C-Γ-hyperideal Theory in Ordered Γ-semihypergroups

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Abstract. Our purpose in this article is to characterize the properties of C-Γ-hyperideals in ordered Γ-semihypergroups. As an application of the results of this paper, the corresponding results of ordered semihypergroups can also be obtained by moderate modification.

Keywords: *C-Γ-hyperideal; Γ-hyperideal, ordered Γ-semihypergroup; regular; semihypergroup;*

1 Introduction

The formal study of semigroups began in the early 20th century. As a generalization of semigroups, the notion of a Γ-semigroup was first introduced by Sen and Saha [1] in 1986. The notion of an ordered Γ-semigroup was first introduced by Sen and Seth [2] in 1993. Many authors studied various aspects of ordered Γ-semigroups, for instance Chinram and Tinpun [3], Dutta and Adhikari [4], Hila [5], Iampan [6], Kehayopulu [7], Kwon [8], Sen and Seth [2], Tang and Xie [9] and many others. In 2010, Hila [5] investigated some properties of quasi-prime and weakly quasi-prime left ideals in ordered Γ-semigroups. In 2013, Tang and Xie [9] studied on α-maximal ideals and C-ideals of ordered Γ-semigroups. Recall from [2] that an ordered Γ-semigroup $(S,\Gamma,\leq)$ is a Γ-semigroup (S,Γ) together with an order relation $\leq$ such that $a \leq b$ implies that $a\gamma c \leq b\gamma c$ and $c\gamma a \leq c\gamma b$ for all $a, b, c \in S$ and $\gamma \in \Gamma$. In [10-12], Fabrici introduced the concept of the covered ideal, which is a proper ideal I of S satisfying $I \subseteq S(S \setminus I)S$, and obtained some properties in terms of maximal ideals of S. In [13], Xie introduced covered ideals in ordered semigroups. In 2016, Changphas and Summaprab [14] discussed the structure of ordered semigroups containing covered ideals.

Algebraic hyperstructures are a suitable generalization of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. The concept of hyperstructure was first introduced by Marty [15] at the eighth Congress of Scandinavian Mathematicians in 1934. Since then, several books have been published on this topic, for example see [16-19]. Ameri and Hoskova [20] studied the fuzzy continuous polygroup as a fuzzy

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polygroup with a continuous membership function. In [21], some properties of hyperlattices are studied and the relationship between prime ideals and prime filters in hyperlattices is discussed. Motivation of compatibility of orderings with hyperoperations can be found in [22,23]. In 2010, Davvaz, *et al.* [24-26] introduced the notion of the Γ-semihypergroup as a generalization of a semigroup, a generalization of a semihypergroup and a generalization of a Γ-semigroup. They proved some results in this respect and presented many examples of Γ- semihypergroups. Since then, properties of Γ- semihypergroups were studied by some mathematicians, for example, see [24,27-29]. Anvariyeh, *et al.*, studied the Γ-hyperideals of Γ-semihypergroups in [24]. Many classical notions of semigroups and semihypergroups have been extended to Γ-semihypergroups.

Davvaz and Omidi [30] investigated several notions, for example, hyperideals, quasi-hyperideals and bi-hyperideals of an ordered semihyperring. The concept of ordered semihypergroups is a generalization of the concept of ordered semigroups. The concept of ordering hypergroups was introduced by Chvalina [31] as a special class of hypergroups. Many authors studied various aspects of ordered semihypergroups, for instance, Davvaz, *et al.*, [32], Gu and Tang [33], Heidari and Davvaz [23], Tang, *et al.* [34], and many others. Explicit study of ordered semihypergroups seems to have begun with Heidari and Davvaz [23] in 2011. We have noticed that the relationships between ordered semigroups and ordered semihypergroups have been already considered by Davvaz, *et al.* [32]. In [33], the authors answered to the open problem given by Davvaz, *et al.* [32].

