On the Interaction between *Trichogramma chilonis* and Jatiroto Flies with Stem Borer Pests in Sugarcane Plantation

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Abstract. Pests in sugarcane plantations are a major cause of damage, which could lead to severe damage to the crop, reduction of sugar quality, and significant economic loss. One of the major pests known in sugarcane plantations is stem borer (*Chilo sacchriphagus*), which attacks the canes. Two primary parasitoids, *Trichogramma chilonis*, which predates the stem borer eggs, and the Jatiroto fly (*Diatraeophaga striatalis*), which predates the stem borer larvae, are discussed here. This paper presents a time-dependent ten-dimensional dynamical mathematical model consisting of four-stage stem borer compartments, three-stage *Trichogramma* compartments, and three-stage Jatiroto compartments. Simulations are presented to describe the phenomenon of *Trichogramma* predation, Jatiroto predation, and simultaneous predation of both predators. It is shown that the release rate of each predator and a combined release of two predators can significantly reduce the infestation levels to a tolerable level for sugarcane production. The oscillatory dynamics of the stem borers and the Jatiroto flies affected the release timing strategy based on the level of infestation in the field. The results are expected to help us better understand the predator-prey phenomenon in the field and improve the forecasting of infestations in the field.

Keywords: *bifurcation; infestation; Jatiroto flies; predator-prey; stem borer; sugarcane plantation; Trichogramma chilonis.*

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1 Introduction

Plant diseases such as Fusarium wilt and red rot have been widely known to severely threaten sugarcane plantations. These diseases cause a decrease in crop yields and the quality of the sugar produced. An integrated control approach between fungicides and biological agents such as Trichoderma has demonstrated effectiveness in reducing their impact [1]. Competition between weeds and sugar cane plants for water, nutrients, and light can cause yield reduction of up to 70%.

Selective herbicides and integrated control strategies are essential to reduce the impact of weeds and pests on sugarcane crops. The dynamics of the pest and disease population and the infestation in different varieties of sugarcane may vary with climate change. This problem is a great challenge for pest and disease management in developing adaptive and responsive approaches [2]. Stem borer pests are commonly found in sugar cane plantations. These pests reduce stem weight, juice quality, and sugar yield. The management of stem borer pests often involves an Integrated Pest Management (IPM) approach that includes using pesticides and biological agents and selecting resistant sugarcane varieties [3].

Sugar production areas in Indonesia are located on the islands of Sumatra, Java, Sulawesi, and Nusa Tenggara. The climate conditions vary in each region and the sugar mills vary in terms of resources. Consequently, sugarcane stem borer parasitoids differ between regions [4]. Likewise, appropriate technology and human resources are used in agronomic practices. Therefore, the management of sugarcane stem borer pests must be unique to each production area. Indonesia has attempted to achieve self-sufficiency in sugar production, with current production at 2.4 million tons of sugar from 453,000 hectares of sugar cane *(Saccharum spp.* hybrid). There are 58 sugarcane factories in Indonesia, which process 30 million tons of sugar cane from 380 to 400,000 ha. More than three-quarters of sugar cane production occurs on the island of Java. During the past 40 years, production has declined. The main causes are diseases and pests, notably stem borers, which significantly affect the crop yields on the island of Java [5]. A study by [6] has shown that significant losses can occur when $\geq 10\%$ of cane stalks are damaged, reaching up to 34% reduction in sugar production.

Sugarcane productivity in Indonesia has stagnated due to less superior varieties and low efficiency of sugar factories. In many sugarcane plantations, stem borers have been found to cause significant harvest losses. The key counteracting strategies implemented include revitalization of existing sugarcane factories and expansion of sugarcane plantations outside Java [7].

The use of synthetic insecticides was discovered in the 1940s and everyone was optimistic that pest control would become very easy. However, it has become increasingly clear that many problems arise from the use of synthetic insecticides. Some types of pests become resistant, non-target living creatures become victims, and pest resurgence occurs. In addition, issues related to the environment and health also arise. Until now, the protection of clothing, food, and shelter from insect pests, diseases, and weeds still depends heavily on the use of chemical pesticides. Pest control methods such as using pest-resistant plants, cultivation techniques, or biological control may reduce the use of pesticides but are not able to eliminate pests thoroughly. Moreover, some types of pests can only be controlled using pesticides.

For this reason, there is a need for a pest management strategy that integrates various methods, also known as IPM. Biological control is essential in IPM programs. Other components are chemical, physical and mechanical control, cultivation techniques, and the use of pest-resistant varieties. The potential and future of biological control are closely related to the development of IPM [8].

More than 100 types of pests can attack sugar cane, such as insects (Lepidoptera, Coleoptera, Hemiptera); fleas (Aphid); mites (Acarina); parrots; and mammals (rats and wild boars). There are three principal sugarcane pests found in different regions in Indonesia whose presence greatly disrupts sugarcane production and productivity, namely: 1) Top borers *(Scirpophaga excerptalis)*. Sugar losses because of death due to top borers, at 5, 4, 3, 2, and 1 months before cutting, reached 77%, 58%, 46%, 24%, and 15%, respectively. Symptoms of attack are newly hatched larvae burrowing into young leaves that have not yet opened. The larvae bore towards the growing point and penetrate the stem. Affected leaves will curl and dry (die). Each infected stem is usually inhabited by one borer. 2) Stem borers *(Chilo sacchariphagus)*. For every percent of segment damage, sugar loss is 0.5%. Losses due to stem borer attacks will cause a decrease in sugarcane weight. Successive attacks of 10%, 20%, and 30% will reduce the sugarcane weight by 16%, 38.8%, and 60.8%, respectively. Symptoms of attack often occur at the growing points and plants' young shoots so young leaves wilt and die. The passageways caused by these borers are very irregular. In one segment, one or more larvae are usually found. 3) Sugarcane white grub *(Lepidiota stigma)*. The white grub is a polyphagous pest. Apart from sugar cane, other plants that are its source of food include cassava, corn, papaya, rubber, mahogany, and others. White grubs like light, sandy soil and are the most voracious at the third instar stage (3 to 4 individuals can consume the roots of 1 cluster of sugar cane). Signs of attack appear clearly during the dry season and prawn plants suffer more. *Lepidiota stigma* is the most damaging to sugarcane roots [9].

Four stem borer species have been identified in Indonesia: *Chilo sacchariphagus* (Bojer), *C. auricilius* Dudgeon, *C. terrenellus* Pagenstecher (Lepidoptera: Crambidae), and *Phragmataecia castanea* (Hübner) (Lepidoptera: Cossidae). In addition, the shoot borer *Scirpophaga excerptalis* (Walker) (Lepidoptera: Pyralidae) can cause significant losses. Three stem borers *C. infuscatelus* Snellen (Lepidoptera: Crambidae), *Tetramoera schistaceana* (Snellen) (Lepidoptera: Tortricidae), and *Sesamia inferens (Walker)* (Lepidoptera: Noctuidae) also attack sugarcane. However, high levels of infestation have not been reported in Indonesia. Indonesia's most dominant and destructive stem borer species is the spotted stem borer, *C. sacchariphagus*. This species is widespread in sugar cane plantations in Sumatra, Java, Sulawesi, and Nusa Tenggara Island [10]. *P. castanea* is currently only found in sugar cane plantations in North Sumatra and Lampung, while *C. terrenellus* is only found in West Papua [11].

Classical biological control, namely introducing and establishing exotic natural enemies against introduced pest species, is a well-known technology and an important component in managing sugarcane pests [12]. This technology has been developed for over 50 years in most sugar cane-producing countries, especially to control stem borer pests that are difficult to reach when the larvae are already in the sugar cane stem. In addition, sugar cane is a very compact crop, sometimes reducing the efficacy of pesticide treatments when sprayed through the air. In this context, sugarcane is treated less with chemicals for insect control than other cropping systems such as cotton or horticulture. Biocontrol in sugarcane includes the use of parasitoids and entomopathogenic fungi. Although parasitoids are primarily used on Lepidoptera, entomopathogenic pathogens are more widely used to control coleopteran and heteropteran species.

