Application of Fibonacci Sequence and Lucas Sequence on the Design of the Toilet Siphon Pipe Shape

Xiaole Ge[1], Hongfeng Wang[1], Shengrong Liu[1], Zhanfu Li[2], Xin Tong[2*] & Jiafei Pu[1]

[1]College of Mechanical & Electronic Engineering, Huangshan University, Huangshan, Anhui, 245041, China
[2]School of Mechanical & Automobile Engineering, Fujian University of Technology, Fuzhou, Fujian, 350118, China
*E-mail: tongxin_hqu_fjut@163.com

Abstract. The purpose of this study was to explore the method for designing the toilet siphon pipe shape to improve flushing performance. The Fibonacci sequence and the Lucas sequence were used to design the structural parameters of the siphon pipe. The flushing processes of the toilet were simulated using the computational fluid dynamics (CFD) method to analyze the flushing performance under different siphon pipe shapes. Experimental studies were conducted to verify the reliability of the simulation results. The results indicated that when the Lucas numbers and the Fibonacci numbers were utilized to regulate the curvature of the siphon pipe in the X_i direction and the Y_j direction respectively, the flushing performance of the toilet was optimal. In order to obtain better flushing performance, the curvature of the siphon pipe should be smooth and have obvious transitions at the connections of different sections. When the overall size of the siphon pipe is kept constant, a short siphon pipe length is helpful for the improvement of toilet flushing performance.

Keywords: *computational fluid dynamic; Fibonacci sequence; flushing performance; Lucas sequence; siphon pipe shape.*

1 Introduction

The toilet is an indispensable sanitary facility in our daily life and it is also one of the most wasteful water products. With the increasingly high quality requirements for toilets, the toilet is developing in the direction of strong flushing performance combined with low water consumption. The siphon toilet, which accomplishes the flushing process by using the siphon effect formed inside a siphon pipe, is the most commonly used on the market today due to having the advantages of deodorization and a quiet and strong flushing performance. The flushing capacity of a siphon toilet depends on the siphon intensity in the siphon pipe and is obviously affected by the shape of the siphon pipe [1]. Therefore, the design of the shape of the siphon pipe to improve flushing performance is a topic worthy of study.

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Several scholars have investigated the influence of the shape of the siphon pipe on flushing performance from different perspectives. Cheng, *et al*. studied the flushing processes of the siphon toilet by means of three different siphon pipes. They compared the changes of water velocity in different siphon pipes and found that the toilet could obtain good flushing performance with an appropriate siphon pipe shape [2]. Zhao, *et al*. analyzed the effect of the siphon pipe's shape on the velocity distribution, pressure distribution and siphon performance [3]. The flushing performance was improved by optimizing the shape of the siphon pipe based on the results of their analysis. A flexible siphon pipe, which can be adjusted to a horizontal shape during the flushing process and restored to the original shape after flushing, was developed by An, *et al.* to enhance flushing performance [4]. The above studies proved that a reasonable shape of the siphon pipe is an important factor in the improvement of flushing performance. However, the design of the structural parameters of the siphon pipe is mainly based on the traditional production experience and lacks specific design guidelines. Therefore, it is still unclear what the most reasonable shape of the siphon pipe is and further research is still needed.

Siphon intensity is an important evaluation index for flushing performance, but it is difficult to measure directly during the flushing process. Fortunately, the computational fluid dynamics (CFD) method has been well applied in the research of toilet flushing performance and its reliability has been proved [5-8]. Therefore, the change of siphon intensity during the flushing process can be monitored conveniently by using the CFD method. The Fibonacci sequence and the Lucas sequence, two famous sequences in mathematics, have been widely used in a variety of engineering designs, yielding ideal effect [9-11].

The aim of this work was to design the siphon pipe shape of a toilet by using the Fibonacci and the Lucas sequence to improve flushing performance. The computational fluid dynamic method was used to simulate the flushing processes and the changes of flow field parameters under different siphon pipe shapes were compared. Furthermore, experimental studies were performed to verify the reliability of the simulation results. The research contents of this paper can provide references for the design of the siphon pipe shape.

2 Design and Simulation

2.1 Design of Siphon Pipe Shape

2.1.1 Fibonacci Sequence and Lucas Sequence

The Fibonacci sequence, also called the golden section series, refers a sequence such as: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55… All the values in the sequence are called Fibonacci numbers. This sequence was introduced by mathematician Leonardoda Fibonacci from the mathematical problem of the reproduction of rabbits, so it is also known as the rabbit series. In nature, there are many plant shapes that are related to the Fibonacci sequence, such as the arrangement of leaves, the number of petals of a sunflower, the wings of dragonflies, the spirals on pineapples, and so on. Therefore, the Fibonacci sequence is in accordance with the laws of natural development. In mathematics, the Fibonacci sequence can be defined by recursive method as below in Eq. (1):

$$F(n) = F(n - 1) + F(n - 2),\ n=3,4,5,\ldots \tag{1}$$

here, $F(1) = 1$, $F(2) = 1$.