In this paper, we give some properties of Γ-hyperideals and C-Γ-hyperideals of ordered Γ-semihypergroups. We make a connection between proper Γ-hyperideals and C-Γ-hyperideals. Moreover, we prove that for a Γ-hyperideal I of a regular ordered Γ-semihypergroup $(S, \Gamma, \leq)$, every C-Γ-hyperideal J of I is also a C-Γ-hyperideal of S.

2 Basic Definitions

In the following, we recall the notion of an ordered Γ-semihypergroup and then we present some definitions and properties that we will need in this paper. Throughout this paper, unless otherwise stated, S is always an ordered Γ-semihypergroup $(S, \Gamma, \leq)$. One can see that ordered Γ-semihypergroups are generalizations of ordered semihypergroups.

Definition 2.1 [35] An algebraic hyperstructure $(S, \Gamma, \leq)$ is said to be an *ordered Γ-semihypergroup* if (S, Γ) is a Γ-semihypergroup and $(S, \leq)$ is a partially ordered set such that: for any $x, y, z \in S$ and $\gamma \in \Gamma$, $x \leq y$ implies

$z\gamma x \preccurlyeq z\gamma y$ and $x\gamma z \preccurlyeq y\gamma z$. Here, $A \preccurlyeq B$ means for any $a \in A$ there exists $b \in B$ such that $a \leq b$, for all non-empty subsets A and B of S.

Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. By a *sub* Γ*-semihypergroup* of S we mean a non-empty subset A of S such that $a\gamma b \subseteq A$ for all $a, b \in A$ and $\gamma \in \Gamma$. Let K be a non-empty subset of S. If H is a non-empty subset of K, then we define:

$$(H]_K := \{k \in K \mid k \leq h \text{ for some } h \in H\}.$$

Note that if $K = S$, then we define:

$$(H] = \{x \in S \mid x \leq h \text{ for some } h \in H\}.$$

Example 1. [35] Let $(S, \circ, \leq)$ be an ordered semihypergroup and Γ a non-empty set. We define $x\gamma y = x \circ y$ for every $x, y \in S$ and $\gamma \in \Gamma$. Then $(S, \Gamma, \leq)$ is an ordered Γ-semihypergroup.

Definition 2.2 [35] Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. A non-empty subset I of S is called a left (resp. right) Γ-hyperideal of S if it satisfies the following conditions:

1. $S\Gamma I \subseteq I$ $(resp.\, I\Gamma S \subseteq I)$;
2. When $x \in I$ and $y \in S$ such that $y \leq x$, imply that $y \in I$.

Equivalent Definition:

1. $S\Gamma I \subseteq I$ *.resp*)$I\Gamma S \subseteq I$);
2. $(I] \subseteq I$.

By *two-sided* Γ*-hyperideal* or simply Γ*-hyperideal,* we mean a non-empty subset I of S that is both a right and a left Γ-hyperideal of S. A Γ-hyperideal I of S is said to be *proper* if $I \neq S$.

3 Main Results

An ordered Γ-semihypergroup $(S, \Gamma, \leq)$ is said to be *simple* if it has no proper Γ-hyperideals, i.e., for any Γ-hyperideal $I \neq \emptyset$ of S, we have $I = S$. A Γ-hyperideal $I \neq S$ of S is said to be *maximal* if for any proper Γ-hyperideal J of S, $I \subseteq J$ implies that $I = J$. A Γ-hyperideal I of S is called a *minimal* Γ*-hyperideal* of S if there is no Γ-hyperideal J of S such that $J \subset I$. The concept of C-ideals in ordered Γ-semigroups was introduced by Tang and Xie [9]. As a continuation of Tang and Xie's works on ordered Γ-semigroups, the aim of this section is to study and characterize Γ-hyperideals and C-Γ-hyperideals of an ordered Γ-semihypergroup.