The most common parasitoids are maintained in biological control laboratories and used for field release. The reintroduction of *T. chilonis* and *D. striatalis* has reduced stem borer populations below economic thresholds in recent years. For example, borer attacks in the early stages of plant growth (one to four months old) can be immediately controlled by releasing *T. chilonis* 10 to 12 times at one-week intervals. Each release must contain at least 100,000 Trichogramma/ha to be effective [13]. Parasitoid larvae such as *C. flavipes* and *D. striatalis* are also released in the field if the infestation number is $\geq 5\%$ when the sugarcane is four months old or more. Pesticides should not be used before, during, and after parasitoid release. In recent years, many sugar factories in Indonesia have closed due to a lack of sugar cane supply, especially sugar factories on the island of Java. As a result, the biological control laboratories of these factories were also closed. However, the production capacity of some laboratories was increased to cover other plantations that lost their parasitoid suppliers.

Planting resistant varieties and managing varieties is the most valuable and recommended preventive control for sugarcane stem borer pests. An outbreak recently occurred in the Lampung region when the dominant variety, ROC 22, was planted, exacerbating the *C. sacchariphagus* infestation. At that time, almost 2,000 ha was badly damaged. Indonesian varieties such as PSJT 941, PS 851, and PS 864 are resistant to stem borer attacks due to the increased hardness of their skin and pubescent leaf sheaths. The planting of resistant varieties and management of the varietal diversity are the most valuable and recommended preventive control measures against sugarcane stem borers. Planting varieties with varying resistance to borers may reduce the risk of outbreaks. A recent outbreak occurred in Lampung area when the dominant variety, ROC 22, was planted and aggravated *C. sacchariphagus* infestations. At that time, almost 2,000 ha were severely damaged. *C. sacchariphagus* also attacked resistant and tolerant varieties close to the susceptible variety. Sugarcane stem borer infestations are influenced by morphological differences among varieties [14]. Indonesian varieties such as PSJT 941, PS 851, and PS 864 are resistant to stem borer infestations due to increased hardness of the rind and pubescent leaf sheaths.

Variety with natural resistance to pests can reduce invading sugarcane borers and, in turn, influence the effectiveness of biological control agents such as parasitoids. Resistant sugarcane varieties can improve the success of biological control by reducing the burden from pest populations, thereby influencing interactions between sugarcane borers and parasitoids in the ecosystem. Additionally, more susceptible varieties may exacerbate population dynamics and challenges in biological control, demonstrating the importance of variety selection in sustainable pest control strategies [15].

Applying pest control strategies against stem borers on sugarcane plantations has great potential but also various challenges. This process involves releasing predators that have been bred in large numbers to effectively reduce pest populations. However, the main challenges in its implementation are the difficulty of obtaining high-quality and stable predators, optimal release times, and adapting the method to changing environmental conditions. According to [16], despite progress in biological augmentation, much work still needs to be done to improve this release method and increase its effectiveness in sugar cane plantations. These difficulties include the need for a better understanding of the interactions between predators and pests and efforts to reduce negative impacts on non-target species and the ecosystem as a whole.

Biological pest control using parasitoids and natural predators has shown positive results in the sustainable control of pest populations. Meanwhile, an integrated approach combining three control methods (cultural, biological, and varietal) for more effective pest management has been proposed by [17]. According to [18], in addition, chemical, biological, and agronomic controls such as the selection of resistant varieties and crop rotation can help significantly reduce infestation by stem borers. These practices increase plant resistance to pests and reduce the need for chemical controls. Studies by [19] and [20] applied various biological control methods in Venezuela, including the introduction of parasitoids and predators to control pests such as sugarcane stem borers and aphids, which reduced 50 to 60% of the pest population and increased productivity by 10 to 20%.

1.1 Pest Control in Sugarcane Plantations

Sugarcane is a vital crop that plays a crucial role in the global economy, providing a significant source of sugar, biofuel, and other valuable by-products. However, sugarcane plantations are vulnerable to many pests that can significantly impact crop yields and quality [21]. Effective pest control strategies are essential to maintain the profitability and sustainability of sugarcane production.

One of the key challenges in sugarcane pest management is the diversity of pests that can affect sugarcane crops, including insects, diseases, and weeds [22]. Among the most significant insect pests are scarabid beetles, commonly known as white grubs, which can cause extensive damage to crops by feeding on the roots and underground stems [23].

Only two parasitoid flies belonging to the Tachinidae family that attack sugarcane stem borer larvae have been found in Asia, namely the Jatiroto fly and *Sturmiopsis inferens*. The Jatiroto fly *(Diatraeophaga striatal)* is a parasitoid of sugarcane stem borer larvae and is commonly used in pest control in Java. In contrast, the *Sturmiopsis inferens* fly is commonly used in Sumatra as a natural enemy of sugarcane stem borer larvae. It was first described by Towsend in 1916. This parasitoid has a high parasitizing ability on *Chilo sp.* of about 52%.

Pest control in plantations can be done biologically through parasitoids and natural predators, which have shown positive results in sustainably controlling pest populations. In [24], an integrated approach was used that combines three different control methods (cultural, biological, and varietal) for more effective pest management. According to [25], in addition, chemical control, biological and agronomic controls such as selecting resistant cultivars and crop rotation can help significantly reduce stem borer infestation.

Biological control has been commonly used in Indonesian sugarcane fields since the 1970s, when key parasitoids (e.g., *Telenomus globosus*, *Trichogramma chilonis*, and *Cotesia flavipes*) were introduced to target *C. sacchariphagus* [6]. Several sugar factories in Indonesia have continued to maintain and release these parasitoids, which helps reduce reliance on chemical control measures. These practices increase crop resistance to pests and reduce the need for chemical control. Studies by [4] and [20] applied various biological control methods in Venezuela, including the introduction of parasitoids and predators to control pests such as sugarcane stem borer and aphids, which reduced 50 to 60% of the pest populations, and productivity increases reached 10 to 20%.

2 Mathematical Model

Here, we consider the interactions between two predators: *Trichogramma chilonis* and Jatiroto flies (*Diatraeophaga striatalis*), and stem borer (*Chilo sacchariphagus*) pests. We will use the abbreviations 'SB' for stem borer, 'TC' for *Trichogramma chilonis*, and 'JF' for Jatiroto flies to simplify notation. The interaction between each predator and prey is as follows. TC and JF are responsible for predating SB's eggs and larvae, respectively. Table 1 shows the corresponding life cycles of SB, TC, and JF.

We use E, L, P, and M to denote the stem borer egg, larva, pupa, and moth population respectively. For the corresponding TC, we use R for the population of eggs, S for the population of larvae and pupa, and T for the population of imagos. Similarly, for the predator JF, we use X, Y, and J. We combine the two-stage larvae and pupa in one compartment, because each stage stays within the hosts (TC eggs and JF larvae, respectively).

We assume that the prey SB grows according to the following logistic model:

$$\begin{aligned}
\frac{dE}{dt} &= \alpha M - \beta E \\
\frac{dL}{dt} &= \beta E(1 - \frac{L}{K(t)}) - \gamma L \\
\frac{dP}{dt} &= \gamma L - \eta P \\
\frac{dM}{dt} &= \eta P - \mu M,
\end{aligned} \tag{1}$$

where the carrying capacity is chosen in the form of

$$K(t) = K_o(\varepsilon + \arctan(0.1t)), \tag{2}$$

and t is the weekly time unit. The choice of $K(t)$ in (2) reflects field conditions where pest attacks begin a few months into the crop cycle—when sugarcane stalks start to appear—and K_0 is the average number of fully grown sugarcanes per hectare. For consistency in the modeling, we assume that the pest attacks are uniformly distributed over all sugarcane trees. Then, all variables E, L, P, M can be interpreted as the number of sugarcane trees attacked by each E, L, P, M. Further, for simplification in writing, we use the terminology 'the number of individuals' for each state variable.

