



Improved Wavelet Threshold De-noising Method Based on GNSS Deformation Monitoring Data

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Abstract. In the process of GNSS deformation monitoring, it is inevitable that the monitoring data are contaminated by noise. Effectively mitigating the impact of noise on the measurements and thus improving the quality of the deformation data is the objective of GNSS data processing. Wavelet analysis can analyse the signal according to different frequencies of the signal. Simulation data can be used to determine the best wavelet basis function and select the appropriate decomposition level. In this paper, an improved threshold de-noising method is proposed, based on an analysis of conventional hard threshold de-noising, soft threshold de-noising and compulsory de-noising methods. The improved method was examined through a simulation analysis and applied in an engineering case. The results show that it effectively removed the noise at high frequencies while retaining data details and mutation. The de-noising ability of the proposed technique was better than that of the conventional methods. Moreover, this method significantly improved the quality of the deformation data.

Keywords: *GNSS deformation monitoring data; improved threshold de-noising method; best wavelet basis function; appropriate decomposition level; threshold de-noising; wavelet analysis.*

1 Introduction

In the GNSS deformation monitoring process, errors (i.e. noise) are inevitably present in the monitoring results because of the influence of external conditions, measurement methods, measurement instruments, data transmission, and other factors. In particular, white noise from the multi-path effect covers almost the entire monitoring range of the GNSS deformation data. Monitoring data consist of two parts, namely real data and noise. Thus, the objective of GNSS deformation data processing is to effectively reduce errors (noise) that affect the data, preventing the real data to be covered by noise. Traditional processing methods for deformation monitoring data have wide applications but are limited

by a number of drawbacks. For example, the commonly used Kalman filter and smoothing method are capable of data de-noising [1-2] but broaden the waveform. Moreover, smoothing the mutation component of the signal leads to loss of important information regarding mutation. Wavelet analysis is a local and multi-scale analysis tool that can decompose a known data signal according to its various frequencies. As the scale changes from large to small, the signal is transformed from coarse to fine observation. The low-frequency part has a high frequency resolution and a low time resolution, whereas the high-frequency part has a low frequency resolution and a high time resolution.

The first step of wavelet de-noising is selecting the best wavelet function. Under normal circumstances, the best wavelet function is selected based on simulation examples [3], after which the de-noising method is selected. The most widely applied method is threshold de-noising, which includes hard threshold de-noising, soft threshold de-noising, and forced de-noising [4]. Finally, based on the high-frequency signal and a given specific threshold, the low-frequency and high-frequency parts are reconstructed [5]. This study performed a systematic analysis of the different threshold de-noising methods and proposes an improved threshold de-noising method that is based on the data processing of a simulation experiment and an engineering example. Compared with conventional de-noising methods, the improved threshold de-noising method can better eliminate noise while effectively retaining the original data details and mutation, thus avoiding the loss of effective data from the high frequency part. Therefore, the proposed method has the potential to improve the quality of GNSS deformation monitoring data.

2 Principle of Wavelet Threshold De-noising and the Improved Method

GNSS deformation monitoring data can be viewed as a set of signals consisting of real data and noise data, where the noise primarily comprises random white noise. The noise appears as high-frequency signals that are mainly contained in the high frequencies of the wavelet. The real data appear as low-frequency signals and some stationary signals.

$$s(t) = x(t) + w(t) \quad (1)$$

In Formula (1), $s(t)$ denotes monitoring data, $x(t)$ denotes real data, $w(t)$ denotes noise data, and $w(t)$ denotes Gaussian white noise.

2.1 Wavelet Analysis

The purpose of wavelet transform is to approximate a signal or function from a series of functions. The series of functions is called the wavelet basis function,

which is obtained by wavelet translation and scaling. The basis function has the function of multi-resolution analysis. It is a window of constant size, although the shape of the window can change the time-frequency analysis method [6].

$$(T^{WAV} f)(a, b) = |a|^{-1/2} \int dt f(t) \psi\left(\frac{t-b}{a}\right) \quad (2)$$

$$\psi^{a,b}(s) = |a|^{-1/2} \psi\left(\frac{s-b}{a}\right) \quad (3)$$

Formula (2) is the wavelet transform model, and Formula (3) is called the mother wavelet. In Formula (3), $\psi^{a,b}(s)$ and ψ are the same functions. $\psi^{a,b}(s)$ is the result of ψ obtained through telescopic translation. a is the telescopic factor, and b is the translation factor. The other wavelet basis functions are all obtained by changing the mother wavelet scaling factor (a) and the translation factor (b).