Lemma 3.1 Let $(S,\Gamma,\leq)$ be an ordered Γ-semihypergroup. If A and B are non-empty subsets of S, then we have:

1. $A \subseteq (A]$;
2. If $A \subseteq B \subseteq S$, then $(A] \subseteq (B]$;
3. $((A]] = (A]$;
4. $(A]\Gamma(B] \subseteq (A\Gamma B]$;
5. $\big((A]\Gamma(B]\big] = (A\Gamma B]$.

Proof. The proof is straightforward.

Let a be an element of an ordered Γ-semihypergroup $(S,\Gamma,\leq)$. We denote by $L_S(a)$ (resp. $R_S(a), I_S(a)$) the left (resp. right, two-sided) Γ-hyperideal of S generated by a.

Lemma 3.2 Let a be an element of an ordered Γ-semihypergroup $(S,\Gamma,\leq)$. Then,

1. $L_S(a) = (a \cup S\Gamma a]$;
2. $R_S(a) = (a \cup a\Gamma S]$;
3. $I_S(a) = (a \cup S\Gamma a \cup a\Gamma S \cup S\Gamma a\Gamma S]$.

Proof. The proof is straightforward.

We continue this section with the following definition.

Definition 3.3 An element a of an ordered Γ-semihypergroup $(\mathrm{S},\Gamma,\leq)$ is *regular* if there exist $x \in S$ and $\alpha,\beta \in \Gamma$ such that $a \leq a\alpha x\beta a$. An ordered Γ-semihypergroup S is said to be regular if every element of S is regular.

Lemma 3.4 Let $(S,\Gamma,\leq)$ be an ordered Γ-semihypergroup. Then, the following statements are equivalent:

1. S is regular.
2. $a \in (a\Gamma S\Gamma a]$ for every $a \in S$.
3. $A \subseteq (A\Gamma S\Gamma A]$ for every $A \subseteq S$.

Proof. The proof is straightforward.

Theorem 3.5 Let $(S,\Gamma,\leq)$ be a regular ordered Γ-semihypergroup. If I is a one-sided Γ-hyperideal of S, then $(I\Gamma I] = I$.

Proof. Let I be a left Γ-hyperideal of S. Then, $(I\Gamma I] \subseteq (S\Gamma I] \subseteq (I] = I$. Now, let $a \in I$. Since S is regular, there exist $x \in S$ and $\alpha,\beta \in \Gamma$ such that $a \leq$

$a\alpha x\beta a$. As I is a left Γ-hyperideal of S, we have $x\beta a \subseteq I$. Hence, $a \leq a\alpha x\beta a \subseteq I\Gamma I$. This means that $a \in (I\Gamma I]$, and so $I \subseteq (I\Gamma I]$. Hence, the theorem is proved.

Let S be a Γ-semihypergroup and $\gamma \in \Gamma$. We define $x \circ y = x\gamma y$ for every $x, y \in S$. Then $(S, \circ)$ is a semihypergroup and we denote it by S_γ. A Γ-semihypergroup S is called a *Γ-hypergroup* if (S_γ, γ) is a hypergroup for every $\gamma \in \Gamma$. An ordered Γ-semihypergroup $(S, \Gamma, \leq)$ is called an *ordered* Γ-*hypergroup* if for every $\gamma \in \Gamma$, (S, γ) is a hypergroup. In fact, for all $a \in S$ and $\gamma \in \Gamma$, we have $a\gamma S = S\gamma a = S$.

Theorem 3.6 Every ordered Γ-hypergroup is a regular ordered Γ-semihypergroup.

Proof. Let $(S, \Gamma, \leq)$ be an ordered Γ-hypergroup and $a \in S$. Then $(a\gamma S] = (S\gamma a] = S$ for all $\gamma \in \Gamma$. Hence, $a \leq a\gamma x$ for some $x \in S$. Again, $x \leq y\gamma a$ for some $y \in S$. Then, $a \leq a\gamma x \leq a\gamma(y\gamma a)$ which implies that $a \in (a\Gamma S\Gamma a]$ *for every* $a \in S$. Therefore, S is a regular ordered Γ-semihypergroup.