It should be noted that this formula is also called the Lucas sequence if $F(1) = 1$ and $F(2) = 3$, namely 1, 3, 4, 7, 11, 18, 29, 47…

Interestingly, when n approaches to infinity, the ratio of $F(n\text{-}1)$ to $F(n)$ becomes more and more close to the golden ratio (0.618). The golden ratio has been widely used as an important design parameter in architectural design, mechanical design and other fields [12].

2.1.2 Design of Structural Parameters

The Fibonacci sequence and the Lucas sequence were adopted to design the structural parameters of the siphon pipe. The design idea is shown in Figure 1.

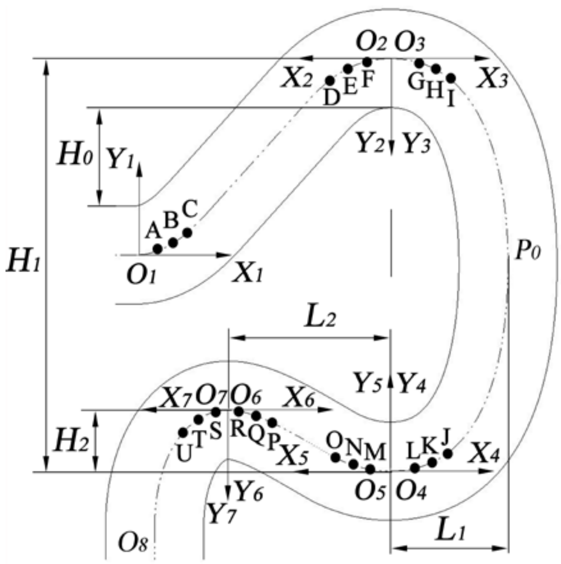


Figure 1 Method used for designing the siphon pipe.

For the convenience of the research, the siphon pipe was divided into 4 parts, namely that O_1O_2, O_3O_4, O_5O_6 and O_7O_8. Seven coordinate systems ($X_1O_1Y_1$, $X_2O_2Y_2$, $X_3O_3Y_3$, $X_4O_4Y_4$, $X_5O_5Y_5$, $X_6O_6Y_6$ and $X_7O_7Y_7$) were set up to connect each part. The curvature of the center line in each coordinate system was controlled by using three points to ensure smooth transitions in the siphon pipe. For instance, points *A*, *B* and *C* were used to control the curvature in $X_1O_1Y_1$, while points *D*, *E* and *F* were set to control the curvature in $X_2O_2Y_2$, and so on. Moreover, the points in each part were connected by spline curves.

In order to search the ideal curvature of the siphon pipe, four siphon pipe shapes were designed using different combinations. In Group 1, Fibonacci numbers were used to regulate the curvature in the X_i (i = 1, 2, 3, 4, 5, 6 and 7) direction and Y_j (j = 1, 2, 3, 4, 5, 6 and 7) direction. In Group 2, Lucas numbers were adopted to restrict the curvature in the X_i direction and the Y_j direction.

In Group 3, Fibonacci numbers were used to connect the curvature in the X_i direction while Lucas numbers were adopted to connect the curvature in the Y_j direction. In Group 4, Lucas numbers were applied to restrict the curvature in the X_i direction while the Fibonacci numbers were utilized to regulate the curvature in the Y_j direction. It should be noted that the points in $X_7O_7Y_7$ were all connected by Fibonacci numbers to ensure the smoothness of the curvature. The detailed parameter settings are listed in Tables 1- 4.

Table 1 Parameter settings for Group 1.

***X*(mm)**	x_1	x_2	x_3	***Y*(mm)**	y_1	y_2	y_3
X_1	13	21	34	Y_1	2	5	13
X_2	13	21	34	Y_2	2	5	13
X_3	13	21	34	Y_3	3	8	21
X_4	13	21	34	Y_4	3	8	21
X_5	13	21	34	Y_5	1	3	8
X_6	13	21	34	Y_6	1	3	8
X_7	8	13	21	Y_7	1	3	8

Table 2 Parameter settings for Group 2.

