflected shape and normal stress distribution for hollow circular and square core slabs.

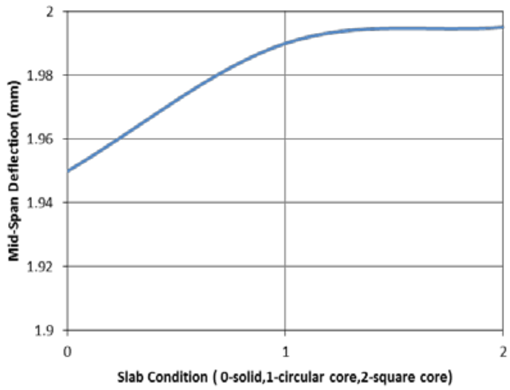


Figure 18 Effect of core shape on mid-span deflection.

Figure 19 Effect of core shape on mid-span moment.

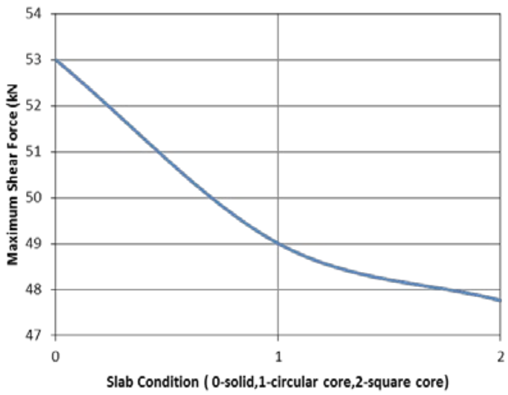


Figure 20 Effect of core shape on mid-span deflection

6.3 Effect of Subgrade Reaction

Different values were assumed for the subgrade reaction parameter (K_z). The slab was simply supported and loaded with concentrated load (128kN) at mid span. The assumed values of subgrade reaction were (0.0,10000 and 50000 kN/m^3). Figure 21 shows the variation of subgrade reaction with slab maximum or mid-span deflection.

The mid-span or maximum deflection had a reduced value when the subgrade reaction was increased. When the subgrade reaction was increased from (0.0 to 50000 kN/m^3), the mid-span deflection decreased by 70% for solid and hollow core slabs.

Figure 22 shows the variation of subgrade reaction with slab mid-span moment. It is obvious that the mid-span moment decreased with increasing subgrade. It was observed that when the soil subgrade coefficient or reaction was increased from (0.0 to 50000 kN/m^3), the mid-span moment decreased by 59% for solid and hollow core slabs.

Figure 23 shows the variation of subgrade reaction with support force of the slab. When the subgrade reaction was increased from (0.0 to 50000 kN/m^3), the support shear force decreased by 85% for solid and hollow core slabs.

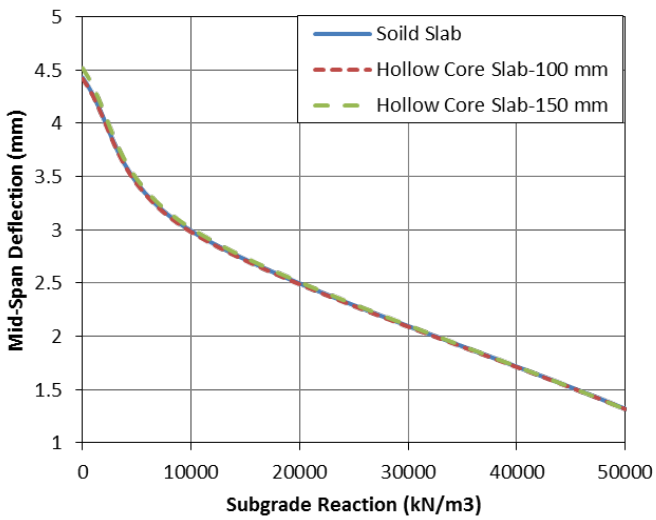


Figure 21 Subgrade reaction effect on mid-span deflection for a slab under a concentrated load.

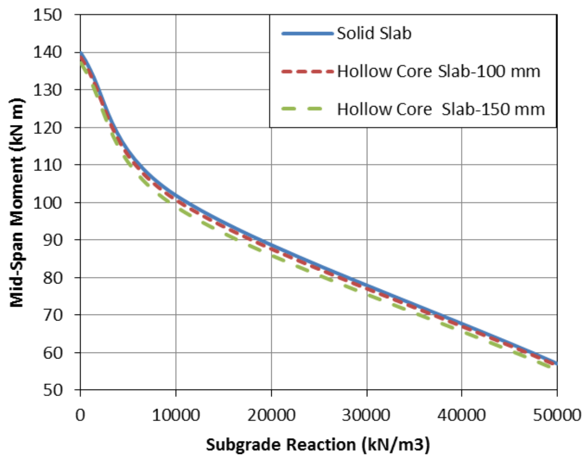


Figure 22 Subgrade reaction effect on mid-span moment for a slab under a concentrated load.