Definition 3.7 A proper Γ-hyperideal I of an ordered Γ-semihypergroup $(S, \Gamma, \leq)$ is called a covered Γ-hyperideal (simply C-Γ-hyperideal) of S if it satisfies $I \subseteq (S\Gamma(S \setminus I)\Gamma S]$.

Note that S itself is not a C-Γ-hyperideal.

Example 2. Let $S = \{a, b, c, d\}$ and $\Gamma = \{\gamma, \beta\}$ be the sets of binary hyperoperations defined as follows:

γ	a	b	c	d
a	a	$\{b,d\}$	c	d
b	$\{b,d\}$	b	$\{b,d\}$	d
c	c	{b,d}	a	d
d	d	d	d	d

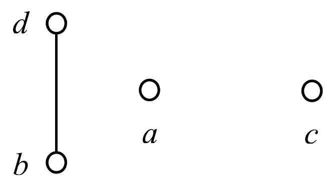
β	a	b	c	d
a	$\{a,c\}$	$\{b,d\}$	$\{a,c\}$	d
b	$\{b,d\}$	b	$\{b,d\}$	d
c	$\{a,c\}$	$\{b,d\}$	$\{a,c\}$	d
d	d	d	d	d

Then S is a Γ-semihypergroup [29]. We have $(S, \Gamma, \leq)$ is an ordered Γ–semihypergroup where the order relation $\leq$ is defined by:

$$\leq := \{(a,a), (b,b), (b,d), (c,c), (d,d)\}.$$

The covering relation and the figure of S are given by:

$$\prec = \{(b,d)\}.$$



It is a routine matter to verify that $I = \{b, d\}$ is a C-Γ-hyperideal of S.

Lemma 3.8 If I, J are two C-Γ-hyperideals of an ordered Γ-semihypergroup $(S, \Gamma, \leq)$, then $I \cap J$ is also a C-Γ-hyperideal of S.

Proof. Let $x \in I$, $y \in J$ and $\gamma \in \Gamma$. Then, $x\gamma y \subseteq I\Gamma J \subseteq I\Gamma S \subseteq I$ and $x\gamma y \subseteq I\Gamma J \subseteq S\Gamma J \subseteq J$. So, $x\gamma y \subseteq I \cap J$ and hence $\emptyset \neq I \cap J \subseteq S$. We have:

$$(I \cap J)\Gamma S \subseteq I\Gamma S \subseteq I \text{ and } S\Gamma(I \cap J) \subseteq S\Gamma J \subseteq J.$$

Similarly, $(I \cap J)\Gamma S \subseteq J$ and $S\Gamma(I \cap J) \subseteq I$. So, we have $(I \cap J)\Gamma S \subseteq I \cap J$ and $S\Gamma(I \cap J) \subseteq I \cap J$. Now, let $x \in I \cap J$, $y \in S$ and $y \leq x$. Since I and J are Γ-hyperideals of S, we obtain $y \in I$ and $y \in J$. So, $y \in I \cap J$. Hence, $I \cap J$ is a proper Γ-hyperideal of S. By hypothesis, we have:

$$I \cap J \subseteq I \subseteq (S\Gamma(S \setminus I)\Gamma S] \subseteq (S\Gamma(S \setminus (I \cap J))\Gamma S].$$

Hence, $I \cap J$ is a C-Γ-hyperideal of S.

Theorem 3.9 Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. If S has a C-Γ-hyperideal, then every C-Γ-hyperideal of S is minimal if and only if the intersection of any two different C-Γ-hyperideals is empty.

