Linear Parameter-Varying Versus Linear Time-Invariant Reduced-Order Controller Design for Turboprop Aircraft Dynamics

Widowati[1], Bambang Riyanto[2] & Hari Muhammad[3]

[1]Mathematics Department, Diponegoro University
Jalan Prof. H. Soedarto, S.H., Semarang, Jawa Tengah 50275, Indonesia
[2]Electrical Engineering Study Program, Institut Teknologi Bandung
Jalan Ganesha No. 10 Bandung, Jawa Barat 40132, Indonesia
[3]Aerospace Engineering Study Program, Institut Teknologi Bandung,
Jalan Ganesha No. 10 Bandung, Jawa Barat 40132, Indonesia
E-mail: widowati_math@undip.ac.id

Abstract. The applicability of parameter-varying reduced-order controllers to aircraft models is proposed. A generalization of the balanced singular perturbation method of the linear time-invariant (LTI) system was used to reduce the order of the linear parameter-varying (LPV) system. Based on the reduced-order model, a low-order LPV controller was designed using the H_∞ synthesis technique. The performance of the reduced-order controller was examined by applying it to the lateral-directional control of a 20th-order aircraft model. Furthermore, the time responses of the closed-loop system with several reduced-order LPV controllers and a reduced-order LTI controller were compared. The simulation results show that an , 8th-order LPV controller can maintain stability and provide the same level of closed-loop system performance as a full-order LPV controller. This was not the case with the reduced-order LTI controller, which cannot maintain stability and performance for all allowable parameter trajectories.

Keywords: *H_∞ synthesis; singular perturbation approximation; reduced-order LPV controller; stability; aircraft dynamics.*

1 Introduction

A generalization of the balanced truncation method for LTI systems to reduce the model order of LPV systems has been published by Zobaidi, *et al.* [1], Goddard [2], and Wood, *et al.* [3]. Next, Widowati, *et al.* [4] have used a generalized balanced truncation to reduce the order of a parameter-varying controller. In paper [4], we have investigated the degradation of LVP closed-loop performance due to the parameter-varying reduced-order LPV controller. A study of the application of a balanced singular perturbation approximation (BSPA) to reduce the controller order of LTI systems has been published by Saragih and Yoshida [5]. Widowati, *et al.* [6] proposed a method to reduce

Received September 13th, 2011, Revised March 30th, 2012, Accepted for publication May 20th, 2012.
 ISSN: 1978-3051, DOI: 10.5614/itbj.eng.sci.2012.44.2.5

unstable LPV systems by generalizing BSPA for LTI systems via contractive right coprime factorization. Further, Saragih and Widowati [7] have presented sufficient conditions for the existence of right coprime factorization of parameter-varying controllers.

In this paper we compare the performance of an LPV closed-loop system using several reduced-order LPV controllers and a reduced-order LTI controller. This paper uses the balanced singular perturbation method, whereas in [1] the balanced truncation method was used to reduce a high-order plant. To verify the validity of the proposed method, it was applied to a model reduction for N-250 aircraft. Aircraft dynamics may be particularly severe in the case of high-performance aircrafts flying over a wide range of operational flight conditions (take-off, cruise, landing, altitude, airspeed). In the early days of automatic flight control systems, most systems were designed by using LTI control. However, aircraft dynamics vary for different flight conditions, whereas LTI controller is only suitable for certain flight conditions, and therefore cannot guarantee performance for other flight conditions. If an LTI controller is not capable of maintaining performance, the controller parameter values need adjustment. It is well known that the variation of some aircraft parameters is strongly related to air data variables, such as airspeed and altitude of the aircraft. It is necessary to rely on adjusting the flight controller parameters to the air data variables, a technique referred to as gain-scheduling.

In an attempt to reduce conservatism in control design and to improve numerical computation for systems significantly affected by measurable time-varying parameters, several new Linear Parameter Varying (LPV) approaches have emerged in the last few years. These LPV approaches explicitly take into account the relationship between real-time parameter variations and control system stability and performance. Therefore, the LPV controller theoretically guarantees performance and robustness for whole ranges of operating conditions. Most of the LPV approaches are based on linear matrix inequalities (LMI) and are solved numerically with some efficiency. While multitudes of theoretical results exist in this area [8-11, 2], only a few aerospace application oriented papers have been published to demonstrate the effectiveness of LPV controllers [12,13]. Related studies by the authors of the current paper on the application of LPV in flight control systems were published [14,15], but these works focused on full-order controller cases. The purpose of this paper is to propose robust gain-scheduling for uncertain LPV systems for lateral-directional control of N-250 aircraft by using reduced-order controllers.

The paper is organized as follows. Section 2 describes a brief review of the linear parameter varying (LPV) system. LPV synthesis is given in Section 3. Order-reduction of the LPV system by using the BSPA method is discussed in

Section 4. In Section 5, the reduced-order LTI and LPV control designs are demonstrated. Concluding remarks are presented in Section 6.

2 Brief Review of LPV System

In this paper, we consider a linear parameter-varying system. For a compact subset $W \subset R^s$, parameter-variation set F_ρ denotes the set of all piecewise continuous mapping *R*(time) into *W* with a finite number of discontinuities in any interval, $F_\rho = \{\rho : R \to W, \rho_{i_{\min}} \le \rho_i \le \rho_{i_{\max}}, i = 1, 2, \cdots, s\}$.

A compact set $W \subset R^s$, along with continuous functions $A: R^s \to R^{n \times n}$, $B: R^s \to R^{n \times n_u}$, $C: R^s \to R^{n_y \times n}$, $D: R^s \to R^{n_y \times n_u}$ represent an n^{th}-order parameter-varying plant, $G(\rho)$, whose dynamics evolve as

$$\dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t), \quad (1)$$
$$y(t) = C(\rho(t))x(t) + D(\rho(t))u(t), \ \forall \rho(t) \in F_\rho,$$

where $x(t) \in R^n$, $y(t) \in R^{n_y}$, $u(t) \in R^{n_u}$. A state space realization of the parameter-varying plant, $G(\rho)$, is written as

$$G(\rho) = \left[\begin{array}{c|c} A(\rho(t)) & B(\rho(t)) \\ \hline C(\rho(t)) & D(\rho(t)) \end{array}\right], \ \forall \rho(t) \in F_\rho.$$

The parameters of LPV system (1) are the functions of parameter ρ with unbounded parameter-variation rates; hence the dependence of system parameters on $\dot{\rho}$ can be omitted.