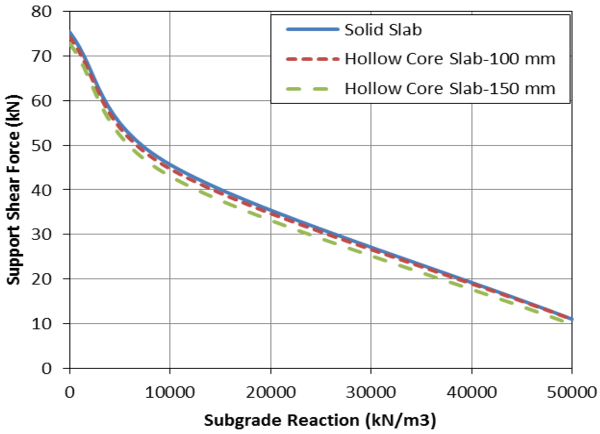


Figure 23 Effect of subgrade reaction on maximum shear force for a slab under a concentrated load.

6.4 Effect of Slab Thickness

To show the effect of slab thickness, different values were considered with the slab placed under a uniform load. The values of slab thickness were (0.2, 0.3 and 0.4 m). Figure 24 demonstrates the variation of slab thickness with slab mid-span deflection. It is obvious that the mid-span vertical displacement or deflection decreased as the slab thickness was increased. When the slab thickness was increased from (0.2 to 0.4 m), the mid-span deflection decreased by 70% for both solid and hollow core slabs. Figure 25 shows the variation of slab thickness with slab mid-span moment. It was observed that when the slab thickness was increased from (0.2 to 0.4 m), the mid-span moment increased by 150%, 160% and 180% for solid, hollow core (diameter = 0.1) and hollow core (diameter = 0.15 m) respectively. Figure 26 shows the variation of slab thickness with slab maximum or support shear force. It was observed that by increasing the slab thickness from (0.2 to 0.4 m), the maximum shear force increased by 102%, 105% and 116% for solid, hollow core (diameter = 0.1) and hollow core (diameter = 0.15 m) respectively.

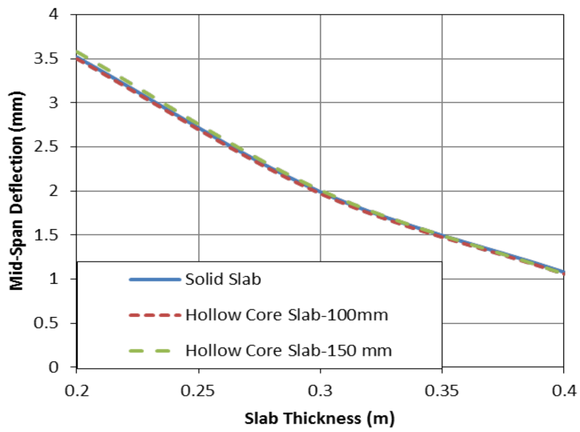


Figure 24 Effect of slab thickness on mid-span deflection for a slab under a uniform load.

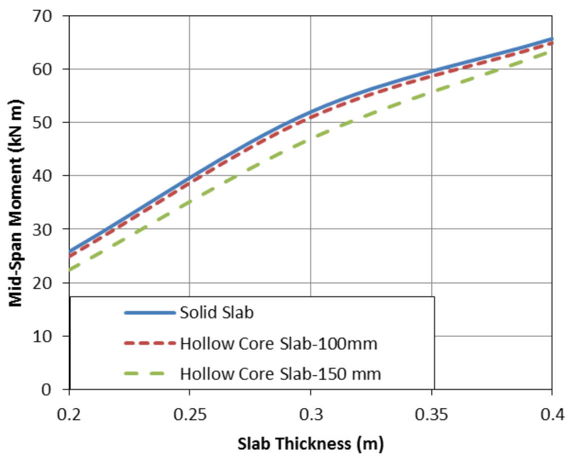


Figure 25 Effect of slab thickness on mid-span moment for a slab under a uniform load.

