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An Efficient General Family of Estimators for Population Mean in the Presence of Non-Response

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Abstract. This paper presents a new general family of estimators to estimate the population mean of study variable y in the presence of non-response when utilizing a known coefficient of variation of study variable y. The expressions for bias, mean squared error (MSE), and minimum mean squared error (MMSE) of the proposed family of estimators are derived up to the first degree of approximation. In addition, a numerical example and a simulation study are presented to explain the performance of the proposed estimators. It was shown that the proposed estimators perform better compared to all other relevant estimators.

Keywords: *auxiliary variables; bias; mean squared error; ratio estimator; study variables.*

1 Introduction

In most sample surveys it is difficult to collect complete information from all the units selected in the sample due to the occurrence of non-response. Non-response occurs when a respondent or a person selected for the survey does not participate in the survey or participates but does not provide complete information. An estimate obtained from incomplete data can lead to inaccurate results. To reduce the effect of non-response, Hansen and Hurwitz [1] first suggested the technique of sub-sampling to estimate the population mean by combining the information available from response and non-response groups.

Let (Y_i, X_i) be the non-negative values for the i^{th} unit of the population $U = (U_1, U_2, \dots, U_N)$ on study variable y and auxiliary variable x with their population means $(\bar{Y}, \bar{X})$. Suppose that the population U of size N is divided into N_1 responding units and $N_2 = N - N_1$ non-responding units. Assume that the non-response is observed only on study variable y, while auxiliary variable x is free from non-response. A sample of size n is drawn from the population of size N by using simple random sampling without replacement (SRSWOR), which observes that there are n_1 responding units and n_2 non-responding units. Further, a sub-sample of size $r = \frac{n_2}{k}; k > 1$ is selected from the n_2 non-

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responding units by SRSWOR, where k is the inverse sampling rate and the information on r units is collected by personal interviews for study variable y. Therefore, the estimator for $\bar{Y}$ based on $n_1 + r$ proposed by Hansen and Hurwitz [1] is defined as:

$$\bar{y}^* = (n_1 / n)\bar{y}_1 + (n_2 / n)\bar{y}_{2r} \tag{1}$$

where $\bar{y}_1 = \sum_{i=1}^{n_1} y_i / n_1$ and $\bar{y}_{2r} = \sum_{i=1}^{r} y_i / r$ are the means of the n_1 responding units and the sub-sampled units respectively. Estimator (1) is unbiased with variance:

$$V(\bar{y}^*) = \varphi S_y^2 + \varphi^* S_{y(2)}^2 \tag{2}$$

where $\varphi = (N-n)/Nn$, $\varphi^* = W_2(k-1)/n$, $W_2 = N_2 / N$,

$S_y^2 = \sum_{i=1}^{N} (y_i - \bar{Y})^2 / (N-1)$, $S_{y(2)}^2 = \sum_{i=1}^{N_2} (y_i - \bar{Y}_2)^2 / (N_2 - 1)$.

For improving the sample survey precision, estimating population mean $\bar{Y}$ can be increased by utilizing information on auxiliary variable x, which is correlated with y, whose population mean $\bar{X}$ is known. For example, the use of the coefficient of variation of study variable y in proposing the estimator for population mean $\bar{Y}$ has been conducted by Searls [2]. Motivated by Searls' [2] work, Das and Tripathi [3] used the coefficient of variation of auxiliary variable x to improve the class of estimators for population mean $\bar{Y}$, which were more efficient than several estimators proposed at that time. Later, Khoshnevisan, *et al*. [4] noticed that several authors had developed estimators to estimate population mean $\bar{Y}$ using known auxiliary variables. Therefore, they proposed a family of estimators to estimate population mean $\bar{Y}$ after substitution of a number of parameters in this family. This was reduced to a number of ratio and product estimators, initially proposed by several authors as mentioned before, as follows:

$$\hat{T} = \bar{y}\left(\frac{a\bar{X} + b}{\alpha(a\bar{x} + b) + (1-\alpha)(a\bar{X} + b)}\right)^g, \tag{3}$$