The parameter-varying system $G(\rho)$ is quadratically stable [2] if there exists a real positive-definite matrix $P = P^T > 0$, such that

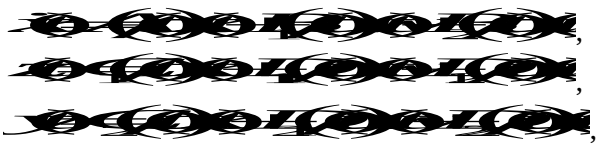
$$A^T(\rho(t))P + PA(\rho(t)) < 0, \ \forall \rho(t) \in F_\rho \quad (2)$$

The induced L_2 norm of a quadratically stable parameter-varying system, $G(\rho)$, with zero initial conditions, is defined as [2]

$$\|G(\rho)\|_{i,2} = \sup_{\rho(t) \in \mathcal{F}_\rho} \sup_{u \neq 0, u \in \mathcal{L}_2} \frac{\|y\|_2}{\|u\|_2}. \quad (3)$$

3 LPV Synthesis

To formulate a performance-oriented parameter-varying output feedback synthesis problem, the parameter-varying plant is written as follows



(4)

where y is the measurement, $u(t)$ is the control input, $w(t)$ is the disturbance, $z(t)$ is the controlled output, $x(t)$ is the state vector. Readers are referred to references [2,3] for the definition of quadratic stabilizability and quadratic detectability. The following assumptions are made for the parameter-varying plant:

1. $D_{22}(\rho(t)) = 0$
2. $B_2(\rho(t))$, $C_2(\rho(t))$, $D_{12}(\rho(t))$, and $D_{21}(\rho(t))$ are parameter-independent.
3. The pairs $(A(\rho(t)), B_2)$ and $(A(\rho(t)), C_2)$ are quadratically stabilizable and quadratically detectable over W, respectively.

The construction of a full-order controller has been developed by Apkarian, *et. al.* [8]. The design objectives are to satisfy H_∞ performance criteria, *i.e.* the parameter-varying closed-loop system is quadratically stable over W and the L_2 gain of the parameter-varying closed-loop system is bounded by γ, $\gamma > 0$ for all possible trajectories ρ. We assume the full-order controller to be m^{th}-order. The parameter-dependent full-order controller has state space realization as follows. For brevity, t is omitted.

(5)

The parameters of the gain-scheduled controller given by (5) are the functions of parameter ρ with an unbounded parameter-variation rate, hence the dependence of controller parameters on $\dot{\rho}$ can be omitted.

Theorem 3.1. [8] Consider a generalized LPV plant (4) with parameter trajectories. There exists a gain-scheduled output feedback controller (5) enforcing internal stability and a bound γ on the L_2 gain of the closed-loop system, whenever there exist symmetric matrices X and Y and a parameter-dependent quadruple of state-space data $(\bar{A}_K, \bar{B}_K, \bar{C}_K, \bar{D}_K)$, such that the

following linear matrix inequality problem holds, (for brevity, the parameter-dependence of state space matrices ρ is omitted).

$$\begin{bmatrix} XA+\bar{B}_K C_2+(*) & * & * & * \\ \bar{A}_K^T & AY+\bar{B}_2\bar{C}_K+(*) & * & * \\ (XB_1+\bar{B}_K D_{21})^T & (B_1+B_2\bar{D}_K D_{21})^T & -\gamma I & * \\ C_1+D_{12}\bar{D}_K C_2 & C_1 Y+D_{21}\bar{C}_K & D_{11}+D_{12}\bar{D}_K D_{21} & -\gamma I \end{bmatrix} < 0, \tag{6}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > \mathbf{O}. \tag{7}$$

Note that the notation (*) is induced by symmetry, for example

$$\begin{bmatrix} M+N+(*) & * \\ K & L \end{bmatrix} = \begin{bmatrix} M+M^T+N+N^T & K^T \\ K & L \end{bmatrix}$$

In such a case, a gain-scheduled (parameter-varying) controller of the form (5) is readily obtained with the following two-step scheme:

1. Solve for N, M, the factorization problem: $I - XY = NM^T$
2. Compute A_K, B_K, C_K, D_K with

4 Order-Reduction of LPV System

In this section, we use a generalization of the BSPA method to reduce the model order of a parameter-varying system (1). This generalization is based on solutions of the parameter-varying Lyapunov inequalities of LPV systems. Consider the n^{th}-order quadratically stable model of the LPV system in equation (1). By using a balancing-state transformation matrix we obtain the transformed controllability and observability Gramians.

$\tilde{P}\tilde{Q} = \mathrm{diag}(\sigma_1, \cdots, \sigma_n)$,

$\tilde{P} = \mathrm{diag}(P_1, \cdots, P_r)$ and

$\sigma_j = \sqrt{\lambda_j(PQ)}, \sigma_j \geq \sigma_{j+1}, j = 1, \cdots, r, r+1, \cdots, n.$

P and Q are solutions of Lyapunov inequalities [2].

A balanced LPV system can be expressed by

$$\dot{x}(t) = \tilde{A}(\rho)x(t) + \tilde{B}(\rho)u(t),$$

$$y(t) = \tilde{C}(\rho)x(t) + \tilde{D}(\rho)u(t). \tag{8}$$

Partition the balanced LPV system conformably, with $\Sigma = diag(\Sigma_1, \Sigma_2)$ is described as follows

$$\begin{aligned} \begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} &= \begin{bmatrix} A_{11}(\rho) & A_{12}(\rho) \\ A_{21}(\rho) & A_{22}(\rho) \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} B_1(\rho) \\ B_2(\rho) \end{bmatrix} u(t), \\ y(t) &= \begin{bmatrix} C_1(\rho) & C_2(\rho) \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \tilde{D}(\rho)u(t), \end{aligned} \tag{9}$$

where $\tilde{x}_1 \in R^r$ and $\tilde{x}_2 \in R^{n-r}$.