Proof. Suppose I and J are two different C-Γ-hyperideals of S. Then I and J are minimal. If $I \cap J \neq \emptyset$, then, by Lemma 3.8, $I \cap J$ is a C-Γ-hyperideal of S. As I and J are minimal, we obtain $I = J$. This is a contradiction. Hence, $I \cap J = \emptyset$. The converse is clear.

Theorem 3.10 Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. If S is not simple, then S contains at least one C-Γ-hyperideal of S.

Proof. Let A be any proper Γ-hyperideal of S. We have:

$$\begin{aligned} S\Gamma(S\Gamma(S \setminus A)\Gamma S] &= (S]\Gamma(S\Gamma(S \setminus A)\Gamma S] \\ &\subseteq (S\Gamma(S\Gamma(S \setminus A)\Gamma S)] \\ &= ((S\Gamma S)\Gamma(S \setminus A)\Gamma S] \\ &\subseteq (S\Gamma(S \setminus A)\Gamma S]. \end{aligned}$$

Similarly, we have $(S\Gamma(S \setminus A)\Gamma S]\Gamma S \subseteq (S\Gamma(S \setminus A)\Gamma S]$. Now, suppose that $x \in (S\Gamma(S \setminus A)\Gamma S]$ and $y \in S$ such that $y \leq x$. Since $x \in (S\Gamma(S \setminus A)\Gamma S]$, it follows that $x \leq u$ for some $u \in S\Gamma(S \setminus A)\Gamma S$. Since $y \leq x$ and $x \leq u$, we get $y \leq u$. So, we have $y \in (S\Gamma(S \setminus A)\Gamma S]$. Hence, $(S\Gamma(S \setminus A)\Gamma S]$ is a Γ-hyperideal of S. Consider $I = A \cap (S\Gamma(S \setminus A)\Gamma S]$. By the proof of Lemma 3.8, I is a Γ-hyperideal of S. Next, we show that I is a C-Γ-hyperideal of S. We have:

$$I \subseteq (S\Gamma(S \setminus A)\Gamma S] \subseteq (S\Gamma(S \setminus I)\Gamma S].$$

Therefore, we conclude that I is a C-Γ-hyperideal of S.

Example 3. Let $S = \{a, b, c\}$ and $\Gamma = \{\gamma, \beta\}$ be the sets of binary hyperoperations defined as follows:

γ	a	b	c
a	$\{a, b\}$	$\{b, d\}$	c
b	$\{a, b\}$	b	c
c	c	c	c

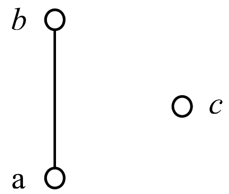
β	a	b	c
a	a	$\{a, b\}$	c
b	$\{a, b\}$	$\{a, b\}$	c
c	c	c	c

Then S is a Γ-semihypergroup. We have $(S, \Gamma, \leq)$ is an ordered Γ–semihypergroup, where the order relation $\leq$ is defined by:

$$\leq := \{(a, a), (a, b), (b, b), (c, c)\}.$$

The covering relation and the figure of S are given by:

$$\prec = \{(a, b)\}.$$



Since $I = \{c\}$ is a Γ-hyperideal of S, it follows that S is not simple. Since $c = c\gamma b\beta c, b \notin I$, we have $I \subseteq (S\Gamma(S \setminus I)\Gamma S]$. Hence, I is a C-Γ-hyperideal of S.

Theorem 3.11 Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. If S is not simple, then every C-Γ-hyperideal of S is minimal if and only if the intersection of any two different C-Γ-hyperideals is empty.

Proof. It follows immediately from Theorems 3.9 and 3.10.

Theorem 3.12 Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. If S contains two different maximal Γ-hyperideals I, J, then none of them is a C-Γ-hyperideal of S.