To simplify the model, each population is normalized by K_0. Furthermore, the coefficient 0.1 in (2) was selected to moderate the growth rate of $K(t)$ over a 52-week period. This ensures that $\arctan(0.1t)$ transitions gradually from near-zero to a higher (but not saturated) value—reflecting the real-world pest emergence timeline without causing an overly steep or overly slow increase in $K(t)$.

The system (1) is invariant within the biological space $\Omega = \mathbf{R}_0^4 \cup \{(0,0,0,0)\}$, since each normal of the vector field at each border space $E = 0$, $L = 0$, $P = 0$, and $M = 0$ is pointing inward. Note that the dynamic of the larva is bounded within $L \leq L^* \in [0, K_0(1+\varepsilon)]$. The rest of the states are bounded following the following linear differential inequality:

$$\frac{d[E,P,M]^T}{dt} \leq A[E,P,M]^T + [\gamma, L^*, 0]^T, \tag{3}$$

where

$$A = \begin{bmatrix} -\beta & 0 & \alpha \\ 0 & -\eta & 0 \\ 0 & \eta & -\mu \end{bmatrix}$$

has all negative eigenvalues.

Table 1 Life cycles of predators and pests

Species	Duration (days)	Source
Stem borer egg	3-10 days	[4]
Stem borer larva	14-42 days	[4]
Stem borer pupa	7-14 days	[4]
Adult Stem borer	1-3 days	[4]
Trichogramma egg	1-3 days	[26]
. *Trichogramma* larva	3-7 days	[26]
Trichogramma pupa	5-10 days	[26]
Adult stem	14-28 days	[26]
Jatiroto egg	2-5 days	[27]
Jatiroto larva	10-14 days	[27]
Jatiroto pupa	7-14 days	[27]
Jatiroto adult	14-28 days	[27]

Table 2 Descriptions of variables given in system (1)

Variable	Definition	Unit
$E(t)$	Stem borer Eggs	Number of eggs
$L(t)$	Stem borer larvae	Number of larvae
$P(t)$	Stem borer pupas	Number of pupas
$M(t)$	Adult stem borer	Number of adults
$R(t)$	*Trichogramma* egg	Number of eggs
$S(t)$	*Trichogramma* larva-pupa	Number of larva-pupa
$T(t)$	Adult *Trichogramma*	Number of adults
$X(t)$	Jatiroto egg	Number of eggs
$Y(t)$	Jatiroto larva-pupa	Number of larva-pupa
$J(t)$	Jatiroto adult	Number of adults

Parasitism is a key biological interaction in which one organism (the parasite) benefits by exploiting another organism (the host), often leading to the host's harm

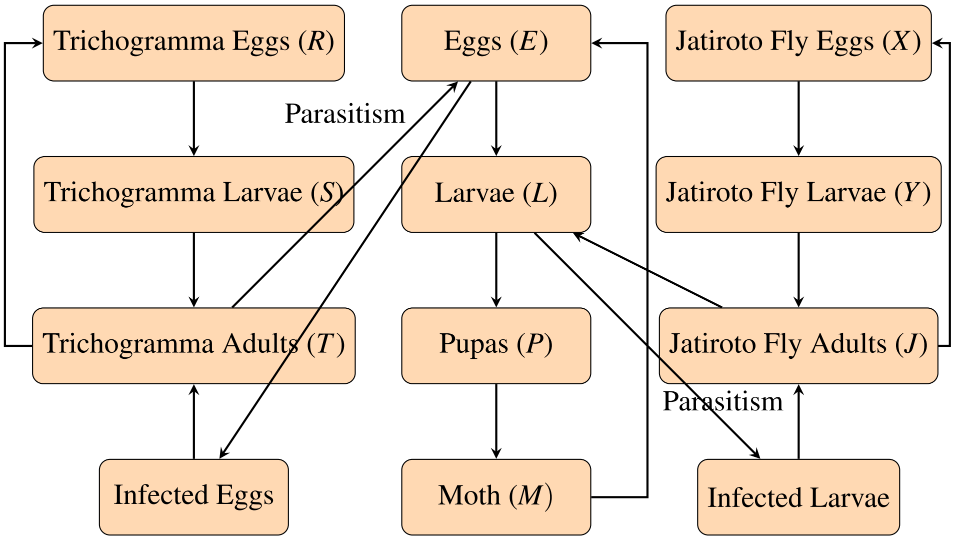


Figure 1 Compartment diagram representing the life cycles of stem borer pests and their biological control agents, Jatiroto flies and *Trichogramma.*

or death. *Trichogramma* wasps serve as effective biological control agents in pest management by parasitizing the eggs of stem borers. Adult *Trichogramma* females lay (T) their eggs inside the stem borer eggs (E), where their larvae (S) develop by consuming the host egg contents. This process prevents the eggs from hatching into larvae, disrupting the early stages of the pests' development. By targeting the egg stage, Trichogramma significantly reduces the pest's ability to progress to more destructive stages, such as larvae or moths.

Similarly, Jatiroto flies (J) target the larvae stage of stem borers (L), one of the most harmful stages to crops. Adult flies deposit their eggs (X) directly next to stem borer larvae. The fly larvae (Y) then enter the gaps between segments of the stem, ultimately leading to the death of the host. This form of parasitism not only decreases the pest population but also supports the growth and reproduction of Jatiroto flies, enhancing their role as natural enemies in the pest control system.

A dynamical model captures the interactions between the stem borer pest and its natural enemies, *Trichogramma* and Jatiroto flies, by describing how their populations change over time. The model incorporates processes such as reproduction, natural mortality, and the effects of parasitism by biological control agents. For example, the parasitism of eggs by *Trichogramma* reduces the pest egg population, while the parasitism of larvae by Jatiroto flies decreases the number of viable larvae. The full system for interaction between the stem borer and the two predators, *Trichogramma* and Jatiroto flies, is given as follows:

$$
\begin{aligned}
\frac{dE}{dt} &= \alpha M - \beta E - \frac{aTE}{r+E} \\
\frac{dL}{dt} &= \beta E\left(1 - \frac{L}{K_0(0.1+\arctan\left(\frac{1}{10}t\right))}\right) - \gamma L - \frac{bYL}{r+L} \\
\frac{dP}{dt} &= \gamma L - \eta P \\
\frac{dM}{dt} &= -M\mu + P\eta \\
\frac{dR}{dt} &= \rho T - \frac{cTE}{r+E} - \kappa R \\
\frac{dS}{dt} &= \frac{dTE}{1+E} - S\delta + S_0 \\
\frac{dT}{dt} &= \delta S - \phi T \\
\frac{dX}{dt} &= \xi J - \zeta X \\
\frac{dY}{dt} &= \zeta X - \frac{eYL}{r+L} - \nu Y \\
\frac{dJ}{dt} &= \frac{fYL}{r+L} - \theta J + J_0.
\end{aligned} \tag{4}
$$

In biological interpretation and simulations, we take $K = 1000$, which measures the average number of sugar cane trees per hectare. All population sizes are measured as the average number of eggs, larvae, pupae, or moths per tree. On this scale, the number of stem borer larvae represents the level of infestation.