***X*(mm)**	x_1	x_2	x_3	***Y*(mm)**	y_1	y_2	y_3
X_1	18	29	47	Y_1	4	11	29
X_2	18	29	47	Y_2	4	11	29
X_3	11	18	29	Y_3	1	3	7
X_4	11	18	29	Y_4	1	3	7
X_5	11	18	29	Y_5	1	3	7
X_6	11	18	29	Y_6	1	3	7
X_7	8	13	21	Y_7	1	3	8

Table 3 Parameter settings for Group 3.

***X*(mm)**	x_1	x_2	x_3	***Y*(mm)**	y_1	y_2	y_3
X_1	13	21	34	Y_1	1	4	11
X_2	13	21	34	Y_2	1	4	11
X_3	13	21	34	Y_3	3	7	18
X_4	13	21	34	Y_4	3	7	18
X_5	13	21	34	Y_5	1	3	7
X_6	13	21	34	Y_6	1	4	11
X_7	8	13	21	Y_7	1	3	8

Table 4 Parameter settings for Group 4.

***X*(mm)**	x_1	x_2	x_3	***Y*(mm)**	y_1	y_2	y_3
X_1	11	18	29	Y_1	2	5	13
X_2	11	18	29	Y_2	2	5	13
X_3	11	18	29	Y_3	2	5	13
X_4	11	18	29	Y_4	2	5	13
X_5	11	18	29	Y_5	1	2	5
X_6	11	18	29	Y_6	1	3	8
X_7	8	13	21	Y_7	1	3	8

Moreover, the overall dimensions in the designed siphon pipes were the same as those of the original siphon pipe and H_0, H_1, H_2, L_1, L_2 and the pipe's diameter were 50 mm, 210 mm, 30 mm, 55 mm, 90 mm and 50 mm, respectively.

2.2 VOF Model

Computational fluid dynamics, a new subject that combines fluid mechanics, mathematics and computer science, has been widely used in ship hydrodynamics, aircraft aerodynamics, ocean engineering, environmental engineering and other fields. The essence of computational fluid dynamics is to solve fluid flow, heat transfer and related transmission phenomena by using a computer. The flushing process of a toilet is a complex and unsteady multiphase flow problem with free surface. This complex flow problem can be solved by taking advantage of the VOF (volume of fluid) model, which is applicable to study the motion of large bubbles in a liquid, the motion of a liquid after a dam break and the steady or transient tracking of any liquid-gas interface. The VOF model deals with the multiphase flow problem by introducing a single fluid model, namely that water and gas in a two phase flow field obey the same set of momentum conservation equations (see in Eq. (2)):

$$\frac{\partial}{\partial t}(\rho\vec{V}) + \nabla\cdot(\rho\vec{V}\vec{V}) = -\nabla p + \nabla\cdot[\mu(\nabla\vec{V} + \nabla\vec{V}^T)] + \rho\vec{g} \quad (2)$$

where $\vec{V}$ is the velocity vector, ρ is fluid density, μ is the viscosity coefficient, $\vec{g}$ is gravitational acceleration, and p is pressure.

Each control volume in the computational domain is filled with water or air, and the sum of their volume fractions is equal to 1 (as shown in Eq. (3)).

$$\alpha_w + \alpha_a = 1 \tag{3}$$

where α_w and α_a are the volume fraction of water and air, respectively.

Therefore, the unknown quantities and characteristic parameters of the water and the gas can be expressed by the weighted average of volume fraction as long as the volume fractions of the water and the gas in the flow field are known. The interface of water and air can be calculated by:

$$\frac{\partial \alpha_w}{\partial t} + u_i \frac{\partial \alpha_w}{\partial x_i} = 0 \tag{4}$$

2.3 Simulation Conditions

In this study, we took a kind of jet siphon toilet as the object of study. The flushing processes of the toilet under different siphon pipe shapes were calculated using the FLUENT software. The pressure-based solver was selected to solve the conservation equations and the VOF multiphase flow model was applied to track the interface between water and air. For the description of the water flow in the toilet, the realizable k-epsilon turbulence model and the standard wall function were adopted. In the light of the model characteristics of the toilet, one pressure-inlet boundary condition and two pressure-outlet boundary conditions were set up in this study, as shown in Figure 2. The pressure reference point *P* was located at the top of the water tank, which was full of air. The pressure-velocity coupling was calculated using the SIMPLE algorithm. For the sake of good convergence, the time step, the maximum number of iterations and the residual were set to 0.0002 s, 20 and 0.0001, respectively.

The water consumption in all simulations was 5.5 L. The initial conditions of the computational model are shown in Figure 2. The simulation experiments were carried out on a DELL workstation based on the parallel computing method in order to improve computing efficiency. The processors and RAM of the workstation were: Intel (R) Xeon (R) CPU E5-2620 v4 @ 2.10 GHz 2.10 GHz (dual-processor) and 32.0 GB, respectively. In order to analyze the flushing performance quantitatively, the flow field parameters of the surface *S* inside the siphon pipe during the whole flushing process were recorded, as shown in Figure 2.