where $\bar{y}$ and $\bar{x}$ are the sample means of the study and the auxiliary variables, respectively. Further, let $a(a \neq 0)$ and b refer to either real numbers or functions of the known parameters of auxiliary variable x, i.e. the standard deviation (σ_x), coefficient of variation (C_x), skewness $(\beta_1(x))$, kurtosis $(\beta_2(x))$ and correlation coefficient (ρ_{yx}) of the population. Here, α and g are suitably chosen scalars such that the mean squared error of $\hat{T}$ is minimum. Therefore, the bias and mean squared error (MSE) of this family of estimators are as follows:

$$Bias(\hat{T}) = \varphi\bar{Y}\left[\frac{g(g+1)}{2}\alpha^2\lambda^2C_x^2 - \alpha\lambda gC_{yx}\right], \quad (4)$$

$$MSE(\hat{T}) = \varphi\bar{Y}^2\left[C_y^2 + \alpha^2\lambda^2g^2C_x^2 - 2\alpha\lambda gC_{yx}\right], \quad (5)$$

where $\lambda = a\bar{X}/(a\bar{X}+b)$, $C_y^2 = S_y^2/\bar{Y}^2$, $C_x^2 = S_x^2/\bar{X}^2$, $C_{yx} = S_{yx}/\bar{Y}\bar{X}$, $S_x^2 = \sum_{i=1}^{N}(x_i-\bar{X})^2/(N-1)$, $S_{y(2)}^2 = \sum_{i=1}^{N_2}(y_i-\bar{Y}_2)^2/(N_2-1)$, $\rho_{yx} = S_{yx}/S_yS_x$,

$$S_{yx} = \sum_{i=1}^{N}(y_i-\bar{Y})(x_i-\bar{X})/(N-1).$$

In the case of incomplete information on study variable y, Cochran [5] extended the technique of Hansen and Hurwitz [1] by proposing ratio and regression estimators to estimate population mean $\bar{Y}$, while information on the auxiliary variable is obtained from all the sample units and the population mean $\bar{X}$ of auxiliary variable x is known. Extending the technique of Hansen and Hurwitz [1] and Searls' [2] work, Khare and Kumar [6] proposed estimators utilizing the coefficient of variation of study variable y in the estimation of population mean $\bar{Y}$ using the benefits of auxiliary variable x in the presence of non-response. Motivated by Khare and Kumar's [6] work, Khare and Rehman [7] used a coefficient of variation of study variable y along with the population mean of auxiliary variable x to improve the efficiency of ratio in regression type estimators in the presence of non-response by proposing the following estimator:

$$\bar{y}_{KR}^* = \gamma\bar{y}^* \quad (6)$$

where $\gamma = [1+(C_y^2/n)[1+(n_2/n)(k-1)]]^{-1}$ is an optimum constant, which makes the MSE of $\bar{y}_{KR}^*$ become the minimum. The bias and MSE of this estimator are respectively:

$$Bias(\bar{y}_{KR}^*) = -A_2\bar{Y}, \quad (7)$$

$$MSE(\bar{y}_{KR}^*) = (1-A_1)\frac{S_y^2}{n} + (1-2A_2)\varphi^*S_{y(2)}^2, \quad (8)$$

where $A_1 = [C_y^2/n][1-n^2\varphi^{*2}]$, $A_2 = [C_y^2/n][1+n\varphi^*]$.

In the present study, a general family of estimators is proposed to estimate the population mean in the presence of non-response by adapting the estimator of

Khoshnevisan, *et al.* [4] using the idea of Khare and Rehman [7], when the value of only a few parameters of auxiliary variable x are known. The expressions for the bias and MSE, including the MMSE of the proposed estimators, are obtained. The performance of the proposed estimator is assessed with that of the existing estimators through a theoretical numerical example and a simulation study.