Natural biological assumptions that are related to the survival of each prey and predator are used as follows:

$$
\begin{aligned}
&\text{survival of Stem borer:} && \alpha > \mu \\
&\text{survival of } \textit{Trichogramma}\text{:} && d > \phi \text{ and } \rho > c \\
&\text{survival of Jatiroto flies:} && f > \delta \text{ and } \xi > e.
\end{aligned} \tag{5}
$$

Using an argument similar to the SB system (1), the system (4) is invariant within $\mathbf{R}_{+}^{10} \cup \{(0,0,0,0,0,0,0,0,0,0)\}$, and we immediately derive the boundedness of the

Table 3 Description of parameters in the system (4)

Parameter	Definition	Unit
α	Egg-laying rate of moths	[time^{-1}]
β	Transition rate from eggs to larvae	[time^{-1}]
γ	Transition rate from larvae to pupas	[time^{-1}]
η	Mortality rate of pupas	[time^{-1}]
μ	Natural mortality rate of moths	[time^{-1}]
a	Parasitism rate of *Trichogramma* on stem borer eggs	[time^{-1}]
b	Parasitism rate of Jatiroto flies on larvae	[time^{-1}]
c	Egg consumption rate by *Trichogramma*	[time^{-1}]
d	Parasitism rate of *Trichogramma* pupas on eggs	[time^{-1}]
δ	Maturation rate of *Trichogramma* pupas to adults	[time^{-1}]
ϕ	Mortality rate of adult *Trichogramma*	[time^{-1}]
ρ	Egg-laying rate of adult *Trichogramma*	[time^{-1}]
κ	Natural mortality rate of *Trichogramma* eggs	[time^{-1}]
ξ	Egg-laying rate of Jatiroto flies	[time^{-1}]
ζ	Transition rate from Jatiroto eggs to pupas	[time^{-1}]
e	Parasitism rate of Jatiroto pupas on larvae	[time^{-1}]
f	Parasitism success rate of Jatiroto on larvae	[time^{-1}]
θ	Mortality rate of adult Jatiroto flies	[time^{-1}]
r	Holling's functional response	Same dimension as corresponding variable
K_0	Carrying capacity of larvae (density-dependent)	[population size]
S_0	Release rate of *Trichogramma* pupas	[population size/time]
J_0	Release rate of Jatiroto adults	[population size/time]

states E, L, and P. With this bound of the SB states, the system (R,S,T,X,Y,J) is bounded by a linear differential inequality:

$$\frac{d[R,S,T,X,Y,J]^T}{dt} \leq B[R,S,T,X,Y,J]^T + [-\kappa, S_0, 0, 0, 0, J_0]^T,$$

where

$$B = \begin{bmatrix} -\kappa & 0 & \rho & 0 & 0 & 0 \\ 0 & -\delta & d & 0 & 0 & 0 \\ 0 & \delta & -\phi & 0 & 0 & 0 \\ 0 & 0 & 0 & -\zeta & 0 & e \\ 0 & 0 & 0 & \zeta & -\delta & 0 \\ 0 & 0 & 0 & 0 & \delta & -\theta \end{bmatrix}.$$

The characteristic polynomial of B is given by:

$$\mathbf{B}(\lambda) = (\lambda+\kappa) \Big(\lambda^3 + (\delta+\theta+\zeta)\lambda^2 + (\delta\,\theta + \zeta\,\delta + \theta\,\zeta)\lambda + \zeta\,\delta\,(\theta - e))(\lambda^2 + (\delta+\phi)\lambda + \delta\,(\phi - d)\Big). \tag{6}$$

All eigenvalues of $\mathbf{B}$ are negative, provided that

$$d < \phi \text{ and } e < \theta. \tag{7}$$

Further, with the stability of the linear system

$$\frac{d[R,S,T,X,Y,J]^T}{dt} = B[R,S,T,X,Y,J]^T + [-\kappa, S_0, 0, 0, 0, J_0]^T, \tag{8}$$

we conclude the boundedness of the non-autonomous system (4), provided that the biological restriction (7) is satisfied. The following reduced autonomous system gives more dynamic behavior to the system. This behavior for the non-survival condition (7) is justified for the autonomous case in the next section with the extinction of predators.

3 Dynamical Analysis

We consider the full system (normalized by K_0) under autonomous conditions, representative of long-term dynamics in a uniform sugarcane plantation, where predators and prey co-exist naturally without human intervention. For analytical and computational convenience, we rescale the dependent variables $\hat{V} = V/K_0$ so that the equations become dimensionless. In what follows, we omit the 'hat" notation (i.e., we write V instead of $\hat{V}$) for clarity and simplicity, keeping in mind that all such variables are in normalized form. We further set the normalized functional response to $r = 1$ to capture the relatively large response corresponding to a slower Holling effect.

3.1 Interaction between Stem Borer and *Trichogramma*

The interaction between stem borer and *Trichogramma* in the autonomous form is given as follows:

$$\begin{aligned}
\frac{dE}{dt} &= \alpha M - \beta E - \frac{aTE}{1+E} \\
\frac{dL}{dt} &= \beta E(1-L) - \gamma L \\
\frac{dP}{dt} &= \gamma L - \eta P \\
\frac{dM}{dt} &= \eta P - \mu M \\
\frac{dR}{dt} &= \rho T - \frac{cTE}{1+E} - \kappa R \\
\frac{dS}{dt} &= \frac{dTE}{1+E} - \delta S \\
\frac{dT}{dt} &= \delta S - \phi T.
\end{aligned} \tag{9}$$

The system (9) has three equilibria, i.e.:

$$SBTC_1 = \left\{ E = 0, L = 0, P = 0, M = 0, R = 0, S = 0, T = 0 \right\} \tag{10}$$

$$SBTC_2 = \left\{ E = \frac{\gamma(\alpha-\mu)}{\beta\mu}, L = \frac{\alpha-\mu}{\alpha}, P = \frac{\gamma(\alpha-\mu)}{\alpha\eta}, M = \frac{\gamma(\alpha-\mu)}{\alpha\mu}, \right.$$

$$
\left. R=0, S=0, T=0 \right\}
$$

$$
\begin{aligned}
SBTC_3 \quad = \quad & \left\{ E=\frac{\phi}{d-\phi}, L=\frac{\beta\,\phi}{\gamma(d-\phi)+\beta\,\phi}, P=\frac{\beta\,\phi\,\gamma}{\eta\,(\gamma(d-\phi)+\beta\,\phi)}, \right. \\
& M=\frac{\beta\,\phi\,\gamma}{\mu\,(\gamma(d-\phi)+\beta\,\phi)}, R=\frac{(d\rho-c\phi)\,\beta\,((d-\phi)\,(\alpha-\mu)\,\gamma-\beta\,\mu\,\phi)}{a\mu\kappa\,(d-\phi)\,(\beta\,\phi+\gamma\,(d-\phi))}, \\
& S=\frac{\phi\,\beta\,d\,((d-\phi)\,(\alpha-\mu)\,\gamma-\beta\,\mu\,\phi)}{a\mu\,\delta\,(d-\phi)\,(\beta\,\phi+\gamma\,(d-\phi))}, \\
& \left. T=\frac{\beta\,d\,(\alpha\,d\gamma-\alpha\,\gamma\phi-\beta\,\mu\,\phi-d\gamma\mu+\gamma\mu\,\phi)}{(\beta\,\phi+d\gamma-\phi\,\gamma)\,(d-\phi)\,a\mu} \right\}
\end{aligned}
$$

The following behavior holds:

1. the trivial equilibrium $SBTC_1$ is unstable;
2. the equilibrium $SBTC_2$ is stable provided that $\gamma\,(\alpha-\mu)\,d-\phi\,(\alpha\,\gamma+\beta\,\mu-\mu\,\gamma)<0$, otherwise $SBTC_2$ becomes unstable;
3. The equilibrium $SBTC_3$ appears as $\gamma\,(\alpha-\mu)\,d-\phi\,(\alpha\,\gamma+\beta\,\mu-\mu\,\gamma)>0$.

Hence, we have a transcritical bifurcation, as d varies in the neigborbood of $d^*=\phi\left(1+\frac{\beta\,\mu}{\gamma(\alpha-\mu)}\right)$. At the coexistence state, the equilibrium of the stem borer larva L only depends on the growth factor of the *Trichogramma* larva S. On the other hand, the equilibrium of the *Trichogramma* larva S depends on the predation factor a and the *Trichogramma* larva growth factor d. The dependence of both larval equilibria is given in Figure 2.