2.2 Principle and Method of Wavelet De-noising

The features of wavelet transform include time-frequency localization, multi-resolution analysis, de-correlation and flexible choice of basis function resolution, de-correlation and flexible choice of basis function. These features allow wavelet transform to effectively remove the noise coefficient while maximizing the retention of the signal coefficients. In this way, optimal estimation of the real data signals can be obtained [7]. Wavelet de-noising can be divided into three main steps [8]:

- (1) Data signal wavelet decomposition. Depending on the need for data smoothing and noise reduction in relation to the adaptability of the model, the best wavelet function and wavelet decomposition level are selected to decompose the data signal; the scale coefficients and wavelet coefficients are extracted; and the high-frequency coefficients are separated from the low-frequency coefficients.
- (2) Threshold quantization analysis of the high-frequency coefficient layer of wavelet decomposition. The high-frequency coefficients are quantified based on different threshold de-noising methods and threshold selection methods.
- (3) Data signal reconstruction. In order to reconstruct the quantized high-frequency coefficients and low-frequency coefficients, the data signals are obtained after de-noising [9-12].
- (4) Hard threshold de-noising. After the data signal is decomposed with the wavelet function, if the high-frequency coefficient of the different layers exceeds a given threshold, then it is reserved; otherwise, it is zero, i.e.:

$$w_\lambda = \begin{cases} w & |w| \geq \lambda \\ 0 & |w| < \lambda \end{cases} \quad (4)$$

(5) Soft threshold de-noising. After the data signal is decomposed with the wavelet function, if the high-frequency coefficient of the different layers exceeds a given threshold, then it should be subtracted from the threshold; otherwise, it is zero, i.e.:

$$w_\lambda = \begin{cases} \text{sign}(w)(|w - \lambda|) & |w| \geq \lambda \\ 0 & |w| < \lambda \end{cases} \quad (5)$$

(6) Compulsory de-noising. To compulsory eliminate the decomposed components of the high-frequency part, the coefficient should be zero, i.e.:

$$D(n) = \text{zeros}(l, \text{length}(d(n))) \quad (6)$$

In Formulas (4), (5), and (6), w_λ denotes the processed data, w denotes the low-frequency data signal, λ is a given threshold, $\text{sign}(w)$ denotes the symbols of w , $D(n)$ represents the high-frequency coefficient, $\text{zeros}(l, \text{length}(d(n)))$ is for setting the high-frequency part to zero.

Choosing the appropriate threshold is the key to effective wavelet threshold de-noising. The classical threshold proposed by Donoho [13] is shown in Formula (7). In the present study, the improved method of Zhao Baozhen was used (Formula (8)).

$$\delta = \sigma_\lambda \sqrt{2 \log N} \quad (7)$$

$$\delta = \sigma_\lambda \sqrt{2 \log N / IN(e + \lambda - 1)} \quad (8)$$

The signal-to-noise ratio (SNR), root mean square error (RMSE), and the similarity between the graphs of the reconstruction and the original data are three effective indicators for evaluating the quality of data processing [14]. To ensure that the reconstructed data are not distorted, the SNR should be increased, RMSE should be small, and the wavelet de-noising effect should be better.

$$SNR = -10 \times \log \frac{\sum_{i=1}^n [I(i) - I_d(i)]^2}{\sum_{i=1}^n I^2(i)} \quad (9)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_i^n [I_d(i) - I(i)]^2} \quad (10)$$

In the formula, SNR is the signal-to-noise ratio, $RMSE$ is the root mean square error, $I(i)$ are the after-processing data, $I_d(i)$ are the original data, and N is the number of data.