2 Proposed Family Estimators

Consider that the population mean $\bar{X}$ of auxiliary variable x is known in advance. Motivated by Khoshnevisan, *et al.* [4] and Khare and Rehman [7], a new general family of population mean estimators useful in the presence of non-response is proposed. It is suggested to replace $\bar{y}$ in Eq. (3) from the Khoshnevisan, *et al.* [4] estimator with $\bar{y}^*_{KR}$ in Eq. (6) from the Khare and Rehman [7] estimator to produce the following formula:

$$T = \bar{y}^*_{KR}\left(\frac{a\bar{X}+b}{\alpha(a\bar{x}+b)+(1-\alpha)(a\bar{X}+b)}\right)^g, \tag{9}$$

A few members of the proposed general family of estimators can be generated by replacing the different values of the constants $a(a \neq 0)$, b, α and g into Eq. (9), for which the details are listed in Table 1.

Table 1 Some members of the proposed family of estimators T.

Ratio estimator ($g = 1$)	**Product estimator** ($g = -1$)	**Values of constants**		
		a	b	α
$T_{R1} = \bar{y}^*_{KR}\left[\bar{X}/\bar{x}\right]$	$T_{P1} = \bar{y}^*_{KR}\left[\bar{x}/\bar{X}\right]$	1	0	1
$T_{R2} = \bar{y}^*_{KR}\left[(\bar{X}+\sigma_x)/(\bar{x}+\sigma_x)\right]$	$T_{P2} = \bar{y}^*_{KR}\left[(\bar{x}+\sigma_x)/(\bar{X}+\sigma_x)\right]$	1	σ_x	1
$T_{R3} = \bar{y}^*_{KR}\left[(\bar{X}+C_x)/(\bar{x}+C_x)\right]$	$T_{P3} = \bar{y}^*_{KR}\left[(\bar{x}+C_x)/(\bar{X}+C_x)\right]$	1	C_x	1
$T_{R4} = \bar{y}^*_{KR}\left[(\bar{X}+\beta_1(x))/(\bar{x}+\beta_1(x))\right]$	$T_{P4} = \bar{y}^*_{KR}\left[(\bar{x}+\beta_1(x))/(\bar{X}+\beta_1(x))\right]$	1	$\beta_1(x)$	1
$T_{R5} = \bar{y}^*_{KR}\left[(\bar{X}+\beta_2(x))/(\bar{x}+\beta_2(x))\right]$	$T_{P5} = \bar{y}^*_{KR}\left[(\bar{x}+\beta_2(x))/(\bar{X}+\beta_2(x))\right]$	1	$\beta_2(x)$	1

In order to study the large sample properties of the proposed estimator, the following notations are used: $e^*_0 = (\bar{y}^* - \bar{Y})/\bar{Y}$ and $e_1 = (\bar{x} - \bar{X})/\bar{X}$, such that $E(e^*_0) = E(e_1) = 0$, and $E(e^{*2}_0) = \varphi C^2_y + \varphi^* C^2_{y(2)}$, $E(e^2_1) = \varphi C^2_x$, $E(e^*_0, e_1) = \varphi C_{yx}$.

Rewriting Eq. (9) in terms of e's as mentioned above, produces the following equation:

$$T = \gamma\bar{Y}(1+e_0^*)(1+\alpha\lambda e_1)^{-g} \tag{10}$$

Expanding the right hand side of Eq. (10), multiplying out and neglecting terms involving powers of e greater than two, results in the following equation:

$$T \cong \gamma\bar{Y}(1+e_0^* - g\alpha\lambda e_1 + \frac{g(g+1)}{2}\alpha^2\lambda^2 e_1^2 - g\alpha\lambda e_0^* e_1) \tag{11}$$

Subtracting $\bar{Y}$ from both sides of Eq. (11), this equation becomes:

$$T - \bar{Y} \cong \gamma\bar{Y}(1+e_0^* - g\alpha\lambda e_1 + \frac{g(g+1)}{2}\alpha^2\lambda^2 e_1^2 - g\alpha\lambda e_0^* e_1) - \bar{Y} \tag{12}$$

Taking expectation of both sides of Eq. (12), one gains the bias of estimator T to the first degree approximation, as:

$$Bias(T) = (1-A_2)\bar{Y}\left[\varphi\left\{g\frac{(g+1)}{2}\alpha^2\lambda^2 C_x^2 - g\alpha\lambda C_{yx}\right\}\right] - \bar{Y}A_2 \tag{13}$$