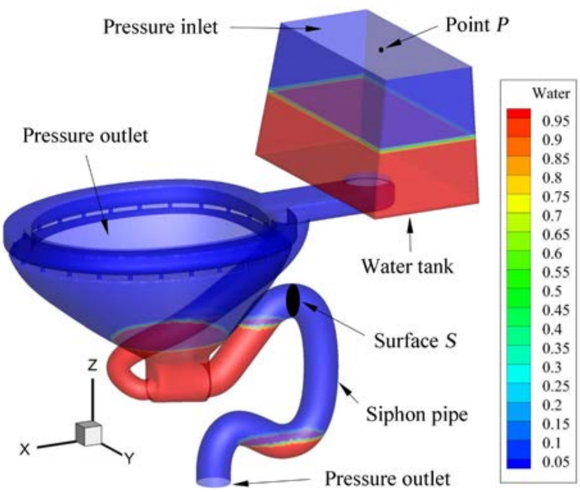


Figure 2 Boundary conditions of the simulation.

2.4 Grid Independence Verification

In the numerical calculation, the grid used is an important factor that affects the accuracy of the calculation results and the number of grids is closely related to the calculation accuracy. According to computational fluid dynamics theory, the numerical discretization of partial differential equations can be done by calculating the physical quantities at a number of nodes and then interpolating them to obtain the values between the nodes. Therefore, in theory, a large number of grids leads to a high accuracy of the calculation results. In our practical application, however, the number of grids could not be increased infinitely due to the limitations of the computing resources of the computer. Moreover, the rounding errors caused by computer floating point arithmetic operations will also increase as the number of grids grows. Hence, the grid independence should be verified before the numerical calculations are done to find an appropriate balance between computational accuracy and computational overhead. The essence of grid independence verification is to validate the sensitivity of the calculation results related to the change of the number of grids.

In this work, the computational model was meshed using the ICEM CFD software and the number of grids was determined by controlling the global mesh size. Based on the structural characteristics of the toilet model, structured grids were utilized in the water tank to reduce the number of grids, while unstructured grids were applied in other components for the convenience of grid partition. Four group simulations with global grid sizes of 9 mm, 7 mm, 5 mm and 4 mm were performed to study the sensitivity of the grid quantity to the simulation results. In order to facilitate the comparison of the calculation results, we took the mass flow rate on surface *S* as the research target. The changes of the mass flow rate over time under different global mesh sizes are shown in Figure 3.

Figure 3 shows that the mass flow rate tends to stabilize when the global grid size decreases, which indicates that a small global grid size may contribute to the improvement of calculation accuracy. When the global grid size is about 5 mm, the deviations of the calculation results are relatively small compared to a global grid size of 4 mm. Furthermore, the reduction of the global grid size brings about an increase in the number of grids and the computation time is increased by 55 hours when the global grid size is changed from 5 mm to 4 mm, as illustrated in Figure 3. On the whole, the number of grids with a global grid size of 5 mm is reasonable considering the simulation precision and computation time. In this work, the global grid size was set to 5 mm; the total number of grids was about 2.24 million.

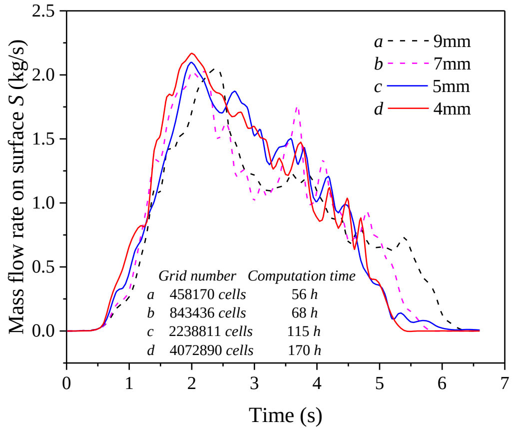


Figure 3 Influence of global mesh size on calculation results.

3 Results and Discussion

3.1 Simulation Results

The flushing process of a siphon toilet is based on the siphon effect, which generates negative pressure inside the siphon pipe. Therefore, the negative pressure is an important parameter to describe the siphon intensity. The lower the negative pressure, the stronger the siphon intensity. In addition, some scholars have evaluated the siphon intensity by using the water velocity and the mass flow rate inside the siphon pipe [13-15]. For a better understanding of the siphon intensity under different siphon pipe shapes, the changes of water flow velocity, negative pressure and mass flow rate on surface *S* during the flushing process were recorded, as shown in Figures 4-6. Furthermore, the integral volume fraction of water, which reflects the duration of siphon intensity, was also monitored, as shown in Figure 7. For ease of comparison, the extreme and average values of the flow field parameters were calculated, as listed in Table 5.