Furthermore, the BSPA method is used to reduce the order of the balanced LPV system (9). The dynamics of the reduced-order model can be expressed in the form

$$\begin{aligned} \dot{x}_1(t) &= \bar{A}(\rho)x_1(t) + \bar{B}(\rho)u(t), \\ y(t) &= \bar{C}(\rho)x_1(t) + \bar{D}(\rho)u(t), \end{aligned} \tag{10}$$

where

$$\begin{aligned} \bar{A}(\rho) &= A_{11}(\rho) - A_{12}(\rho)A_{22}^{-1}(\rho)A_{21}(\rho), \\ \bar{B}(\rho) &= B_1(\rho) - A_{12}(\rho)A_{22}^{-1}(\rho)B_2(\rho), \\ \bar{C}(\rho) &= C_1(\rho) - C_2(\rho)A_{22}^{-1}(\rho)A_{21}(\rho), \\ \bar{D}(\rho) &= \tilde{D}(\rho) - C_2(\rho)A_{22}^{-1}(\rho)B_2(\rho), \end{aligned} \tag{11}$$

by asumming $A_{22}(\rho)$ nonsingular $\forall \rho \in F_\rho$.

If the LPV system is not quadratically stable, but must be quadratically stabilizable and detectable, then it cannot be directly reduced by using the above method. If the unstable LPV system is stabilizable via parameter-varying state feedback, then we can construct a quadratically stable contractive right coprime factorization (CRCF). Consider the following theorem.

Theorem 4.1. [3] Let $G(\rho)$ have continuous, quadratically stabilizable, quadratically detectable realizations, then the contractive right graph symbol of $G(\rho)$ is given by

$$\mathrm{G}(\rho) := \begin{bmatrix} N(\rho) \\ M(\rho) \end{bmatrix} = \left[\begin{array}{c|c} A(\rho)+B(\rho)F(\rho) & B(\rho)S^{-1/2}(\rho) \\ \hline C(\rho)+D(\rho)F(\rho) & D(\rho)S^{-1/2}(\rho) \\ F(\rho) & S^{-1/2}(\rho) \end{array}\right], \tag{12}$$

where $F(\rho) = -S^{-1}(\rho)B(\rho)X + D^T(\rho)C(\rho)$, $S(\rho) = I + D^T(\rho)D(\rho)$, $R(\rho) = I + D(\rho)D^T(\rho)$, and X = $X^T > 0$ is a constant solution of the generalized control Riccati inequality (GCRI)

$$\begin{aligned} &(A(\rho) - B(\rho)S^{-1}(\rho)D^T(\rho)C(\rho))^T X + X(A(\rho) - B(\rho)S^{-1}(\rho)D^T(\rho)C(\rho)) \\ &-XB(\rho)S^{-1}(\rho)B^T(\rho)X + C^T(\rho)R^{-1}(\rho)C(\rho) < 0, \forall \rho \in F_\rho. \end{aligned} \tag{13}$$

Furthermore, we can reduce the coprime factor realization by using a generalization of the BSPA method [6] with the following procedure. $G(\rho)$ is written as $G(\rho) = N(\rho)M^{-1}(\rho)$, where $(N(\rho), M(\rho))$ represents a CRCF of $G(\rho)$, $N(\rho)$ and $M(\rho)$ are quadratically stable. Let $Q=X$ and $P=(I+YX)^{-1}Y$ be the observability and controllability Gramians of $\mathrm{G}(\rho)$, with X solves GCRI and Y solves a generalized filtering Riccati inequality (GFRI),

$$\begin{aligned} &(A(\rho) - B(\rho)D^T(\rho)R^{-1}(\rho)C(\rho))\ Y + Y(A(\rho) - B(\rho)D^T(\rho)R^{-1}(\rho)C(\rho))^T \\ &-YC^T(\rho)S^{-1}(\rho)C(\rho)Y + B(\rho)S^{-1}(\rho)B^T(\rho) < 0, \forall \rho \in F_\rho. \end{aligned} \tag{14}$$

Furthermore, $\mathrm{G}(\rho)$ is balanced by using a state transformation matrix T, such that the transformed Gramians $\tilde{P} = \tilde{Q} = \Sigma$. Next, BSPA method is applied to obtain

$$\mathrm{G_r}(\rho) := \begin{bmatrix} N_r(\rho) \\ M_r(\rho) \end{bmatrix},$$

which has r^{th}-order, $r < n$. Moreover, we obtain the reduced-order LPV system,

$$\overline{G}_r(\rho) = N_r(\rho)M_r^{-1}(\rho).$$

5 Computational Issues

The constraints given by linear matrix inequalities (6) and (7) are parameter-dependent, *i.e.* there is an infinite set of linear matrix inequalities for every parameter value. These linear matrix inequalities can be solved by a polytopic technique. The parameter vector, $\rho(t)$, takes values in a box of R^s with corners

$\{\rho_{v_i} : i = 1,2,\cdots,r\}, r = 2^s$. In other words, the parameter vector varies in polytope with vertices $\rho_{v_1}, \rho_{v_2}, \cdots, \rho_{v_r}$, so that the parameter can be written by convex combination,

$$\rho(t) = Co\{\rho_{v_1}, \rho_{v_2}, \cdots, \rho_{v_r}\} := \sum_{i=1}^{r} \alpha_i \rho_i : \alpha_i \geq 0, \sum_{i=1}^{r} \alpha_i = 1, \ \forall t \geq 0.$$

The problem of linear matrix inequalities (LMIs) (6) and (7) is approximated by solving them at each of the vertices. The constraints given by these LMIs at each of the vertices, can be solved using the interior point method in convex programming. The consequence of this approximation is that the number of LMIs becomes $2^{s+1}+1$ and the number of decision variables becomes $n_x(n_x+1)$, with n_x as number of states. The variables X, Y in LMIs (6)-(7) are approximated by convex interpolation of solution at each of the vertices.