The *Trichogramma* larva S arises only after a successful parasitization of stem borer eggs (see Figure 2(a)) , while the stem borer larva L itself signals the level of infestation in the crop (see Figure 2(b)). Therefore, examining the ratio L/S provides insight into the relative abundance of the pest (stem borer) compared to its natural enemy (*Trichogramma*). A higher L/S indicates a greater infestation level relative to the available biological control, suggesting an insufficient parasitoid presence to suppress the pest population. Conversely, a lower L/S ratio implies that *Trichogramma* larvae are effectively controlling the stem borer population, leading to a more balanced pest management dynamic. As shown in Figure 2(b), the stem borer larva decreases (L) in coexistence as the growth rate of predation increases, whereas the corresponding *Trichogramma* larva increases.

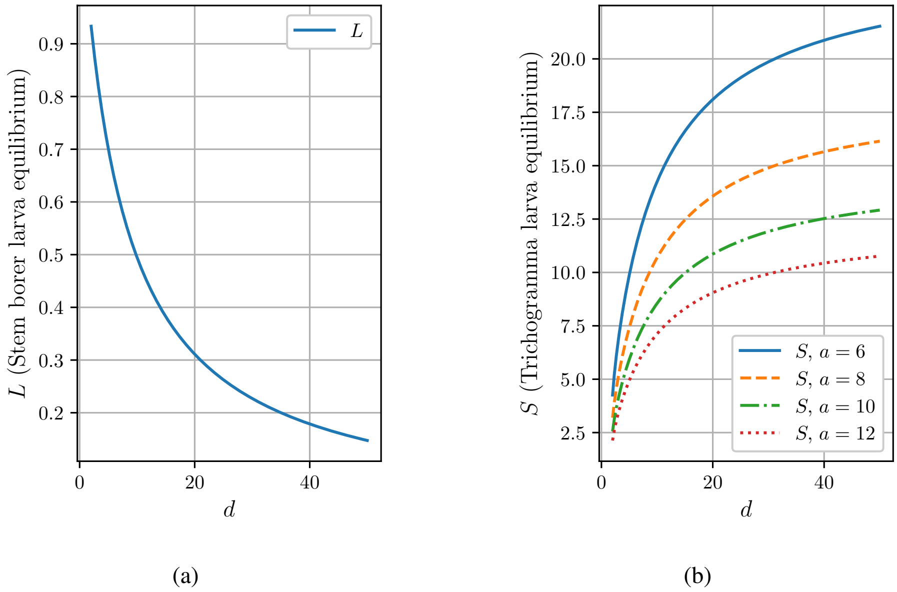


Figure 2 (a) The stem borer larvae and (b) the *Trichogramma* larvae in the coexistence state.

The ratio of the equilibria L/S is given as

$$\frac{L}{S} = \frac{a\mu\,(d-\phi)\,\delta}{d\,((d-\phi)\,(\alpha-\mu)\,\gamma-\beta\,\mu\,\phi)} \tag{11}$$

where graph of L/S is given in Figure 3. As shown in Figure 3, the ratio L/S at the end increases as a increases. This increase is contributed by decreasing the *Trichogramma* larvae in co-existence.

The stability of $SBTC_3$ is tedious; instead, we show some analysis within the range of values of biological parameters.

3.2 Interaction between Stem Borer and Jatiroto Fly

The interaction between the stem borer and the Jatiroto fly is given as follows:

$$\begin{aligned}
\frac{dE}{dt} &= \alpha M - \beta E \\
\frac{dL}{dt} &= \beta E(1-L) - \gamma L - \frac{bYL}{1+L}
\end{aligned}$$

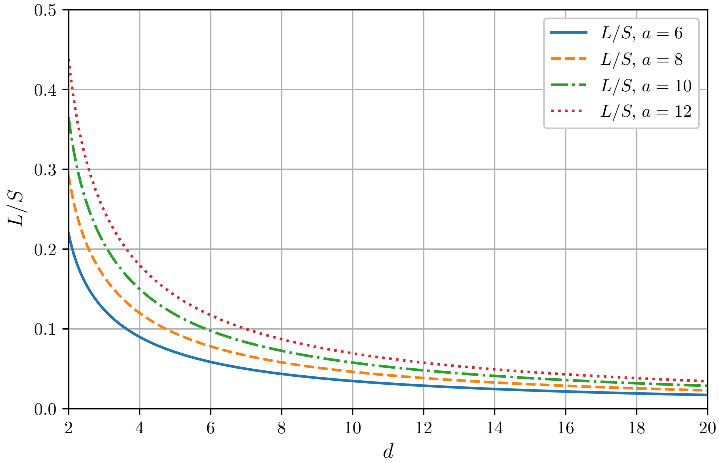


Figure 3 Ratio L/S at the coexistence equilibrium for $a = 6,\ 8,\ 10,$ and 12.

$$\begin{aligned}
\frac{dP}{dt} &= \gamma L - \eta P \\
\frac{dM}{dt} &= \eta P - \mu M \\
\frac{dX}{dt} &= \xi J - \zeta X \\
\frac{dY}{dt} &= \zeta X - \frac{eYL}{1+L} - \nu Y \\
\frac{dJ}{dt} &= \frac{fYL}{1+L} - \theta J.
\end{aligned} \tag{12}$$

The following three equilibria of (12) are:

$$\begin{aligned}
SBJT_1 &= \{E = 0, L = 0, P = 0, M = 0, X = 0, Y = 0, J = 0\} \\
SBJT_2 &= \left\{ E = \frac{\gamma(\alpha-\mu)}{\beta\mu}, L = \frac{\alpha-\mu}{\alpha}, P = \frac{\gamma(\alpha-\mu)}{\alpha\eta}, M = \frac{\gamma(\alpha-\mu)}{\alpha\mu}, \right. \\
&\qquad \left. X = 0, Y = 0, J = 0 \right\} \\
SBJT_3 &= \left\{ E = \frac{\alpha\gamma\nu\theta}{\beta\mu\mathscr{D}_1}, L = \frac{\nu\theta}{\mathscr{D}_1}, M = \frac{\gamma\nu\theta}{\mu\mathscr{D}_1}, P = \frac{\gamma\nu\theta}{\eta\mathscr{D}_1}, X = \frac{f\gamma\nu\xi\mathscr{D}_2}{b\mu\zeta\mathscr{D}_1^2}, \right. \\
&\qquad \left. Y = \frac{\gamma(f\xi - e\theta)\mathscr{D}_2}{b\mu\zeta\mathscr{D}_1^2}, J = \frac{f\nu\gamma\mathscr{D}_2}{b\mu\zeta\mathscr{D}_1^2} \right\},
\end{aligned} \tag{13}$$

where

$$\begin{aligned}\mathscr{D}_1 &= f\xi - \theta(\nu + e) \\ \mathscr{D}_2 &= (\alpha - \mu)\mathscr{D}_1 - \alpha\nu\theta.\end{aligned} \tag{14}$$

The equilibrium $SBJT_3$ exists when

$$\mathscr{D}_2 > 0. \tag{15}$$

Similar to the previous case, the trivial equilibrium $SBJT_1$ is always unstable and a transcritical bifurcation occurs at $SBJT_2$ as the parameter f passes the bifurcation point:

$$f^* = \frac{\theta(e+\nu)}{\xi} + \frac{\alpha\theta\nu}{\xi(\alpha - \mu)}. \tag{16}$$

The eigenvalues of equilibrium point $SBJT_3$ can be tracked numerically as follows:

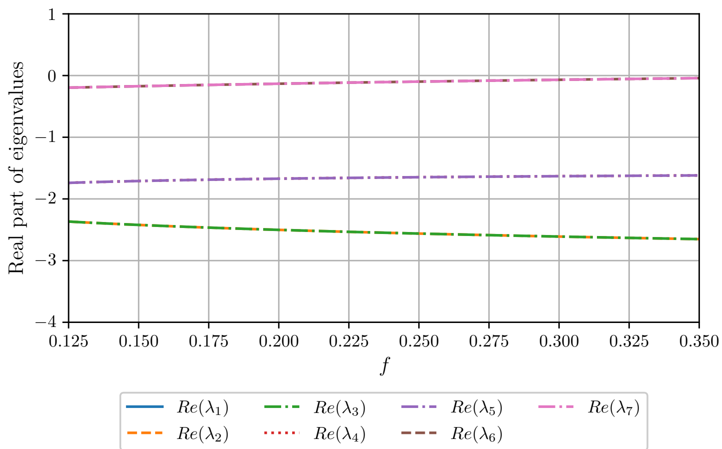
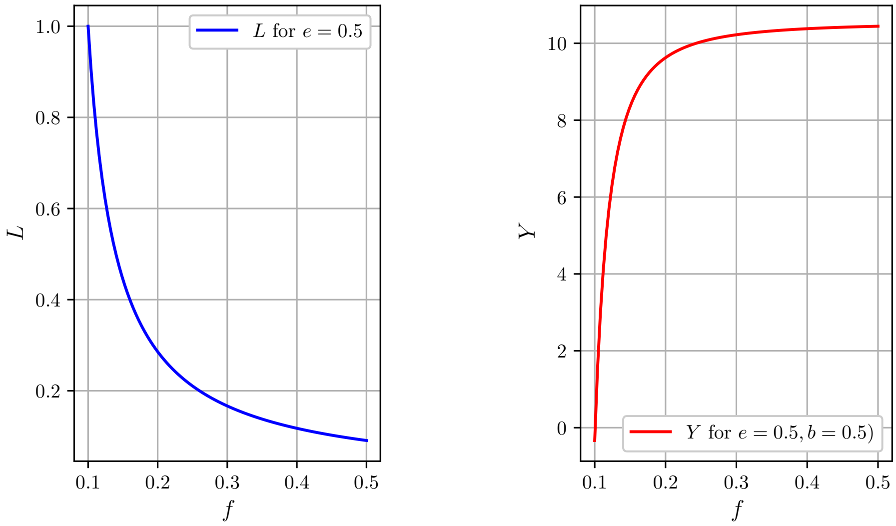


Figure 4 Numerical tracking of eigenvalues for equilibrium point $SBJT_3$.

The dynamic of system (12) is more complex than the dynamic of system (9). At the end of the coexistence, the stem borer larva Y depends on the survival factor of the Jatiroto larva, as shown in Figure 5*a*. On the other hand, the Jatiroto larva grows fast with an increase in its survival growth, see Figure 5*b*.



(a) (b)

Figure 5 (a) The larvae of stem borer and (b) the larvae of Jatiroto at the coexistence state.

The ratio of coexistence L/Y is given as:

$$\frac{L}{Y} = \frac{b\nu\theta\mu\zeta\mathscr{D}_1}{\gamma(f\xi - e\theta)\mathscr{D}_2}. \tag{17}$$

The ratio (17) shows a typical predator-prey behavior with oscilatory dynamics.

3.3 Interaction between Stem Borer and *Trichogramma* and Jatiroto Fly

The full autonomous system, where *Trichogramma* and the Jatiroto fly as predators simultaneously predate the stem borer, is given below

$$\begin{aligned}
\frac{dE}{dt} &= \alpha M - \beta E - \frac{aTE}{1+E} \\
\frac{dL}{dt} &= \beta E(1-L) - \gamma L - \frac{bYL}{1+L} \\
\frac{dP}{dt} &= \gamma L - \eta P \\
\frac{dM}{dt} &= -M\mu + P\eta
\end{aligned}$$

$$
\begin{aligned}
\frac{dR}{dt} &= \rho T - \frac{cTE}{1+E} - \kappa R \\
\frac{dS}{dt} &= \frac{dTE}{1+E} - S\delta \\
\frac{dT}{dt} &= \delta S - \phi T \\
\frac{dX}{dt} &= \xi J - \zeta X \\
\frac{dY}{dt} &= \zeta X - \frac{eYL}{1+L} - \nu Y \\
\frac{dJ}{dt} &= \frac{fYL}{1+L} - \theta J.
\end{aligned} \tag{18}
$$

The system (19) has five equilibria:

$$
\begin{aligned}
SBTJ_1 &= \{E=0, J=0, L=0, M=0, P=0, R=0, S=0, T=0, X=0, Y=0\} \\
SBTJ_2 &= \left\{ E = \frac{\gamma(\alpha-\mu)}{\beta\mu}, L = \frac{\alpha-\mu}{\alpha}, P = \frac{\gamma(\alpha-\mu)}{\alpha\eta}, M = \frac{\gamma(\alpha-\mu)}{\alpha\mu}, \right. \\
&\qquad \left. R=0, S=0, T=0, X=0, Y=0, J=0 \right\} \\
SBTJ_3 &= \left\{ E = \frac{\alpha\gamma\nu\theta}{\beta\mu D_1}, L = \frac{\nu\theta}{\mathscr{D}_1}, P = \frac{\gamma\nu\theta}{\eta\mathscr{D}_1}, M = \frac{\gamma\nu\theta}{\mu\mathscr{D}_1}, R=0, S=0, \right. \\
&\qquad \left. T=0, X = \frac{\gamma\xi\mathscr{D}_2\mathscr{D}_3}{b\mu\xi\mathscr{D}_1^2}, Y = \frac{\gamma\mathscr{D}_2\mathscr{D}_3^2}{bf\mu\nu\mathscr{D}_1^2}, J = \frac{\mathscr{D}_2\mathscr{D}_3}{b\mu\mathscr{D}_1^2} \right\} \\
SBTJ_4 &= \left\{ E = \frac{\phi}{d-\phi}, L = \frac{\beta\phi}{\mathscr{D}_4}, P = \frac{\beta\gamma\phi}{\eta\mathscr{D}_4}, M = \frac{\beta\gamma\phi}{\mu\mathscr{D}_4}, R = \frac{\beta(d\rho - c\phi)\mathscr{D}_5}{a\kappa\mu(d-\phi)\mathscr{D}_4}, \right. \\
&\qquad S = \frac{d\beta\phi\mathscr{D}_5}{a\mu\delta(d-\mu)\mathscr{D}_4}, T = \frac{d\beta\mathscr{D}_5}{a\mu(d-\phi)\mathscr{D}_4}, \\
&\qquad \left. X=0, Y=0, J=0 \right\} \\
SBTJ_5 &= \left\{ E = \frac{\phi}{d-\phi}, L = \frac{\nu\theta}{\mathscr{D}_1}, P = \frac{\gamma\nu\theta}{\eta\mathscr{D}_1}, M = \frac{\gamma\nu\theta}{\mu\mathscr{D}_1}, R = \frac{(c\phi - d\rho)\mathscr{D}_6}{a\kappa\mu\phi(d-\phi)\mathscr{D}_1}, \right. \\
&\qquad S = \frac{d\mathscr{D}_6}{b\zeta\theta\delta(d-\phi)\mathscr{D}_1}, T = \frac{d\mathscr{D}_6}{a\mu\phi(d-\phi)\mathscr{D}_1}, \\
&\qquad \left. X = \frac{\xi(f\xi - e\theta)\mathscr{D}_7}{b\zeta\theta\nu(d-\phi)\mathscr{D}_1}, Y = \frac{(f\xi - e\theta)^2\mathscr{D}_7}{bf\theta\nu^2(d-\phi)\mathscr{D}_1}, J = \frac{d\mathscr{D}_7}{b\theta\nu\phi(d-\phi)\mathscr{D}_1} \right\}.
\end{aligned} \tag{19}
$$

where

$$
\begin{aligned}
\mathscr{D}_3 &= f\xi - e\theta \\
\mathscr{D}_4 &= \beta\phi + \gamma(d-\phi) \\
\mathscr{D}_5 &= \gamma(\alpha-\mu)(d-\phi) - \beta\phi\mu \\
\mathscr{D}_6 &= \gamma\alpha\theta\nu(d-\phi) - \beta\,\mu\,\phi\,\mathscr{D}_1 \\
\mathscr{D}_7 &= \beta\phi\mathscr{D}_8 - \gamma\nu\theta(d-\phi) \\
\mathscr{D}_8 &= f\xi - \theta(e+2\nu).
\end{aligned}
\tag{20}
$$

The two predators, *Trichogramma* and Jatiroto flies, contribute to the more complex dynamics. At the end of the coexistence, the stem borer larva is not affected by the Trichogramma. Instead, the *Trichogramma* and Jatiroto larvae are affected by the predation factors of both predators. In addition to the survival condition (20), we assume that $\mathscr{D}_2 > 0$, which implies $\mathscr{D}_1 > 0$ and $\mathscr{D}_3 > 0$.