Taking expectation of both sides of Eq. (12), one gains the MSE of estimator T to the first degree approximation, as:

$$MSE(T) = (1-2A_2)\bar{Y}^2\left[\varphi\left\{C_y^2 + g^2\alpha^2\lambda^2 C_x^2 - 2g\alpha\lambda C_{yx}\right\} + \varphi^* C_{y(2)}^2\right] \tag{14}$$

Eq. (14) depends on three unknown constants, g, α and λ. Keep the values of g and λ fixed. To obtain the value of α that minimizes the MSE of T, one takes the partial derivative of Eq. (14) with respect to α and equates it to zero as follows:

$$\frac{\partial MSE(T)}{\partial\alpha} = (1-2A_2)\bar{Y}^2\varphi(2g^2\alpha\lambda^2 C_x^2 - 2g\lambda C_{yx}) = 0$$

$$\alpha = \frac{2g\lambda C_{yx}}{2g^2\lambda^2 C_x^2}$$

$$= \frac{\rho_{yx} C_y C_x}{g\lambda C_x^2}$$

$$= \frac{\rho_{yx} C_y}{g\lambda C_x}$$

$$\alpha = \frac{\theta}{g\lambda} = \alpha_{opt.} \tag{15}$$

where $\theta = \rho_{yx} C_y / C_x$.

To replace the optimum value of α from Eq. (15) into Eq. (14), the minimum MSE of T is as follows:

$$MSE_{\min}(T) = (1-2A_2)\bar{Y}^2\left[\varphi C_y^2(1-\rho_{yx}^2) + \varphi^* C_{y(2)}^2\right] \tag{16}$$

In addition, it is observed that the expression of the first degree approximation of bias and MSE of the given member of the family can be obtained by merely substituting the values of constants $a(a \neq 0)$, b, α and g in Eq. (13) and Eq. (14) respectively. Therefore, for the ratio estimators as given in Table 1, one can express the bias and MSE for these estimators by the following equations:

$$Bias(T_{Ri}) = \begin{cases} (1-A_2)\bar{Y}\left[\varphi(C_x^2 - C_{yx})\right] - \bar{Y}A_2 & ; i = 1 \\ (1-A_2)\bar{Y}\left[\varphi(\lambda_{(i-1)}^2 C_x^2 - \lambda_{(i-1)} C_{yx})\right] - \bar{Y}A_2 & ; i = 2,\dots,5 \end{cases} \tag{17}$$

$$MSE(T_{Ri}) = \begin{cases} (1-2A_2)\bar{Y}^2\left[\varphi(C_y^2 + C_x^2 - 2C_{yx}) + \varphi^* C_{y(2)}^2\right] & ; i = 1 \\ (1-2A_2)\bar{Y}^2\left[\varphi(C_y^2 + \lambda_{(i-1)}^2 C_x^2 - 2\lambda_{(i-1)} C_{yx}) + +\varphi^* C_{y(2)}^2\right] & ; i = 2,\dots,5 \end{cases} \tag{18}$$

For the product estimators, the bias and MSE are given by the following equations:

$$Bias(T_{Pi}) = \begin{cases} (1-A_2)\bar{Y}\left[\varphi C_{yx}\right] - \bar{Y}A_2 & ; i = 1 \\ (1-A_2)\bar{Y}\left[\varphi \lambda_{(i-1)} C_{yx}\right] - \bar{Y}A_2 & ; i = 2,\dots,5 \end{cases} \tag{19}$$

$$MSE(T_{Pi}) = \begin{cases} (1-2A_2)\bar{Y}^2\left[\varphi(C_y^2 + C_x^2 + 2C_{yx}) + \varphi^* C_{y(2)}^2\right] & ; i = 1 \\ (1-2A_2)\bar{Y}^2\left[\varphi(C_y^2 + \lambda_{(i-1)}^2 C_x^2 + 2\lambda_{(i-1)} C_{yx}) + \varphi^* C_{y(2)}^2\right] & ; i = 2,\dots,5 \end{cases} \tag{20}$$

where $\lambda_1 = \bar{X}/(\bar{X} + \sigma_x)$, $\lambda_2 = \bar{X}/(\bar{X} + C_x)$, $\lambda_3 = \bar{X}/(\bar{X} + \beta_1(x))$, $\lambda_4 = \bar{X}/(\bar{X} + \beta_2(x))$.