The parameter-varying controller (5) with state space matrices $A_K(\rho)$, $B_K(\rho)$, $C_K(\rho)$, and $D_K(\rho)$ range in a polytope of matrices, whose vertices are the images of vertices $\rho_{v_1}, \rho_{v_2}, \cdots, \rho_{v_r}$, that is,

$$\begin{bmatrix} A_K(\rho) & B_K(\rho) \\ C_K(\rho) & D_K(\rho) \end{bmatrix} = Co\left\{ \begin{bmatrix} A_{K_i} & B_{K_i} \\ C_{K_i} & D_{K_i} \end{bmatrix}, i = 1,2,\cdots,r \right\} := \sum_{i=1}^{r} \alpha_i \begin{bmatrix} A_{K_i} & B_{K_i} \\ C_{K_i} & D_{K_i} \end{bmatrix},$$

$$\alpha_i \geq 0, \sum_{i=1}^{r} \alpha_i = 1.$$

6 N-250 LPV Control Design and Simulation

To verify the capability of the singular perturbation approximation, simulations were conducted by using it to reduce an aircraft model [15]. The performances of the closed-loop systems with the reduced-order LPV controllers and LTI-reduced controller were compared.

6.1 Modeling of N-250 Aircraft

The N-250 is a turboprop aircraft with two engines and is equipped with a three-axis fly-by-wire (FBW) system. The FBW system is designed to deflect the aircraft control surface, comprising flap, spoiler, rudder, aileron, and T-tail elevator, by applying electric signals to actuators (see Figure 1). The N-250 is a regional commuter aircraft, which has a capacity of 64-70 passengers, a cruise speed of 300 knots (556 km/hour), a maximum velocity of 330 knots, and flight range of 1.482 km.

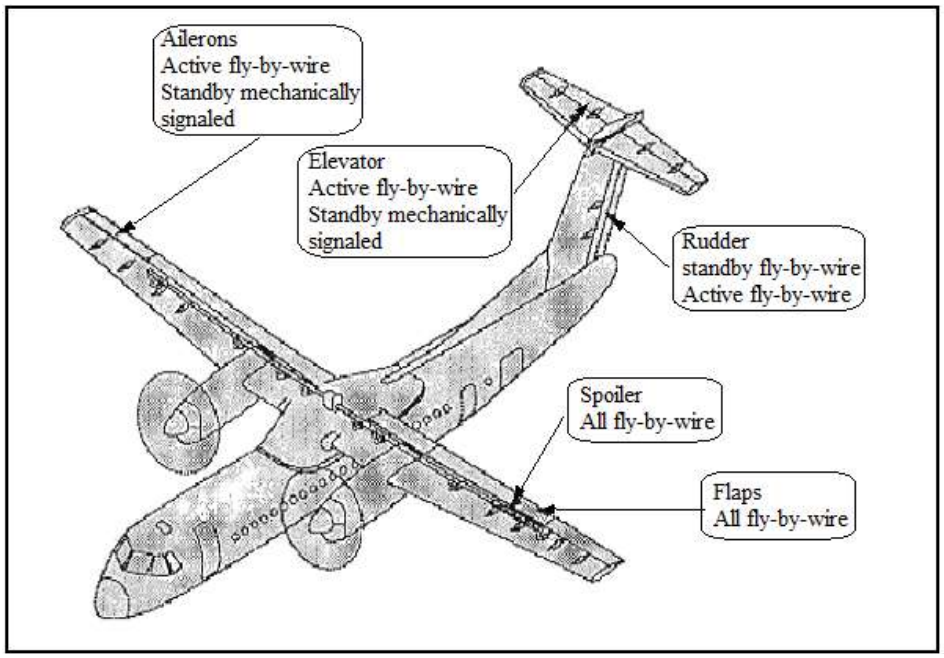


Figure 1 N250 Aircraft control surface.

For the purpose of simulation, the N250 is classified as a Class II aircraft, for which the whole mission is dominated by flight phase B. Consequently, the ideal model is chosen from the minimum requirements of flight phase B for Class II aircraft. The linear equation of the N-250 PA-2 used in this research is derived from the non-linear equation of the motion of the aircraft. Aerodynamic forces and moment coefficients were obtained from N-250 PA-2 wind-tunnel tests conducted by PT Dirgantara Indonesia (formerly PT IPTN). The linearization process of the equation of motion was carried out by introducing small perturbations over all variables of the equation, a technique referred to as small perturbation method. In the linearized form, the resulting equation of motion consists of two independent motions: longitudinal and lateral-directional motion. The linearized model is fine-tuned using flight test data and a parameter identification process based on the flight test data analysis introduced by Laban [16] and Lean [17]. Figure 2 shows the technique with which the identification process was applied to the problem of estimating the aerodynamic coefficients based on flight test data. During the flight test program, a specially-designed dynamic maneuver was applied to the test aircraft, and various aircraft dynamic variables were measured. The aircraft control surface deflections were measured and were applied to the dynamic aircraft system model. Basically, this process consists of two major phases (see Figure 2), given the real-time presentation of

the raw flight test measurements, which in our case were provided by a telemetric system at the former Flight Test Center, IPTN. The first phase is data reconstruction/compatibility check of measurement data from aircraft response due to input given by the pilot.