2.3 De-noising Method of Improved Threshold

Using different threshold de-noising methods can lead to different de-noising results. In forced de-noising, the high-frequency coefficient is set to zero. Although the de-noising effect is obvious, useful data from the high-frequency part is lost. As a result, hard threshold and soft threshold de-noising methods are more widely used [15-18]. However, Formulas (3) and (4) show that when the high-frequency coefficient is below the threshold, both methods make it zero. Consequently, useful information from the high-frequency part is lost. Therefore, we propose an improved threshold de-noising method. If the high-frequency coefficient exceeds a given threshold, then it is reserved; otherwise, this part will be multiplied by a constant (a). When $a = 0$, this method is equivalent to hard threshold de-noising, i.e.:

$$w_\lambda = \begin{cases} w & |w| \geq \lambda \\ \text{sign}(w) \times a \times |w| & |w| < \lambda \end{cases} \quad (11)$$

Through reverse thinking, arbitrary values of a can be considered to establish the mathematical relationship between SNR and a , and consequently establish a mathematical model of the two. Simulation experiments revealed that the a value and the SNR form a parabola. The mathematical expression is then fitted in Matlab to obtain the derivative of the formula in order to find the value of a . When the mathematical relationship reaches the maximum value, the value of a is the value chosen by the improved method.

3 Simulation Test

3.1 Selection of Wavelet Base Function

The selection of the wavelet basis function is mainly based on the de-noising effect on the actual data. This study utilized Matlab as the platform. Block data were used, and random white noise was added to the data. Then, the basis functions sym4, sym8, db3, and db6 were used to de-noise. The SNR and $RMSE$ of the data processing were compared for the four kinds of wavelet basis functions. Finally, the most suitable wavelet basis function was selected.

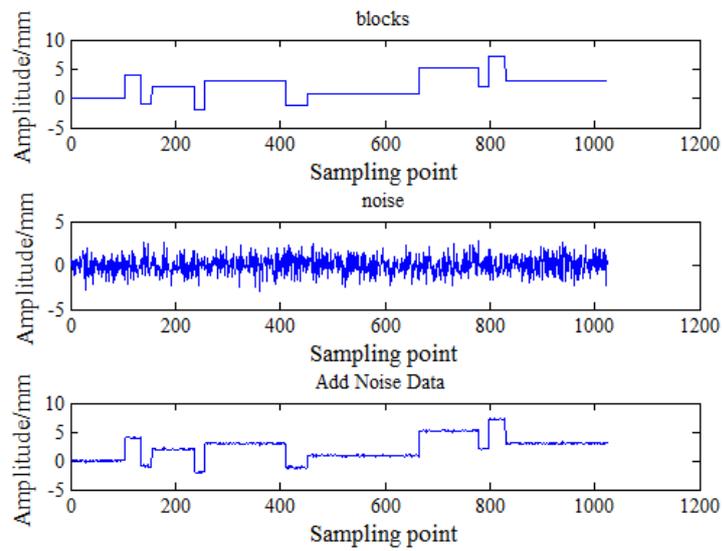


Figure 1 Simulation of data preparation.

In Figure 1, the data available in the blocks call Matlab (Figure 1 – top), and Matlab were used to generate a random white noise $\text{noise} = 0.1 \times \text{randn}(\text{size}(\text{blocks}))$ (Figure 1 – middle), and the noise was added to the known data generated (Figure 1 – bottom).

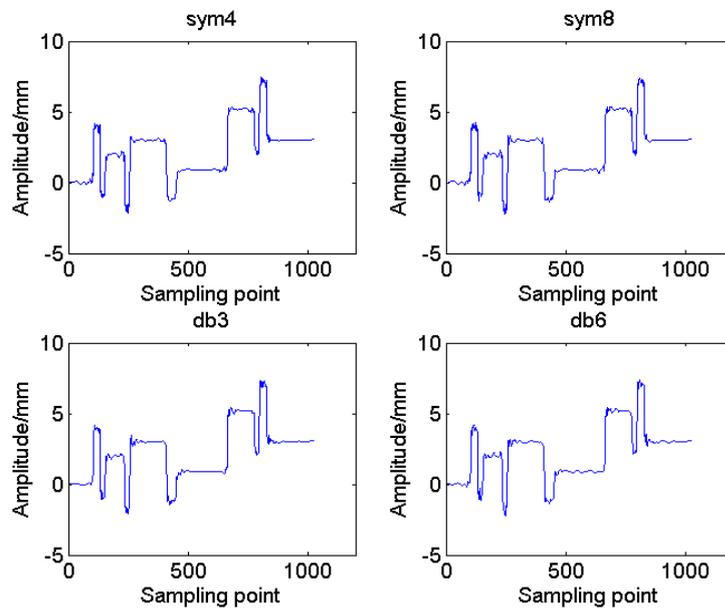


Figure 2 Data processing of different wavelet basis functions.