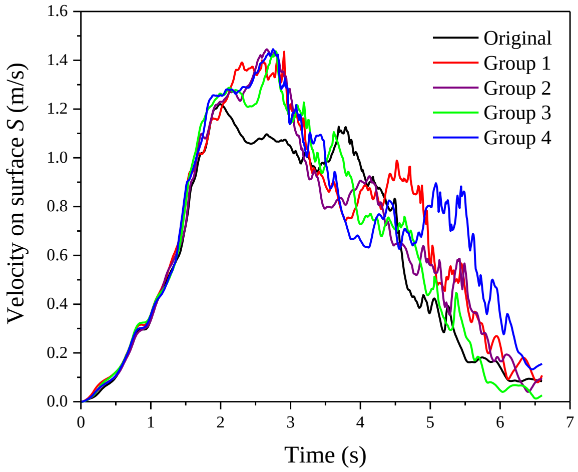


Figure 4 Changes of water flow velocity over time.

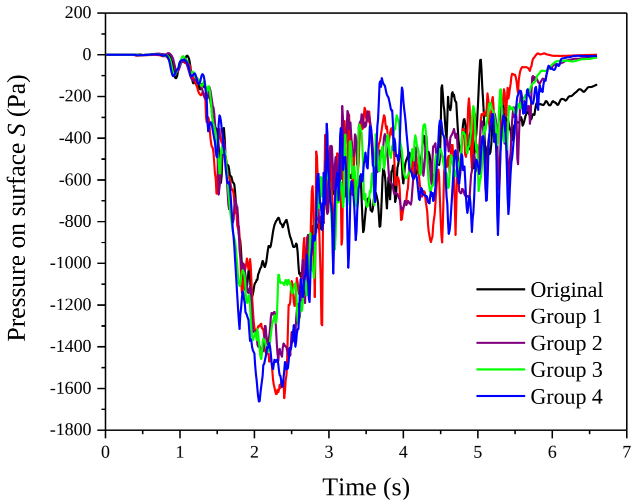


Figure 5 Changes of negative pressure over time.

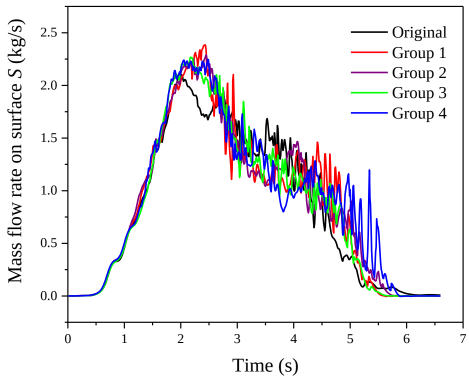


Figure 6 Changes of mass flow rate over time.

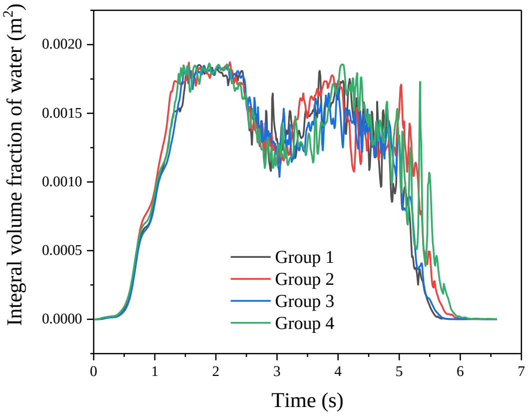


Figure 7 Changes of integral volume fraction of water over time.

Table 5 Extreme values and average values of flow field parameters.

Group number	Original	Group 1	Group 2	Group 3	Group 4
Maximum flow velocity (m/s)	1.2201	1.4351	1.4454	1.4365	1.4457
Average flow velocity (m/s)	0.6058	0.6914	0.6568	0.64288	0.7177
Minimum negative pressure (Pa)	-1166.7171	-1646.7962	-1467.1719	-1459.4333	-1662.5238
Average negative pressure (Pa)	-429.9278	-440.6120	-451.2841	-443.3144	-485.9841
Maximum mass flow rate (kg/s)	2.1042	2.3856	2.2890	2.2682	2.2450
Average mass flow rate (kg/s)	0.8387	0.8693	0.8748	0.8564	0.9016