4 Simulation

Trichogramma is periodically released in the form of parasitized egg cards attached to the sugarcane leaves. Each card contains a certain number of eggs and immediately produces *Trichogramma* imagos. We assume that the release is already in the form of adult *Trichogramma*. In the following simulations, we use 6 to 10 *Trichogramma* per week per sugarcane tree, which is equivalent to 24 to 40 egg cards per hectare per month.

Adult Jatiroto parasites are released and lay their eggs inside the borer holes next to the stem borer larvae. In the model, we formulate a direct interaction between the adult Jatiroto and the stem borer larva.

4.1 *Trichogramma* vs Stem Borer

In the following simulation, *Trichogramma* (*Jatirotos*) are released at a weekly rate of 6, 7, 8, and 10 per time unit and the number of stem borer larvae, expressed as the average larvae per tree, is converted into a percentage of infestation. As shown in Figure 6, lines labeled If $= 5\%, 10\%, 15\%$ represent critical infestation thresholds, indicating the pest proportion at which further intervention measures become necessary. The results suggest that releasing 10 Jatirotos per week is sufficient to reduce the infestation level to 5% by the end of the 50-week planting period. Figure 6 shows that the release of 10 Jatiroto eggs per week achieve 5%

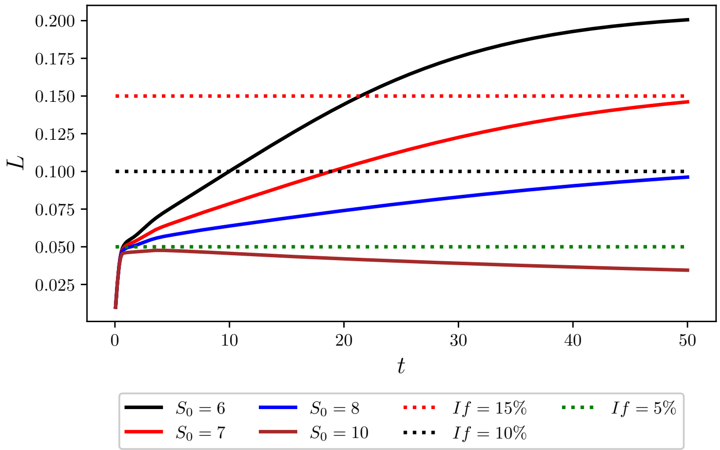


Figure 6 Numerical simulation showing the continuous release of 6 to 10 *Trichogramma* eggs to target stem borer eggs over time. The lines labeled If = 5%, 10%, 15% denote **infestation thresholds**, indicating the pest proportion at which intervention measures become critical. The model parameters used are: $\alpha = 150$, $\beta = \frac{7}{6}$, $\gamma = \frac{7}{36}$, $\eta = \frac{7}{9}$, $\mu = \frac{7}{3}$, $\delta = \frac{7}{10}$, $\kappa = \frac{7}{5}$, $\rho = 10$, $\phi = \frac{7}{5}$, $a = 10$, $c = 12$, $d = 10$.

infestation at the end of the 50-week planting period.

4.2 *Trichogramma* vs Jatiroto Flies

Adult Jatiroto flies are released weekly and produce eggs. Soon after the eggs hatch, we have predator-prey interaction between *Trichogramma* larvae and stem borer larvae. This type of predation creates an oscillatory behavior in each orbit. Figures 7 and 8 represent the release of 0, 0.5, and 1 Jatiroto flies per week. The simulation in Figure 7 shows the oscillator dynamics of the stem borer larvae. The increase in Jatiroto release affects the oscillation damping.

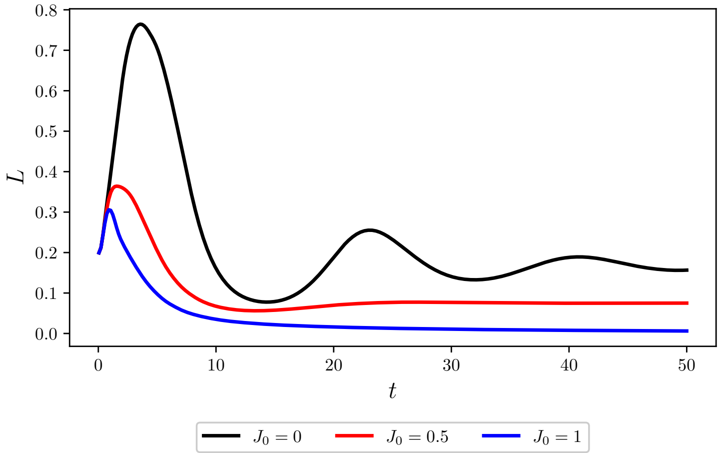


Figure 7 Numerical simulation showing the continuous release of $0, 0.5, 1$ Jatiroto flies to target stem borer larvae over time. The model parameters used are: $\alpha = 150.0, b = 0.5, \beta = \frac{7}{6}, \delta = \frac{7}{10}, e = 0.5, \eta = \frac{7}{9}, f = 0.3, \gamma = \frac{7}{36}, \mu = \frac{7}{3}, \nu = 1, \phi = \frac{7}{5}, \theta = 1, \xi = 25, \zeta = \frac{7}{3}$

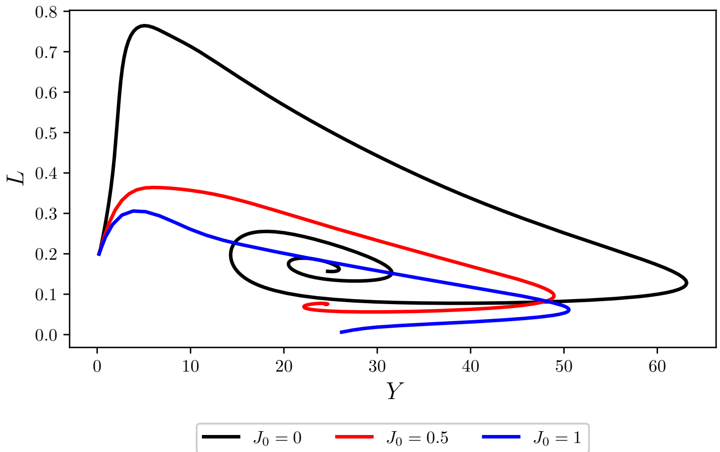


Figure 8 Numerical simulation showing the continuous release of 6 to 10 Jatiroto flies to target stem borer larvae over time. The model parameters used are: $\alpha = 150.0, b = 0.5, \beta = \frac{7}{6}, \delta = \frac{7}{10}, e = 0.5, \eta = \frac{7}{9}, f = 0.3, \gamma = \frac{7}{36}, \mu = \frac{7}{3}, \nu = 1, \phi = \frac{7}{5}, \theta = 1, \xi = 25, \zeta = \frac{7}{3}$

The simulation in Figure 8 shows the interaction of the orbit $L-Y$, where a high number of stem borer larvae corresponds to a low number of Jatiroto larvae, and vice versa. This dynamic occurs due to the direct predator-prey interaction between the Jatiroto larvae and the stem borer larvae. Understanding the interaction behavior is important in management control and selecting the proper time to release the predators.

4.3 Full System

Pest control using more than one predator is commonly done in sugarcane plantations. The combined release of *Trichogramma* and Jatiroto flies is shown here. Each predator attacks a different target and is expected to reduce the stem borer population more effectively. A combination of release rates is shown to give a better result in reducing the infestation.