Table 1 SNR and RMSE of Processing Data by Different Wavelet Bases.

The Type Of Wavelet	SNR/dB	RMSE/mm
sym4	24.6242	0.1744
sym8	23.8359	0.1910
db3	24.7156	0.1726
db6	23.2220	0.2050

As shown in Figure 2 and Table 1, de-noising the same data using different wavelet basis functions leads to different results of data signal processing. Systematic analysis was conducted on four kinds wavelets, namely, sym4, sym8, db3, and db6, which are the most commonly used in deformation data processing. As shown in Table 1, the SNR of the maximum db3 value was 24.7156, whereas the minimum value was 23.2220 db6; the RMSE of the maximum db6 value was 0.2050, whereas the minimum value was 0.1726 db3. Therefore, db3 is more suitable for data signal de-noising.

3.2 Selection of Wavelet Decomposition Level

In the process of wavelet de-noising, the selection of the optimal decomposition level is one of the main factors that influence the de-noising effect [19-25]. In this study, the best decomposition layer was selected based on an analysis of the leleccum data signal 2~9 layer and on a systematic analysis of the SNR and RMSE curves of the hierarchical reconstruction data.

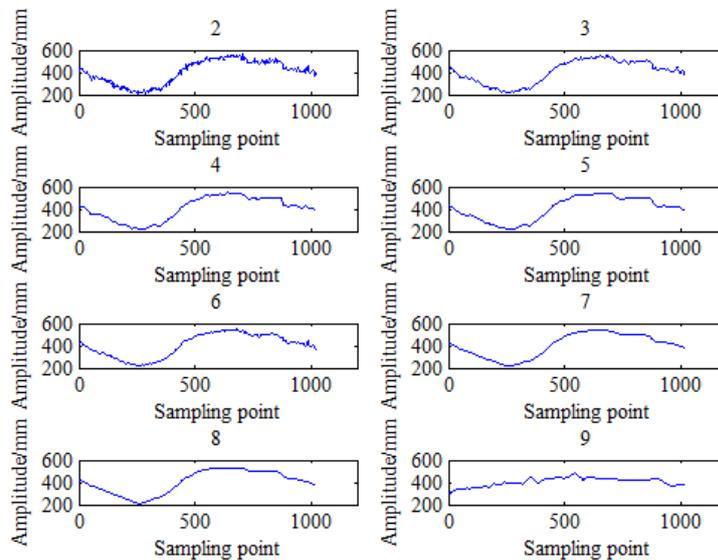
**Figure 3** Influence of different decomposition levels on data de-noising.

Table 2 SNR and RMSE of Different Decomposition Levels on Data De-noising.

Decomposition level	SNR/dB	RMSE/mm
2	32.9312	9.4213
3	34.9907	7.4325
4	35.6229	6.9108
5	35.3708	7.1143
6	34.6478	7.7319
7	33.8591	8.4668
8	33.9070	8.4202
9	13.3250	90.0378

As shown in Figure 3 and Table 2, when the decomposition levels were not the same, the data de-noising effect was not the same. Figure 3 shows that when the decomposition level was 9, the reconstructed data contained graphic distortion. The reason is that the wavelet analysis regards the real data signal as noise that should be eliminated. When the decomposition level was 2, although the reconstructed data signal was not distorted, the noise was not eliminated very well. Therefore, to ensure that the data signal is without distortion, the optimal decomposition level must be selected based on SNR and RMSE. In this study, the decomposition layer was 4.

3.3 Improved Threshold De-noising Method

This study adopted the db3 wavelet function for the simulation data signal by a four-layer wavelet decomposition. Suppose a value of a . Using the improved threshold de-noising method presented in this paper, different values of a were calculated corresponding to SNR, as shown in Table 3.