3 Efficiency Comparisons

In this section, the efficiency of the proposed general family estimators is compared with existing estimators by considering expressions of MSE from these estimators up to the first degree of approximation. The details are as follows:

$$MSE(\bar{y}_{KR}^{*})-MSE_{\min}(T)=(1-A_1)-(1-2A_2)\varphi(1-\rho_{yx}^{2})>0 \tag{21}$$

$$MSE(T_{R1})-MSE_{\min}(T)=(C_x-C_y\rho_{yx})^2>0 \tag{22}$$

$$MSE(T_{R2})-MSE_{\min}(T)=(\lambda_1 C_x-C_y\rho_{yx})^2>0 \tag{23}$$

$$MSE(T_{R3})-MSE_{\min}(T)=(\lambda_2 C_x-C_y\rho_{yx})^2>0 \tag{24}$$

$$MSE(T_{R4})-MSE_{\min}(T)=(\lambda_3 C_x-C_y\rho_{yx})^2>0 \tag{25}$$

$$MSE(T_{R5})-MSE_{\min}(T)=(\lambda_4 C_x-C_y\rho_{yx})^2>0 \tag{26}$$

$$MSE(T_{P1})-MSE_{\min}(T)=(C_x+C_y\rho_{yx})^2>0 \tag{27}$$

$$MSE(T_{P2})-MSE_{\min}(T)=(\lambda_1 C_x+C_y\rho_{yx})^2>0 \tag{28}$$

$$MSE(T_{P3})-MSE_{\min}(T)=(\lambda_2 C_x+C_y\rho_{yx})^2>0 \tag{29}$$

$$MSE(T_{P4})-MSE_{\min}(T)=(\lambda_3 C_x+C_y\rho_{yx})^2>0 \tag{30}$$

$$MSE(T_{P5})-MSE_{\min}(T)=(\lambda_4 C_x+C_y\rho_{yx})^2>0 \tag{31}$$

When conditions Eq. (21) to Eq. (31) are satisfied, one can infer that the proposed family of estimators T at its optimum will be more efficient than all other relevant estimators listed in Table 1.

4 Numerical Result

For numerical illustration, the data provided in Khare and Sinha [8] were considered. The data belong to the population census of 96 villages in a rural area published by the government of India for the West Bengal state in 1981. The 25% villages whose area was greater than 160 ha were considered the non-response group of the population. The number of agricultural in the village was taken as the study variable (y), while the area of the village was taken as the auxiliary variable (x). The values of the parameters of the population under this study were as follows:

$N = 96,\ n = 24,\ \bar{Y} = 137.93,\ \bar{X} = 144.87,\ W_2 = 0.25,\ S_y = 182.50,$

$S_{y(2)} = 287.42,\ C_x = 0.81,\ C_{x(2)} = 0.94,\ \rho_{yx} = 0.77,\ \rho_{yx(2)} = 0.72,$

$\beta_1(x) = 0.55,\ \beta_2(x) = 3.06.$

For the purpose of an efficient comparison of the proposed family of estimators, the percent relative efficiencies (PREs) of the estimators T were computed with respect to Khare and Rehman [2] for different values of k. The results are compiled in Table 2.

Table 2 PREs of the different estimators T with respect to $\bar{y}^*_{KR}$.