It can clearly be seen from Table 5 that Group 4 has the largest maximum flow velocity at 1.4457 m/s, while the maximum flow velocity of the original group is only 1.2201 m/s. At the same time, the average flow velocity of Group 4 reaches a maximum value of 0.7177m/s, which is 0.1119 m/s higher than that of the original group. As can be observed from Figure 4, the occurrence time of the maximum flow velocity of the improved groups is later than that of the original group, which indicates that the siphon intensity of the improved siphon pipe is higher. Furthermore, the minimum negative pressure and the average negative pressure of Group 4 all obtain the minimum value, which illustrates that the siphon intensity is the best at this time. From Figure 6 and Table 5 we can notice that the maximum mass flow rate of Group 1 and the average mass flow rate of Group 4 get the largest value. This implies that Group 1 and Group 4 have better siphon effect and the average siphon intensity of Group 4 during the entire flushing process is optimal. From another perspective, the duration of the siphon intensity of Group 4 is the longest, which means that the siphon effect is the strongest under such condition, as shown in Figure 7. Based on the above analysis, we can conclude that Group 4 has the best siphon intensity and the best flushing performance, which reveals that the structural parameters of the siphon pipe of Group 4 are reasonable.

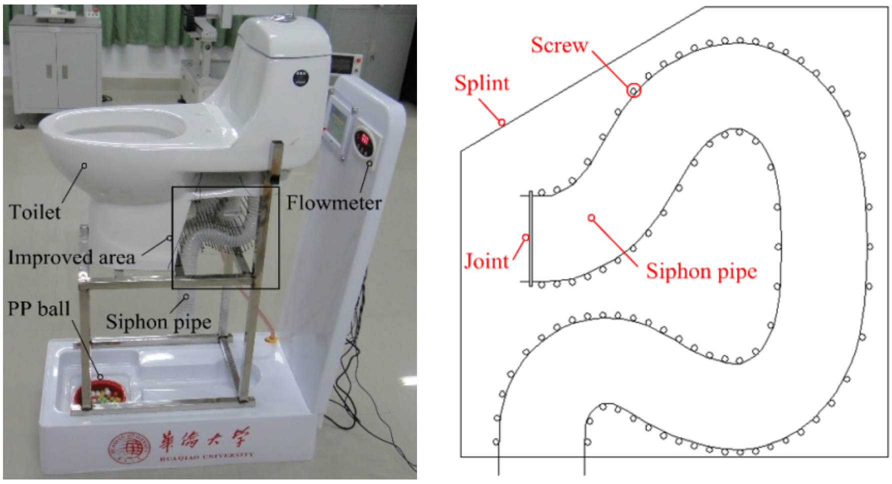
3.2 Experiment

The simulation results show that the improved toilet based on the Fibonacci sequence and the Lucas sequence has larger siphon intensity and stronger flushing performance than the original toilet. In order to verify the reliability of the simulation results, flushing experiments with a toilet were carried out based on an experimental platform (see Figure 8(a)) of a siphon toilet according to Chinese national standards. The water consumption was 5.5 L, which was transported to the water tank automatically by using a motor and a flow meter. In this work, 100 polypropylene balls, whose diameter and density were 17.5 mm and 910 kg/m^3 respectively, were adopted to conduct the flushing experiments. In order to describe the flushing effect reasonably, the flushing process of each group was repeated six times and the average number of polypropylene balls that were washed out of the toilet was counted. For quantitative description of the flushing performance, the average flushing efficiency was defined as:

$$F_A = \frac{\sum_{i=1}^{6} B_i}{6B_A} \times 100\% \qquad (5)$$

where B_i is the number of polypropylene balls that are washed out of the toilet in the *i*-th flushing experiment and B_A is the total number of polypropylene balls used in the flushing experiment.

The flushing performance of the original toilet was tested first. Subsequently, the original siphon pipe was removed and replaced by the improved siphon pipes. The material of the improved siphon pipes was rubber hose, which could be adjusted to different shapes. In order to keep the siphon pipe in the right shape under different experiments, splints and screws were used to fix the position of the siphon pipe, as shown in Figure 8(b). The shapes of the arrangement of screws were the same as those of the siphon pipe shapes in the simulations. Moreover, the improved siphon pipes were connected to the toilet at the joint. The shapes of the siphon pipe improved by different sequence combinations are shown in Figure 9. Finally, the toilet flushing processes under different siphon pipe shapes were performed. The experimental results are listed in Table 6.



(a) Experimental process (b) Fixing device of siphon pipe

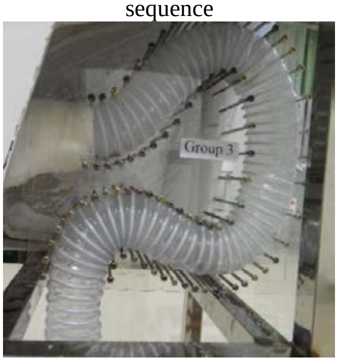
Figure 8 Experimental platform of the toilet.