Proof. Let I and J be two different maximal Γ-hyperideals of S. Then, $I \cup J = S$. Hence, $S \setminus I \subseteq J$ and $S \setminus J \subseteq I$. If I is a C-Γ-hyperideal of S, then $I \subseteq (S\Gamma(S \setminus I)\Gamma S] \subseteq (S\Gamma J\Gamma S] \subseteq (J] = J$. Since $I \cup J = S$, it follows that $J = S$, which is a contradiction. Therefore, I is not a C-Γ-hyperideal of S. If J is a C-Γ-hyperideal of S, then $J \subseteq (S\Gamma(S \setminus J)\Gamma S] \subseteq (S\Gamma I\Gamma S] \subseteq (I] = I$. Since $I \cup J = S$, it follows that $I = S$, which is a contradiction. Therefore, J is not a C-Γ-hyperideal of S.

Theorem 3.13 Let $(S, \Gamma, \leq)$ be an ordered Γ-semihypergroup. If I, J are two C-Γ-hyperideals of S, then $I \cup J$ is a C-Γ-hyperideal of S.

Proof. Clearly, $I \cup J \neq \emptyset$. We have $(I \cup J)\Gamma S = I\Gamma S \cup J\Gamma S \subseteq I \cup J$. So, for each $a \in I \cup J$, $x \in S$ and $\gamma \in \Gamma$, $a\gamma x \subseteq I \cup J$. Similarly, $x\gamma a \subseteq I \cup J$. Now, let $a \in I \cup J$, $y \in S$ and $y \leq a$. Since I, J are Γ-hyperideals of S, we obtain $y \in I \cup J$. Hence, $I \cup J$ is a Γ-hyperideal of S. Let I, J be two C-Γ-hyperideals of S; we need to prove that $I \cup J \subseteq (S\Gamma(S \setminus (I \cup J))\Gamma S]$. By hypothesis, we have:

$$I \subseteq (S\Gamma(S \setminus I)\Gamma S] \text{ and } J \subseteq (S\Gamma(S \setminus J)\Gamma S].$$

If $a \in I$, then there exists $x \in S \setminus I$ such that $a \in (S\Gamma x\Gamma S]$. Now, one of two following cases happens:

Case 1. If $x \in S \setminus (I \cup J)$, then $a \in (S\Gamma(S \setminus (I \cup J))\Gamma S]$.

Case 2. If $x \in (S \setminus I) \cap J$, then $x \in J$. Thus, there exists $b \in S \setminus J$ such that $x \in (S\Gamma b\Gamma S]$. If $b \in I$, then $x \in (S\Gamma b\Gamma S] \subseteq (S\Gamma I\Gamma S] \subseteq (I] = I$. This is a contradiction. This means that $b \in S \setminus I$, and so $b \in (S \setminus I) \cap (S \setminus J) = S \setminus (I \cup J)$. We have:

$$\begin{aligned} a \in (S\Gamma x\Gamma S] &\subseteq (S\Gamma(S\Gamma b\Gamma S]\Gamma S] \\ &\subseteq (S\Gamma b\Gamma S] \\ &\subseteq \big(S\Gamma\big(S \setminus (I \cup J)\big)\Gamma S\big]. \end{aligned}$$

Therefore, $I \subseteq \big(S\Gamma\big(S \setminus (I \cup J)\big)\Gamma S\big]$. Similarly, we have $J \subseteq \big(S\Gamma\big(S \setminus (I \cup J)\big)\Gamma S\big]$. Hence the proof is completed.

We now describe the relationship between the proper Γ-hyperideal and the C-Γ-hyperideal.

Theorem 3.14 Let $(S, \Gamma, \leq)$ be a regular ordered Γ-semihypergroup. If for any proper Γ-hyperideal I and for every $I_S(a) \subseteq I$, there exists $b \in S \setminus I$ such that $I_S(a) \subseteq I_S(b)$, then every proper Γ-hyperideal of S is a C-Γ-hyperideal of S.