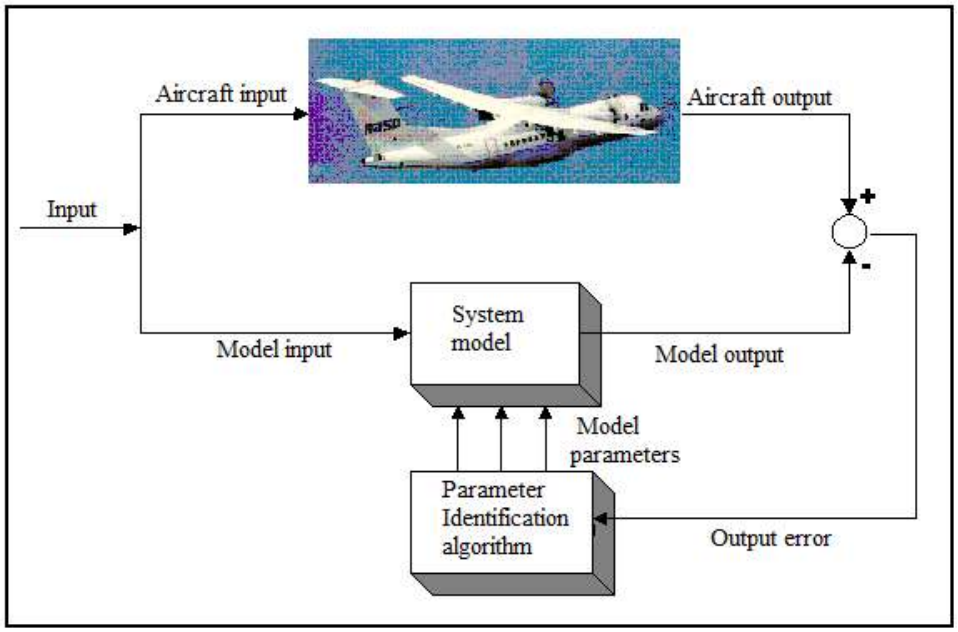


Figure 2 N250 Aircraft identification process.

During the data reconstruction process, a matching mechanism was performed on input and output data recorded during flight tests, based on aircraft kinematics equations. The extended Kalman filter was applied to suppress noise that arises from atmospheric conditions and inaccuracy/uncertainty in flight parameter measurement devices. In the second phase, an optimization was performed to obtain values of aerodynamic coefficients using a least-square method [18], which minimizes output mismatch. Modeling the N250 aircraft motion based on flight testing took several months, and various flight test maneuvers had to be conducted to cover most of the flight envelope.

The primary aircraft control surface consists of aileron and rudder for lateral-directional motion and elevator for longitudinal motion. The secondary control surface consists of spoiler and flaps. In this paper, we consider only the lateral-directional dynamics; the longitudinal dynamics are assumed to remain at equilibrium. The lateral-directional dynamic of the aircraft can be expressed as an unstable LPV system with the following state space model:

$$\begin{bmatrix}\dot{v}\\ \dot{p}\\ \dot{r}\\ \dot{\phi}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix} Y_v^{'}(V,f) & Y_p^{'}(V,f) & Y_r^{'}(V,f) & g\cos\theta & 0\\ L_v^{'}(V,f) & L_p^{'}(V,f) & L_r^{'}(V,f) & 0 & 0\\ N_v^{'}(V,f) & N_p^{'}(V,f) & N_r^{'}(V,f) & 0 & 0\\ 0 & I & Tan\theta & 0 & 0\\ 0 & 0 & Sec\theta & 0 & 0\end{bmatrix}\begin{bmatrix}v\\ p\\ r\\ \phi\\ \psi\end{bmatrix}+\begin{bmatrix} Y_{da}^{'}(V,f) & Y_{dr}^{'}(V,f)\\ L_{da}^{'}(V,f) & L_{dr}^{'}(V,f)\\ N_{da}^{'}(V,f) & N_{dr}^{'}(V,f)\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta_A\\ \delta_R\end{bmatrix},$$

$$\begin{bmatrix}\beta\\ p\\ r\\ \phi\\ \psi\\ a_y\end{bmatrix}=\begin{bmatrix} \frac{57,3}{V} & 0 & 0 & 0 & 0\\ 0 & 57,3 & 0 & 0 & 0\\ 0 & 0 & 57,3 & 0 & 0\\ 0 & 0 & 0 & 57,3 & 0\\ 0 & 0 & 0 & 0 & 57,3\\ Y_v^{''}(V,f) & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}v\\ p\\ r\\ \phi\\ \psi\end{bmatrix},$$

where

$$Y_v'(V,f)=\frac{\zeta u_e S}{2m}C_{y_\beta}(V,f) \qquad Y_p'(V,f)=\frac{\zeta u_e S}{4m}C_{y_p}(V,f) \qquad Y_r'(V,f)=\frac{\zeta u_e S}{4m}C_{y_r}(V,f)$$

$$L_v'(V,f)=\frac{L_v}{I_{xxs}+I_{xzs}N_v}(V,f) \qquad L_p'(V,f)=\frac{L_p}{I_{xxs}+I_{xzs}N_p}(V,f) \qquad L_r'(V,f)=\frac{L_r}{I_{xxs}+I_{xzs}N_r}(V,f)$$

$$L_v=\frac{\zeta u_e^2 S_b}{2I_{xx}}C_{l_\beta}(V,f) \qquad L_p=\frac{\zeta u_e S_b^2}{4I_{xx}}C_{l_p}(V,f) \qquad L_r=\frac{\zeta u_e S_b^2}{4I_{xx}}C_{l_r}(V,f)$$

$$N_v=\frac{\zeta u_e^2 S_b}{2I_{zz}}C_{n_\beta}(V,f) \qquad N_p=\frac{\zeta u_e S_b^2}{4I_{zz}}C_{n_p}(V,f) \qquad N_r=\frac{\zeta u_e S_b^2}{4I_{zz}}C_{n_r}(V,f)$$

$$N_v'(V,f)=\frac{N_v}{I_{zzs}+I_{xzs}L_v}(V,f) \qquad N_p'(V,f)=\frac{N_p}{I_{zzs}+I_{xzs}L_p}(V,f) \qquad N_r'(V,f)=\frac{N_r}{I_{zzs}+I_{xzs}L_r}(V,f)$$

$$Y_{da}'(V,f)=\frac{Y_{da}}{m}(V,f) \qquad Y_{dr}'(V,f)=\frac{Y_{dr}}{m}(V,f) \qquad L_{da}'(V,f)=\frac{L_{da}}{I_{xxs}+I_{xzs}N_{da}}(V,f)$$