It can be seen from Table 6 that the average flushing efficiency of the improved toilet was higher than that of the original toilet. Furthermore, Group 4 got the best flushing performance, which is consistent with the simulation results. This means that the numerical simulation method can be used to study the flushing performance of a toilet to shorten the product development cycle. Table 6 also indicates that the improved siphon pipe based on the Fibonacci sequence and the Lucas sequence could enhance the flushing performance of the toilet effectively. When Lucas numbers were adopted to restrict the curvature in the X_i

direction and Fibonacci numbers were applied to regulate the curvature in the Y_j direction, the shape of the siphon pipe was the most reasonable.

(a) X_i: Fibonacci sequence; Y_i: Fibonacci sequence

(b) X_i: Lucas sequence; Y_i: Lucas sequence



(c) X_i: Fibonacci sequence; Y_i: Lucas sequence

(d) X_i: Lucas sequence; Y_i: Fibonacci sequence

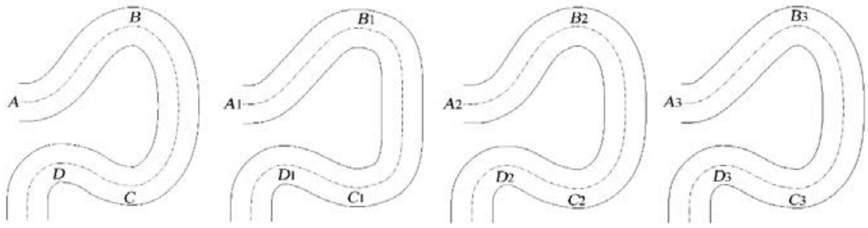
Figure 9 Shapes of the siphon pipe improved by different sequence combinations.

Table 6 Flushing performance of toilet under different siphon pipe shapes.

Group number	**Original**	**Group 1**	**Group 2**	**Group 3**	**Group 4**
F_A (%)	76	85	80	82	92

3.3 Shape Characteristics of Improved Siphon Pipe

The shapes of the improved siphon pipe are shown in Figure 10. Table 6 shows that Group 4 reached the maximum average flushing efficiency and Group 2 obtained the minimum average flushing efficiency, which indicates that the shape of Group 4 is more reasonable than that of Group 2. From Figure 10(b) and 10(d) we can see that the curvature of A_3B_3 is smoother than the curvature of A_1B_1, which shows that a smooth A_iB_i contributes to the improvement of the flushing performance. The flushing performance of Groups 1 and 4 was better than that of Groups 2 and 3.



(a) Group 1 (b) Group 2 (c) Group 3 (d) Group 4

Figure 10 Shapes of the siphon pipe in different groups.

We can see that the shapes of *BC* and B_3C_3 are similar and there are obvious transitions at point *B*, point B_3, point *C* and point C_3 by observing Figure 10(a) (Group 1) and Figure 10(d) (Group 4). This is an indication that the curvature should be smooth and have obvious transitions at the connections of different sections when we design the shape of B_iC_i to improve the siphon intensity inside the siphon pipe during the flushing process. Similarly, this principle should also be followed when designing the structural parameters of C_iD_i. In order to explore the relationship between siphon pipe length and flushing performance, the length of the center line of the improved siphon pipes was calculated, as listed in Table 7.

Table 7 Center line length of different groups.

Group number	Group 1	Group 2	Group 3	Group 4
Centerline length (mm)	632.5069	645.4138	637.5775	632.0697

As can be seen from Table 7, the length of Group 4 is the shortest, followed by the length of Group 1 and the length of Group 2 is the longest. This finding suggests that when the overall size is unchanged, the shorter the length of the siphon pipe, the better the flushing performance. This is mainly because the frictional resistance of the water flow in a shorter siphon pipe is smaller than in a longer siphon pipe, which leads to lower pressure loss in the shorter pipe during the flushing process. This is the reason why Group 4 had the best flushing performance. Hence, the length of the siphon pipe should be reduced as much as possible after selecting the overall size when designing the shape of the siphon pipe to reduce frictional resistance and improve flushing performance.