Estimator	$1/k$		
	1/4	1/3	1/2
$\bar{y}^*_{KR}$	100.00	100.00	100.00
T_{R1}	144.88	156.95	186.97
T_{R2}	137.18	145.54	165.45
T_{R3}	144.83	156.87	186.80
T_{R4}	144.85	156.90	186.86
T_{R5}	144.67	156.64	186.34
T_{P1}	87.70	82.13	74.45
T_{P2}	102.31	99.25	95.18
T_{P3}	87.87	82.33	74.67
T_{P4}	87.82	82.26	74.60
T_{P5}	88.34	82.86	75.29
T	155.01	170.54	213.64

From the numerical illustration in Table 2 it is clear that the proposed family of estimators T have the highest percent relative efficiency (PRE) values compared to other estimators under consideration with different values of $1/k$. When considering the overall PRE values, the PREs of estimators T_{R1}, T_{R2}, T_{R3}, T_{R4}, and T_{R5} were found to increase as $1/k$ increased, whereas for the estimators T_{P1}, T_{P2}, T_{P3}, T_{P4}, and T_{P5} they decreased with the increase of the $1/k$ value. However, estimator T_{R1} seemed to be a more appropriate estimator in comparison to the other estimators, because the PREs of estimator T_{R1} were close to the PREs of estimators T. From the results, one can conclude that apart from estimators T, estimator T_{R1} is also an appropriate choice among the estimators under the numerical study.

5 Simulation Study

The performance of the proposed family of estimators through an extensive numerical simulation study was assessed by using R-statistical software. 10,000 simulations were conducted to study the performance of the proposed family of estimators using a few parameters of the auxiliary variable. The details throughout this simulation are described as follows: a population of N = 500 values (Y_i, X_i) was generated from a bivariate normal distribution with means (50, 50), and standard deviation (10, 10), while the correlation coefficient between (Y_i, X_i) was fixed at 0.95. From this population, a sample of size n = 100 was selected by simple random sampling without replacement (SRSWOR).

The following table shows the results of the studies performed under a simulation study to compare the efficiency of the proposed family of estimators based on the PRE criterion. The results are assembled in Table 3.

Table 3 PREs of different estimators T with respect to $\bar{y}^*_{KR}$ under simulation study.

Estimator	**1/*k***		
	1/4	**1/3**	**1/2**
$\bar{y}^*_{KR}$	100.00	100.00	100.00
T_{R1}	143.50	151.96	184.18
T_{R2}	137.90	142.37	160.32
T_{R3}	143.27	151.72	183.80
T_{R4}	143.30	151.76	183.91
T_{R5}	142.34	150.08	179.60
T_{P1}	77.25	73.47	69.13
T_{P2}	84.18	80.28	75.69
T_{P3}	77.89	74.03	69.56
T_{P4}	77.75	73.89	69.42
T_{P5}	80.00	76.12	71.60
T	153.34	169.83	204.09

When examining Table 3, it can be seen that the proposed family of estimators T has the highest PRE values compared to the other estimators. For trends of the PRE values in this section, the same results were found as in the previous section. The PREs of estimators T_{R1}, T_{R2}, T_{R3}, T_{R4}, and T_{R5} increased as the value of k decreased, but for estimators T_{P1}, T_{P2}, T_{P3}, T_{P4}, and T_{P5} it decreased with the decrease of the k value. When considering the PRE values of any estimator in each situation, it was found that the PREs of estimator T_{R1} were closer to the PREs of estimators T in comparison to other estimators.

Therefore, it is clear that apart from estimators T, estimator T_{R1} is more justifiable in comparison with the numerical study and may be recommended for practical application.

6 Conclusions

In this paper, a general family of estimators to estimate the population mean by adapting the Khoshnevisan, *et al*. [4] estimator and using the concept of the Khare and Rehman [7] estimators was proposed. This was achieved by the presence of non-response when non-response occurs on study variable *y* only. The proposed family of estimators and their properties, such as bias and MSE under study conditions, were obtained and studied. Furthermore, the authors also investigated the relation between the proposed family of estimators and other estimators in terms of the percent relative efficiency. In conclusion, the theoretical numerical result and simulation study revealed that the efficiency of the proposed family of estimators was apparently better than that of the other estimators. Therefore, it can be recommend to use the proposed family of estimators in practice when non-response occurs on study variable *y* only and values of population parameters of auxiliary variable *x* are known.

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