Table 3 Different a Corresponding to SNR

a	SNR/dB	a	SNR/dB
1	27.1133	0.1	37.2911
0.9	27.9905	0.04	37.3235
0.8	28.9525	0.005	37.1876
0.7	30.0123	0.007	37.1983
0.6	31.1826	0.0001	37.1600
0.5	32.4702	0.00005	37.1597
0.4	33.8620	0.00001	37.1595
0.3	35.2882	0.000005	37.1595
0.2	36.5536	0	37.1595

As shown in Figure 4, to calculate the function expression (12), a is set as the independent variable and SNR as the dependent variable. The corresponding value of a is calculated when the expression reaches the maximum value.

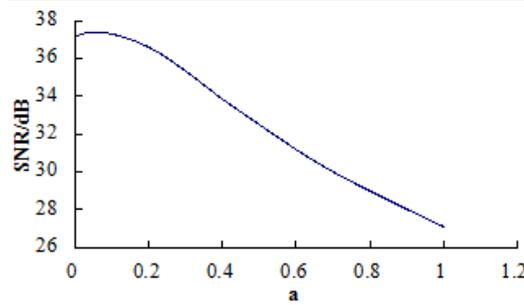


Figure 4 The mathematical relationship between a and SNR.

$$f(x) = 20.59x^5 - 78.52x^4 + 114.7x^3 - 74.57x^2 + 7.791x + 37.15 \quad (12)$$

$$f'(x) = 102.9717x^4 - 314.0858x^3 + 344.0005x^2 - 149.1466x + 7.791 \quad (13)$$

The solution of the mathematical expression: $X_1 = 1.311$, $X_2 = 0.8393 + 0.5051i$, $X_3 = 0.8393 - 0.5051i$, and $X_4 = 0.0601$. Therefore, $a = 0.0601$ ($0 \leq a < 1$).

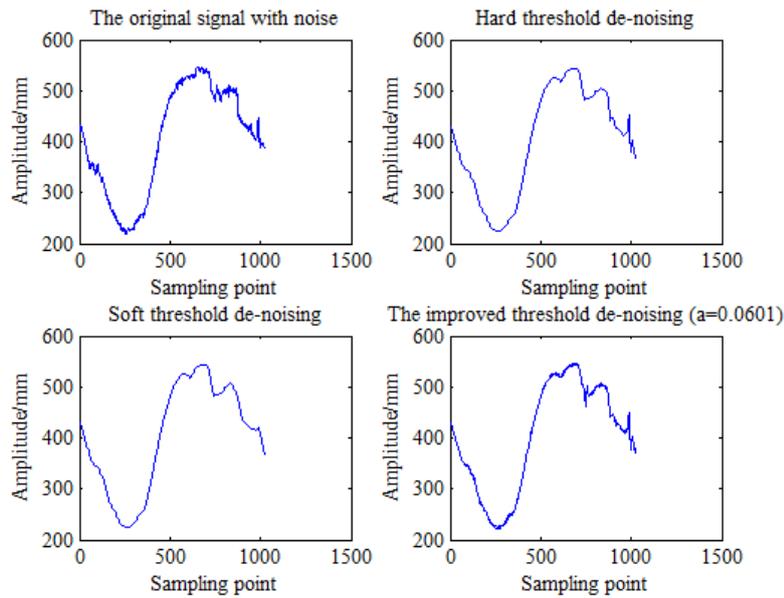


Figure 5 Data comparison of different threshold de-noising methods and improved threshold de-noising method.

Figure 5 shows the data reconstruction process. Although the processed data of the hard threshold and soft threshold de-noising were not distorted and the smoothness of the data was maintained, original data details and mutation were lost. The improved threshold de-noising method not only assures the absence of

distortion but also well preserves data details and mutation. As can be seen in Figure 5, the data signal obtained by the improved threshold de-noising method was closer to the original data signal than that obtained by the other two methods for processing the data signal. As shown in Table 4, the SNR of the improved threshold de-noising was 38.5541 of the maximum, and its RMSE was 4.9314 of the minimum, suggesting that the improved method is superior to the conventional threshold de-noising methods.

Table 4 Different Threshold De-noising Methods and Improved Threshold De-noising Method for SNR and RMSE.