$$L_{dr}'(V,f)=\frac{L_{dr}}{I_{xxs}+I_{xzs}N_{dr}}(V,f) \qquad N_{da}'(V,f)=\frac{N_{da}}{I_{zzs}+I_{xzs}L_{da}}(V,f)$$

$$Y_v''(V,f)=\frac{Y_v V}{mg}(V,f) \qquad N_{dr}'(V,f)=\frac{N_{dr}}{I_{zzs}+I_{xzs}L_{dr}}(V,f)$$

in which the state vector consists of lateral velocity (v), roll rate (p), yaw rate (r), roll angle (φ), and azimuth angle (ψ); the input vector consists of aileron deflection (δ_A) and rudder deflection (δ_R); the measurement vector consists of side-slip (β), roll rate (p), yaw rate (r), roll angle (φ), azimuth angle (ψ), and lateral acceleration (a_y). ζ : air density, g: gravity, m: mass of the aircraft, S: surface area of the aircraft, b: wing span. For details about the symbols, refer to

Etkin [19]. All states are available for supplying feedback to control the aircraft. The dynamics of the N-250 aircraft under consideration vary greatly as a function of speed (*V*) and flap setting (*f*). Signal *V* and *f* are measured in real time. The take-off aircraft speed ranges between 80 KEAS (Knot Equivalent Air Speeds) and 320 KEAS. The flap settings vary between 0, 20, 30, and 40 deg. The nominal data of the aircraft are given in Table 1.

Table 1 The nominal data of the aircraft at take-off flight conditions.

Parameters	values
Speed (knots) : KEAS	80 - 320
Center of Gravitation(%) :cg	26.7
Aircraft Mass : kg	20267.5
Altitude : feet	1250
Flap setting : deg	0, 20, 30, 40

The parameter-dependent state space data are obtained by trimming and linearizing the nonlinear model at speed V = {80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 135, 140, 145, 150, 155, 160, 170, 175, 180, 185, 190, 195, 200, 205, 210, 215, 220, 225, 230, 235, 240, 245, 250, 255, 260, 265, 270, 275, 280, 285, 290, 295, 300, 305, 310, 315, 320} KEAS and flap setting f = {0, 20, 30, 40} deg. Next, we choose entries of parameter-dependent state space data as the vertices of the polytopic plant. However, choosing all the entries as the vertices would result in a large number of LMI constraints to be checked and can cause computational impracticalities. Hence, we consider only those 5 entries of state space data that are most significantly affected by the aircraft speed and flap setting. By observing singular value plots of the plants with respect to variations of state space data that are affected by the true aircraft speed and flap setting, it turns out that entries $L_v^{'}, L_p^{'}, N_v^{'}, N_p^{'}, N_r^{'}$ produce significant changes in the plant. The variations of *V* and *f* result in a parameter range of $\rho_1 = L_v^{'} \in [-0.3132, -0.0181]$, $\rho_2 = L_p^{'} \in [-5.6951, -1.2095]$, $\rho_3 = N_v^{'} \in [-0.0220, 0.0237]$, $\rho_4 = N_p^{'} \in [-1.4561, 0.9450]$, and $\rho_5 = N_r^{'} \in [-1.1020, -0.2057]$. The high-order plant consists of the lateral-directional dynamic and weighting functions that are incorporated to satisfy the controller design specifications. The plant order of this aircraft model is 20^{th} order.

6.2 Reduced-Order Controller Design

The reduced-order model is calculated by using the BSPA method as suggested in the preceding section. Based on the reduced-order model, the low-order controllers are designed by using H_∞ synthesis. Note that the parameters $L_v^{'}, L_p^{'}, N_v^{'}, N_p^{'}, N_r^{'}$ enter the state space matrices in an affine way and are

considered to represent a convex polytope with $32 = 2^5$ vertices. Allow the parameters to be time-varying with no rate bound, which makes it necessary to find solutions for the LMIs (6) and (7), evaluated at each of the 32 vertices. The *X*, *Y* solutions of these LMIs are approximated by convex interpolation of solution at each of the vertices.

Furthermore, states space data of the full-order and reduced-order LPV controllers were constructed using convex interpolation of their values at the 32 vertices. The upper bound of the parameter-varying closed-loop performance for all parameter variations is 50.0003. The LMI Control Toolbox for MATLAB [20] was used for numerical computation. The aim of the controller design is to satisfy the following specifications:

1. Frequency domain specification: the closed-loop system has a bandwidth of 10 rad/s, low sensitivity if frequency $\leq$ 8 rad/s, in the high frequency range the measurement noise is attenuated around 2 dB.
2. Time domain specification: Steady state errors are within 7% tolerance, overshoot $\leq$ 10% and transient response in 4-8 seconds, control surface magnitude and rates do not exceed actuator saturation limits, system response to command meets level-1 flying quality.

Table 2 Maximum deflection and rate of aileron and rudder.

	Maximum deflection	**Maximum rate**
Aileron	± 22 degree	50 degree/second
Rudder	± 20 degree	37 degree/second

The controlled output consists of aileron position, rudder position, aileron rate, rudder rate, side-slip error, and yaw-rate error. In this case, to simulate the time responses of the gain-scheduled controller along parameter trajectories, we chose the following spiral parameter trajectories,