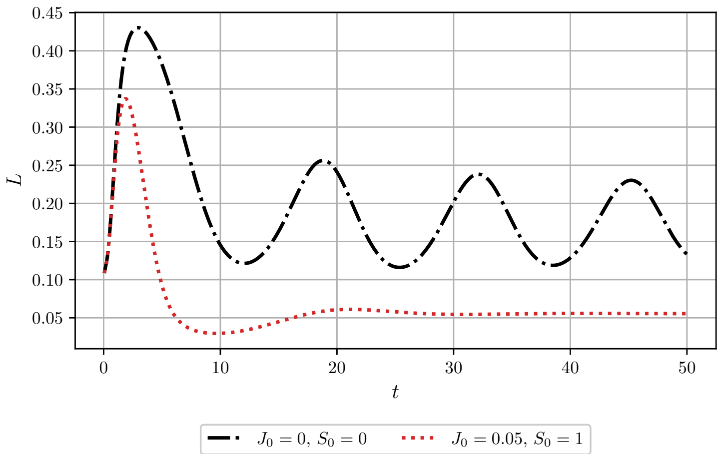
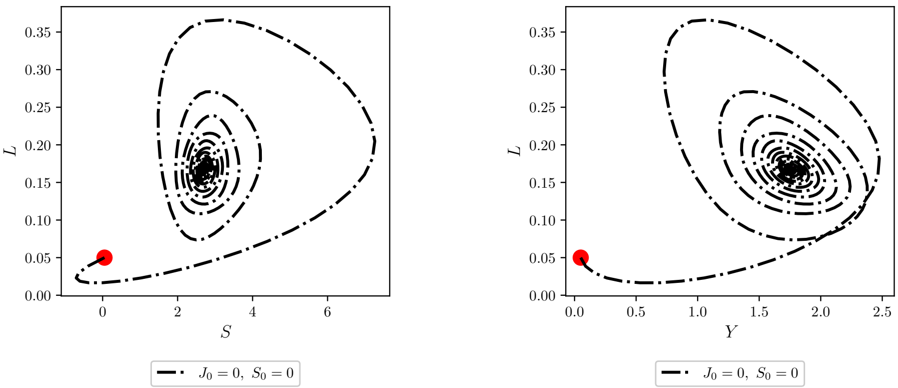


Figure 9 Numerical simulation showing the continuous release of *Trichogramma* and Jatiroto flies to target stem borer eggs and larvae over time. The model parameters used are: $\alpha = 150.0, b = 0.5, \beta = \frac{7}{6}, \delta = \frac{7}{10}, e = 0.5, \eta = \frac{7}{9}, f = 0.3, \gamma = \frac{7}{36}, \mu = \frac{7}{3}, \nu = 1, \phi = \frac{7}{5}, \theta = 1, \xi = 25, \zeta = \frac{7}{3}$

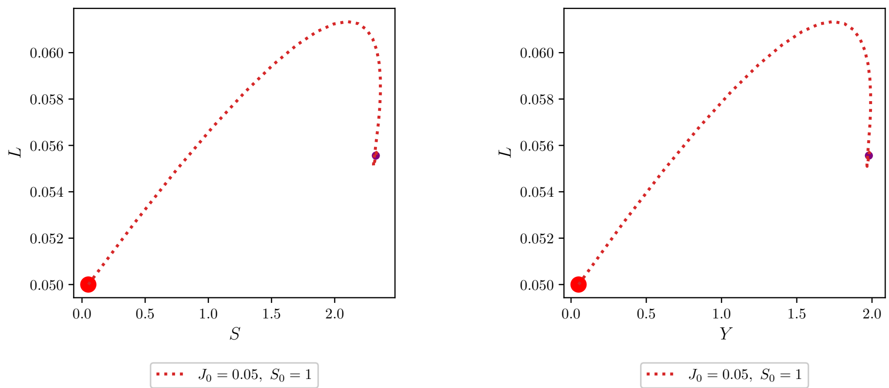
Figure 9 shows that a combination of 1 *Trichogramma* and 0.05 Jatiroto releases is already able to affect 5% infestation and also dampen the dynamic of the stem borer larva.

Fast dynamic of stem borer larva in Figure 9 with the corresponding fast dynamic of *Trichogramma* and Jariroto larvae as shown in Figure 10.



(a) (b)

Figure 10 *Trichogramma* larva vs stem borer larva (left), and Jatiroto larva vs stem borer larva (right) in the coexistence state. The model parameters used are: $\alpha = 150.0, b = 0.5, \beta = \frac{7}{6}, \delta = \frac{7}{10}, e = 0.5, \eta = \frac{7}{9}, f = 0.3, \gamma = \frac{7}{36}, \mu = \frac{7}{3}, \nu = 1, \phi = \frac{7}{5}, \theta = 1, \xi = 25, \zeta = \frac{7}{3}$



q

(a) (b)

Figure 11 *Trichogramma* larva vs stem borer larva (left), and Jatiroto larva vs stem borer larva (right) at the coexistence state. The model parameters used are: $\alpha = 150.0, b = 0.5, \beta = \frac{7}{6}, \delta = \frac{7}{10}, e = 0.5, \eta = \frac{7}{9}, f = 0.3, \gamma = \frac{7}{36}, \mu = \frac{7}{3}, \nu = 1, \phi = \frac{7}{5}, \theta = 1, \xi = 25, \zeta = \frac{7}{3}$

Figure 11 shows the effect of the combination of *Trichogramma* and Jatiroto releases.

With the relatively smaller composition of both rates, we obtain a significant reduction in infestation and damping of predator-prey dynamics.

5 Conclusion

This paper presents a mathematical model representing pest control in sugarcane plantations. The interaction between the stem borer and its biological predators, *Trichogramma* and Jatiroto flies, was formulated in a dimensional system of ordinary differential equations with constant releases of both predators. A carrying capacity is given in the dynamic of sugarcane larvae, representing the sugarcane tree's growth as the borer's logistic resource during the planting period.

Dynamical analysis was performed for the autonomous case by freezing the carrying capacity and with zero release of the predators. This analysis was shown to explain the ultimate behavior of the interaction between prey and predators in the field before the release mechanism takes place. In the interaction between stem borer and *Trichogramma*, the adult *Trichogramma* inject their eggs directly into the host stem borer eggs. A transcritical bifurcation occurs, along with the coexistence of both predator and prey, as the growth factor of the *Trichogramma* larva passes a bifurcation point. As expected, the stem borer larvae decrease in number, which implies a decrease in infestation as the growth factor of *Trichogramma* larva increases. On the other hand, the *Trichogramma* larvae depend on the predation growth of *Thichogramma* and the growth rate of the *Trichogramma* larvae.

The stem borer larvae and Jatiroto larvae interact and carry a typical oscillatory predator-prey behavior. At the end of coexistence, the stem borer larvae are affected by predation growth and the adult growth rate of the Jatiroto flies.

In the two-predators-prey model, we identified the complexity of the dynamic. In the coexistence state, the stem borer larvae remain the same as in the interaction between the stem borers and the Jatiroto flies. However, the *Trichogramma* and Jatiroto larvae are affected by all non-linear parameters.

Simulations representing the field of pest control were shown. With weekly rates of 6, 7, 8, and 10 *Trichogramma* releases, we showed the larvae level per tree, representing the percentage of infestation level. With 10 *Trichogramma* releases per tree, the simulation showed a reduction of infestation below 5%.

In the case of the release of Jatiroto, in which the prey and predator larvae interact, oscillatory dynamics showed fast and slow dynamics in their cycle. Starting with high oscillation in their orbits before the release of Jatiroto, as the release increased, we saw a dampening of the oscillation and a reduction of the infestation at the end of the planting period. The full system combining the two predators showed a more effective control with fewer *Trichogramma* and Jatiroto releases. Using a relatively low composition of release rates for both predators, yielded a significant infestation reduction with dampening dynamics.

These results and discussion are expected to give insight into the behavior of the complicated phenomenon of pest control in the field, especially in the selection and scheduling of the release rates of biological predators.

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