Proof. Let S be regular. Clearly, $(S\Gamma S] \subseteq S$. Since S is regular, we have $S \subseteq (S\Gamma S\Gamma S] \subseteq (S\Gamma S]$. Thus, we have $S = (S\Gamma S]$. Let I be any proper Γ-hyperideal of S and $a \in I$. Then $I_S(a) \subseteq I$. So, there exists $b \in S \setminus I$ such that $I_S(a) \subseteq I_S(b) \subseteq S$. Since $S = (S\Gamma S]$, it follows that $S = (S\Gamma S\Gamma S]$. Thus, $b \leq x\alpha u\beta y$ for some $x, y, u \in S$ and $\alpha, \beta \in \Gamma$. If $u \in I$, then $b \leq x\alpha u\beta y \subseteq S\Gamma I\Gamma S$, which is a contradiction. This leads to $u \notin I$. We have:

$$a \in I_S(a) \subseteq I_S(b) \subseteq (S\Gamma u\Gamma S] \subseteq (S\Gamma(S \setminus I)\Gamma S].$$

Hence, $I \subseteq (S\Gamma(S \setminus I)\Gamma S]$. This completes the proof.

We now give the main result of this paper as below.

Proposition 3.15 Let I be a Γ-hyperideal of a regular ordered Γ-semihypergroup $(S, \Gamma, \leq)$. Then, any C-Γ-hyperideal J of I is a C-Γ-hyperideal of S.

Proof. Since I is a Γ-hyperideal of S, it follows that I is a sub Γ-semihypergroup of S. Let $a \in I \subseteq S$. Then, there exist $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a \leq a\alpha x\beta a \leq a\alpha x\beta(a\alpha x\beta a) = a\alpha(x\beta a\alpha x)\beta a$. Since I is a Γ-hyperideal of S, it follows that $x\beta a\alpha x \subseteq S\Gamma I\Gamma S \subseteq I$. Hence, $a \leq u$ for some $u \in a\alpha(x\beta a\alpha x)\beta a \subseteq a\Gamma I\Gamma a$. This means that $a \in (a\Gamma I\Gamma a]_I$. Hence, I is a regular sub Γ-semihypergroup of S.

Let $a \in J \subseteq I$ and $s \in S$. Then, $a\gamma s \subseteq I$ where $\gamma \in \Gamma$. Now, suppose that $v \in a\gamma s \subseteq I$. Then, there exist $y \in I$ and $\lambda, \mu \in \Gamma$ such that $v \leq v\lambda y\mu v \subseteq (a\gamma s)\lambda y\mu(a\gamma s) \subseteq J\Gamma(S\Gamma I\Gamma I)\Gamma S \subseteq J\Gamma I\Gamma S \subseteq J\Gamma I \subseteq J$. Since J is a Γ-hyperideal of I, it follows that $a\gamma s \subseteq J$. Similarly, we have $s\gamma a \subseteq J$. If $e \in J \subseteq I$ and $f \in S$ such that $f \leq e$, then $f \in I$. Since J is a Γ-hyperideal of S, it follows that $f \in J$. Hence, J is a Γ-hyperideal of S. By hypothesis, we have $J \subseteq (S\Gamma(I \setminus J)\Gamma S]$. As $\emptyset \neq I \setminus J \subseteq S \setminus J$, we obtain:

$$J \subseteq (S\Gamma(I \setminus J)\Gamma S] \subseteq (S\Gamma(S \setminus J)\Gamma S].$$

Therefore, J is a C-Γ-hyperideal of S.

4 Conclusion

In this paper, we introduced and studied some properties of C-Γ-hyperideals of ordered Γ-semihypergroups. In particular, we described a connection between proper Γ-hyperideals and C-Γ-hyperideals. Furthermore, we proved that in regular ordered Γ-semihypergroups, every C-Γ-hyperideal of a Γ-hyperideal is also a C-Γ-hyperideal. The results of this paper can also be used on ordered semihypergroups by some moderate modifications. We hope that this work will offer the foundation for further study of the theory on ordered Γ-semihypergroups.

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