$$\rho_1 = L_v^{'} = -0.12685 + 0.1007\exp(-4t)\cdot\cos(100t)$$

$$\rho_2 = L_p^{'} = -2.8805 + 1.71195\exp(-4t)\cdot\sin(100t)$$

$$\rho_3 = N_v^{'} = 0.0041 + 0.0173\exp(-4t)\cdot\sin(100t)$$

$$\rho_4 = N_p^{'} = -0.15395 + 0.760\exp(-4t)\cdot\cos(100t)$$

$$\rho_5 = N_r^{'} = -0.5832 + 0.287\exp(-4t)\cdot\sin(100t)\,.$$

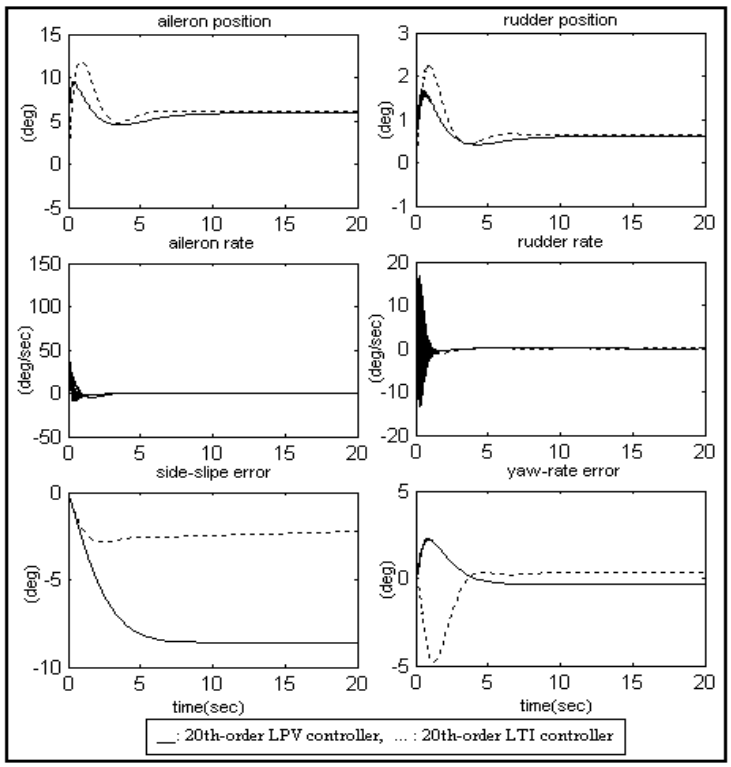


Figure 3 Time responses of the closed-loop system with the 20th-order LPV controller and LTI controller for all parameter trajectories.

The time responses of the closed-loop system with the 20th-order (full-order) LPV and fixed LTI controller are described in Figure 3. The LTI controller is designed for fixed parameter values of the following aircraft flight conditions: speed = 90 KEAS, center of gravity = 26.7%, mass = 20,267.5 kg, altitude = 1250 feet, and flap setting = 40 degrees.

Next, we compared the parameter-varying closed-loop performance with the full-order and reduced-order LPV controllers and the LTI controller. Time responses of the closed-loop system with the 20th-, 10th-, 9th-, and 8th-order LPV controllers for all parameter trajectories with respect to impulse input are given in Figure 4. Figure 3 and Figure 4 show that the closed-loop time responses achieve stability within 6 seconds, except in the closed-loop with the LTI controller, where time responses of the aileron and rudder position achieve stability within 7.5 seconds.

The step responses have a settling time of less than 8 seconds. These figures also show that all saturation and rate limits for aileron and rudder from the design specifications are satisfied.

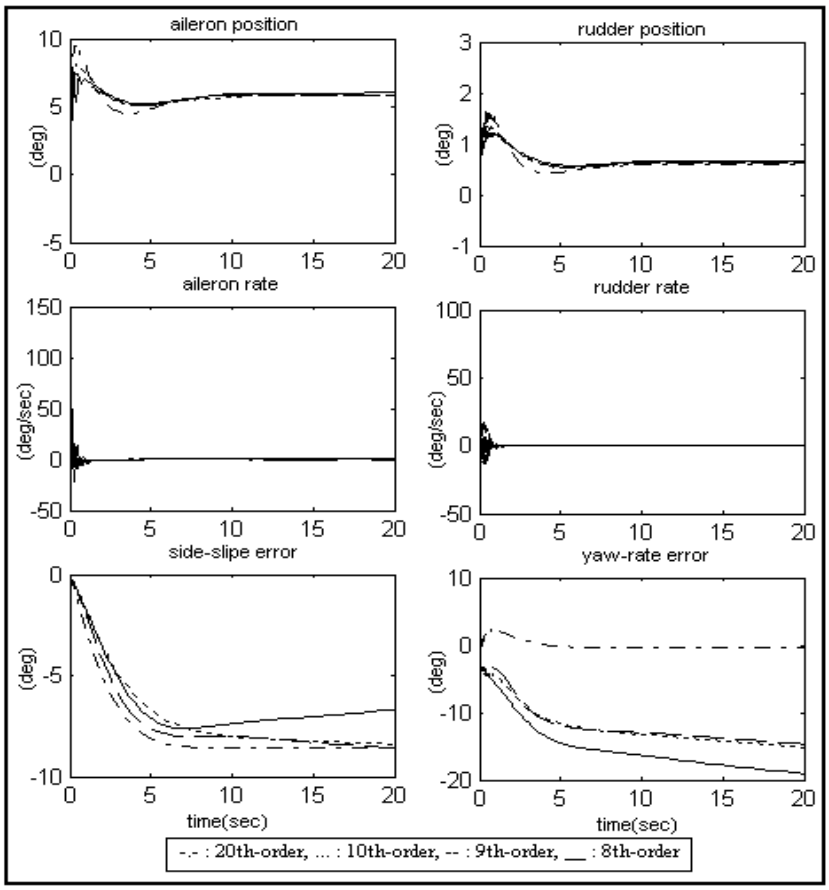


Figure 4 Time responses of the closed-loop system with the full-order and reduced-order LPV controllers for all parameter trajectories.

The time responses of the parameter-varying closed-loop systems with a reduced (19^{th})-order LTI controller are depicted in Figure 5. In this case, as can be seen in Figure 5, the closed-loop time responses cannot be made stable. From the above results, the order of the LPV plant found by the BSPA method can be as low as 8th while maintaining closed-loop system stability and providing the same level of closed-loop system performance as the 20^{th}-order LPV controller.