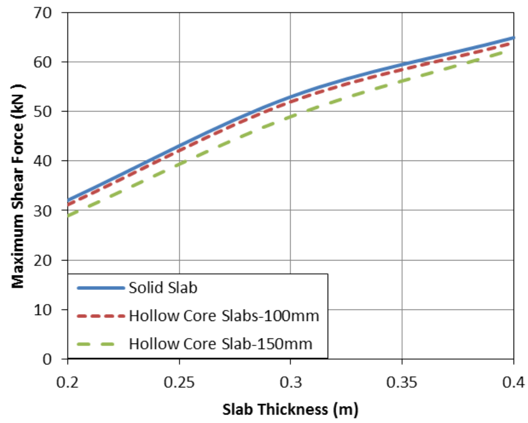


Figure 26 Effect of slab thickness on maximum shear force for a slab beam under a uniform load.

6.5 Boundary Conditions

To show the effect of the boundary conditions, fixed, simply supported and free end conditions were considered. The slab was assumed under uniform load ($q = 40$ kN/m^2) and the modulus or subgrade reaction was (10000 kN/m^3). Figure 27 shows the effect of loading type on slab mid-span deflection. The mid-span deflection increased for free ends compared to simply supported ends by 141%, 139% and 132% for solid, hollow core (diameter = 0.1 m) and hollow core (diameter = 0.15m) respectively.

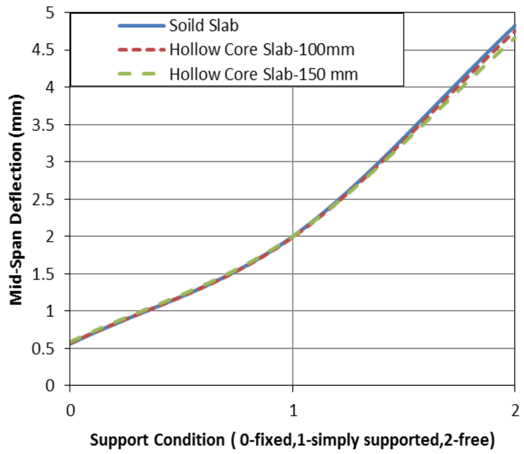


Figure 27 Effect of the boundary conditions on mid-span deflection for a slab under a uniform load.

Figure 28 shows the effect of loading type on slab mid-span moment. The mid-span moment decreased for free ends compared to simply supported ends by 96%, 97% and 98% for solid, hollow core (diameter = 0.1 m) and hollow core (diameter = 0.15m) respectively.

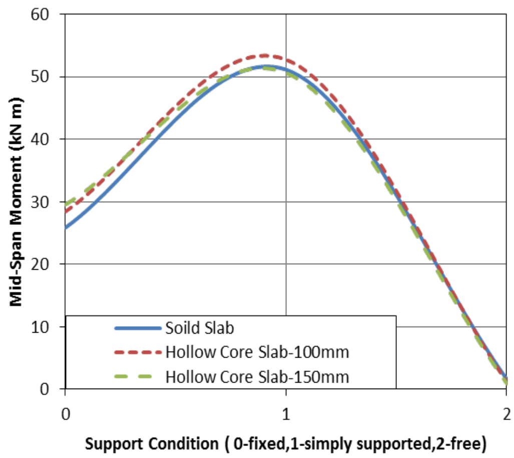


Figure 28 Effect of boundary conditions on mid-span moment for a slab under a uniform load.

7 Conclusions

From this study, the main conclusions are as follows. Good agreement was obtained by the adopted finite difference (FD) and finite element (FE) methods when analyzing the hollow core slab on a Winkler foundation. The maximum obtained difference with the exact solution for deflection, moment and shear and the proposed finite difference method were (1.5%), (3.5%) and (1.8%) and the finite element method were (3.6%), (6%) and (3.7%) respectively. The effect of increasing core diameter on the stress resultants (bending moment and shear resistances) was found to be more significant than the effect on deflection. When the core diameter or void percentage was increased from (0 to 22%), the percentages were (2%), (10%), and (7%) for deflection, moment and shear respectively. The effect of changing the core shape for a constant hole area on the stress resultants (bending moment and shear resistances) was found to be more than the effect on the deflection. When the core shape was changed from circular to square, the percentages were (0.15%), (2%), and (2.5%) for deflection, moment and shear respectively and may be neglected. The effect of increasing subgrade reaction on the behavior of the beam was found to be significant for the concentrated load case. When Kz ranged from (0.0 to 50000 kN/m3), the decrease percentages for deflection, moment and shear were (70%), (59%) and (85%) respectively. The effect of increasing slab thickness on the stress resultants (bending moment and shear resistances) was found to be more significant than the effect on deflection. In the case of a simply supported uniformly loaded hollow core slab (0.15 m core diameter), when the slab thickness (h) was increased from (0.2 to 0.4 m), the percentages were (70%), (180%), and (116%) respectively. The effect of the boundary conditions was found to be significant on deflection (132%) and moment (98%) in the case of free ends compared to simply supported ends.

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