4 Conclusions

In this paper, the Fibonacci sequence and the Lucas sequence were used to design the shape of the siphon pipe of a flushing toilet. The toilet flushing performance under different siphon pipe shapes was studied by performing numerical simulations and experiments. The main conclusions are as follows:

1. The Fibonacci sequence and the Lucas sequence can be used to design the structural parameters of the siphon pipe of a toilet. When the Lucas numbers are utilized to restrict the curvature of the siphon pipe in the X_i direction and the Fibonacci numbers are adopted to design the curvature of the siphon pipe in the Y_j direction, the flushing performance of the toilet is optimal.
2. To obtain strong siphon intensity and a good flushing effect, the curvature of the siphon pipe should be smooth and have obvious transitions at the connections of different sections. When the overall size of the siphon pipe remains unchanged, a short siphon pipe length helps to improve the flushing performance of the toilet.
3. It should be pointed out that the shape design of a siphon pipe was based on a fixed overall size of the siphon pipe in this work. The next step is to design the siphon pipe shape under different overall structural parameters to further improve the flushing performance of the toilet.

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References

[1] An, I.Y., Lee, Y.L., Jung, E.D. & Cho, W.S., *A Study on Performance Optimization of a Toilet by Measuring Accumulated Flow Rate of a Trapway*, Journal of Mechanical Science and Technology, **28**(4), pp. 1319-1326, 2014.

[2] Cheng, W.S., Lee, R.T., Liu, C.H. & Hsia, C.W., *A Study on Evacuation Performance of Siphon-Type Water Closets*, The 10th Asian International Conference on Fluid Machinery, **1225**, pp. 75-80, 2010.

[3] Zhao, S.Y., Liu, Z.J. & Peng, Z.W., *Optimized Design of Toilet Siphon Pipeline Based on Fluent and Its Validated Test*, Journal of System Simulation, **20**(16), pp. 4412-4416, 2008. (Text in Chinese and Abstract in English).

[4] An, I.Y., Lee, Y.L. & Kim, J.H., *A Study of the Characteristics of a Super Water-saving Toilet with Flexible Trapway by Measuring Accumulated Flow Rate*, Journal of Mechanical Science and Technology, **28**(8), pp. 3067-3074, 2014.

[5] Cheng, W.S. & Jhang, C.Y., *Velocity Measurement and Flow Field Simulation of a Sit-Type Water Closet*, The 4th International Meeting of Advances in Thermofluids, **1440**(1), pp. 1068-1077, 2012.

[6] Wang, Y.H., Xiu, G.J. & Tan, H.S., *CAD and CAE Analysis for Siphon Jet Toilet*, Physics Procedia, **19**(1), pp. 472-476, 2011.

[7] Peng, Z.W., Liu, Z.J. & Huang, D.L., *Study of Water Saving and Wastewater Minimization Technology Based on Computational Fluid Dynamics*, Technology of Water Treatment, **35**(8), pp. 72-75, 2009. (Text in Chinese and Abstract in English)

[8] Pan, X.T., Sun, Y.S., Zhan, J.M. & Li, J., *The Determinant Method of Ceramic Sanitary Ware Forming Siphon Phenomenon Based on VOF Model*, Acta Scientiarum Naturalium Universitatis Sunyatseni, **46**(4), pp. 36-40, 2007. (Text in Chinese and Abstract in English)

[9] Li, Z.F. & Tong, X., *Applications of the Discrete Element Method and Fibonacci Sequence on a Banana Screen*, Journal of Engineering, Design and Technology, **15**(1), pp. 2-12, 2017.

[10] Tashtoush, Y., Darwish, O. & Hayajneh, M., *Fibonacci Sequence Based Multipath Load Balancing Approach for Mobile Ad Hoc Networks*, Ad Hoc Networks, **16**, pp. 237-246, 2014.

[11] Adam, M., Assimakis, N. & Farina, A., *Golden Section, Fibonacci Sequence and the Time Invariant Kalman and Lainiotis Filters*, Applied Mathematics and Computation, **250**, pp. 817-831, 2015.

[12] Livio, M., *The Golden Ratio: The Story of Phi*, The World's Most Astonishing Number, Broadway Books, 2002.

[13] Liu, F.M. & Pan, X.T., *Three-phase Flow Analysis for Siphon Jet Toilet Based on VOF Model*, China Ceramics, **47**(11), pp. 69-72, 2011. (Text in Chinese and Abstract in English).

[14] Xiu, G.J., *Three-dimensional Flow Numerical Simulation and Rapid Manufacturing for Toilet Based on RE*, Master Dissertation, School of Mechanical and Automobile Engineering, South China University of Technology, Guangzhou, 2011. (Text in Chinese and Abstract in English)

[15] Zhai, L.X., Wu, J.Q., Zhan, Y. & Zhang, L., *Flow Characteristic and Analysis of Closet Pan Outfall*, Water & Wastewater Engineering, **40**(5), pp. 78-81, 2014. (Text in Chinese and Abstract in English).