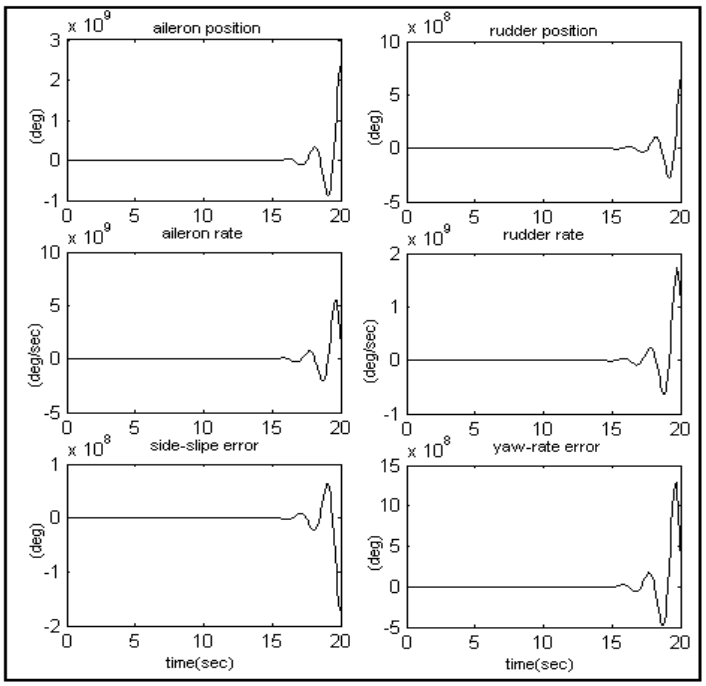


Figure 5 Time responses of the closed-loop system with a 19th-order LTI controller for all parameter trajectories.

7 Concluding Remarks

A 20th-order turboprop aircraft model has been reduced using the balanced singular perturbation method. An effective H_∞ controller based on the reduced-order model to control lateral-directional motion of aircraft was proposed. A reduced-order LTI controller cannot maintain performance for all allowable parameter trajectories. Reduced-order LPV controllers guarantee performance and robustness for whole ranges of operating conditions. Reduced-order LPV controllers are able to do better over the entire operating range of the turboprop aircraft than reduced-order LTI controllers.

Acknowledgements

We would like to thank the reviewers for their detailed comments and suggestions.

References

[1] El-Zobaidi, H.M.H. & Jaimoukha, I., *Robust Control and Model and Controller Reduction of Linear Parameter Varying Systems*, Proc. of the

37th IEEE Conference on Decision and Control, Tampa, Florida USA **3**, pp. 3015-3020, 1998.

[2] Goddard, P.J., *Performance-Preserving Controller Approximation*, PhD Dissertation, Trinity College Cambridge, 1995.

[3] Wood, D., Goddard, P.J. & Glover, K., *Approximation of Linear Parameter Varying Systems*, Proceedings of 35th IEEE Conference on Decision and Control, Kobe, Japan, 1996.

[4] Widowati, Nababan, S.M., Riyanto, B., Saragih, R., *Reduced-order of LPV Control with Guaranteed Closed-loop Performance*, Journal of Indonesian Mathematics Society, **12**(1), 2006.

[5] Saragih, R. & Yoshida, K., *Reduced-Order of Structural Control using Singular Perturbation Approach*, Proc. of the 4th Int. Conf. on MOVIC, **3**, pp. 539-544, 1998.

[6] Widowati, Riyanto, B., Saragih, R. & Nababan, S.M., *Model Reduction for Unstable LPV Systems Based on Coprime Factorizations and Singular Perturbation*, Proceedings of The 5th Asian Control Conference, Melbourne, Australia, pp. 962-969, 2004.

[7] Saragih, R. & Widowati, *Coprime Factor Reduction of Parameter Varying Controller*, International Journal of Control, Automation, and Systems, **6**(6), pp. 836-844, 2008.

[8] Apkarian, P. & Adam, R.J., *Advanced Gain Scheduling Techniques for Uncertain Systems*, Proceedings of the American Control Conference, 1997.

[9] Becker, G. & Packard, A., *Robust Performance of Linear Parametrically Varying Systems Using Parametrically-Dependent Linear Feedback*, System and Control Letters, **23**, pp. 205-215, 1994.

[10] Bendotti, P. & Bodenheimer, B., *Linear Parameter Varying versus Linear Time Invariant Control Design for a Pressurized Water Reactor*, Research Article, California Institute of Technology, EE Dept., Pasadena, USA, 1998.

[11] Bruzelius, F., *Linear Parameter Varying Systems*, PhD Dissertation, Departement of Signal and Systems, Chalmers University of Technology, Goteborg, Sweden, 2004.

[12] Shin, J.Y., Balas, G.J. & Kaya, A.M., *Blending Methodology of Linear Parameter Varying Control Synthesis of F-16 Aircraft System*, NASA/CR-2001-211237, NASA Langley Research Center, 2001.

[13] Spillman, M., *Robust Longitudinal Flight Control Design Using Parameter-Varying Feedback*, Journal of Guidance, Control and Dynamics, 2001.

[14] Riyanto, B., *Dissipative Control of Linear Parametrically Varying Systems Using Output Feedback: an LMI Approach*, International Conference Modeling, Identification and Control, Austria, 1999.

[15] Riyanto, B., *Self-Scheduled H_∞ Control Design for N250 Aircraft Dynamics Based on Polytopic LPV Model*, Proceedings of the IASTED International Conference Control and Applications, Banff, Canada, pp. 52-57, 1999.
[16] Laban, M., *On-line Aircraft Aerodynamic Model Indentification,* Ph.D Thesis, Delft University of Technology, 1990.
[17] Mc. Lean, D., *Automatic Flight Control Systems*, Prentice-Hall, 1990.
[18] Muhammad, H., Hariowibowo, H., Legowo, A. & Jenie, S.D., *Aerodynamic Parameter Identification of the N-250-PA1 Gatotkoco*, TR-01/1998/FTC-DT, IPTN Flight Test Center, Bandung 1998.
[19] Etkin, B. & Reid, L.D., *Dynamic of Flight: Stability and Control*, Third Edition John Wiley & Sons, 1996.
[20] Gahinet, P., Nemirovski, A., Laub, A. & Chilali, M., *The LMI Control Toolbox*, *The MathWorks*, Inc.: Natick, MA, 1995.