De-noising Type	SNR/dB	RMSE/mm
Original signal with noise	27.5134	17.5793
Hard threshold de-noising	37.5087	5.5621
Soft threshold de-noising	36.6976	6.1065
Improved threshold de-noising ($a = 0.0601$)	38.5541	4.9314

4 An Engineering Example

To verify the de-noising effect of the improved threshold de-noising method in practice, it was applied to an Anshan mine waste dump for open-pit GNSS deformation monitoring data processing. The receiver was composed of GPS, GLONASS, COMPASS. The plane precision was $\pm (2.5 + 1 \times 10^{-6} D)$ mm, and the elevation accuracy was $\pm (5 + 1 \times 10^{-6} D)$ mm. In this experiment, mainly monitoring with the rapid static method was used, once every five days. Each observation time was 30 min, with a sampling interval of 10 s, and each observation time was strictly conducted at the same time every day.

All phase noise data were de-noised using hard threshold de-noising, soft threshold de-noising and the improved threshold de-noising according to the abovementioned method to establish the mathematical relationship between a and SNR. As shown in Table 5, $a = 0.2838$. After wavelet analysis was applied to analyze the monitoring data, the data point error was improved compared with that of the untreated data. The results show that the effect of hard threshold de-noising was better than that of soft threshold de-noising. We propose an improved threshold de-noising method to minimize errors.

Table 5 Median Error of a Different Cycle Threshold Data Processing Method of No. 18 Monitoring Points.

Processing method	Median error
Original data	0.1692
Hard threshold de-noising	0.0807
Soft threshold de-noising	0.0949
Improved method for de-noising	0.0658

Hard threshold de-noising, soft threshold de-noising, and improved threshold de-noising were used to process the 31 cycle data.

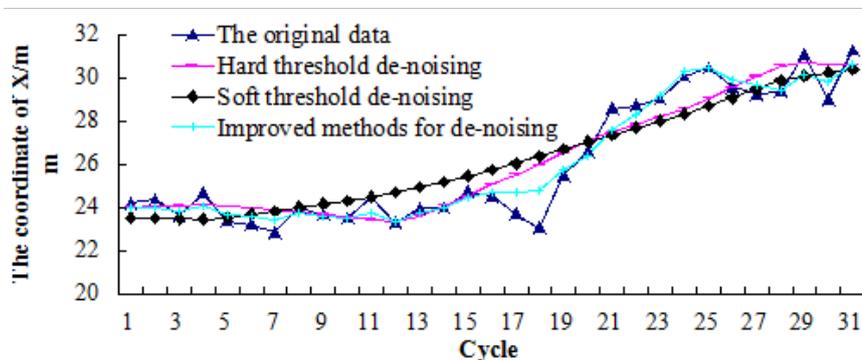


Figure 6 Line chart of data processing with different threshold de-noising methods.

Table 6 SNR and RMSE of Different Threshold De-noising Methods.

Processing method	SNR	RMSE
Hard threshold de-noising	30.1265	0.1804
Soft threshold de-noising	29.2616	0.2027
Improved method for de-noising	33.4764	0.0967

As shown in Figure 6, the data signal obtained by the improved wavelet threshold de-noising method was closer to the original data signal than that obtained by hard threshold de-noising and soft threshold de-noising. Although hard threshold de-noising also removed the noise very well and effectively smoothed the data signal, useful data were missing from the high-frequency part. Consequently, the data signal lost its original details and mutation. However, in our investigation, the improved threshold de-noising method preserved the details and mutation of the high-frequency part, thereby making the after-processing data closer to the original data. Moreover, as shown in Table 6, the improved threshold de-noising method presented in this paper performed better than both other methods in image de-noising for improving the quality of the data.

5 Conclusion

In this paper, an improved threshold de-noising method was proposed based on data simulation and an engineering example, as well as systematic analysis of the selection of the wavelet de-noising basis function and the optimal decomposition level. The best wavelet basis function can be selected by

analyzing the results of actual data processing through Matlab data signal simulation of the quantitative and qualitative analyses of four kinds of the most commonly used deformation data processing wavelet bases. The selected wavelet base in this paper was db3. The decomposition level of the data signal wavelet analysis was one of the other main factors that affected the data processing results. In this study, the selected number of layers of decomposition was 4. Analysis of hard threshold de-noising, soft threshold de-noising, and forced de-noising revealed that the quality of the data processing of the improved threshold de-noising method was the best. Data simulation and an engineering example revealed that the improved method yielded a better de-noising effect than the conventional methods.

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