Restricted Size Ramsey Number Involving Matching and Graph of Order Five

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Abstract. Harary and Miller (1983) started the research on the (restricted) size Ramsey number for a pair of small graphs. They obtained the values for some pairs of small graphs with order not more than four. In the same year, Faudree and Sheehan continued the research and extended the result to all pairs of small graphs with order not more than four. Moreover, in 1998, Lortz and Mengenser gave the size Ramsey number and the restricted size Ramsey number for all pairs of small forests with order not more than five. Recently, we gave the restricted size Ramsey number for a path of order three and any connected graph of order five. In this paper, we continue the research on the (restricted) size Ramsey number involving small graphs by investigating the restricted size Ramsey number for matching with two edges versus any graph of order five with no isolates.

Keywords: *graph with no isolates; matching; restricted size Ramsey number.*

1 Introduction

Let G be a graph with the vertex set, edge set, order, and size are $V(G)$, $E(G)$, $v(G)$, and $e(G)$, respectively. We denote the degree of a vertex $v \in V(G)$ by $d(v)$ and the minimum (resp. maximum) degree of vertices in G by $\delta(G)$ (resp. $\Delta(G)$). Let $H \subseteq G$. A graph $G - H$ is obtained from G by deleting all edges in H. Further terminology related to graphs can be found in [1].

The *size Ramsey number* of graphs G and H, $\hat{r}(G,H)$, is the smallest size of graph F such that for any red-blue coloring of all edges of F we have a subgraph G in red color or a subgraph H in blue color. If the order of F in the size Ramsey number must be equal to $r(G,H)$, then we call it the *restricted size Ramsey number*, $r^*(G,H)$. The *Ramsey number* of graphs G and H, $r(G,H)$, is the minimum order r of K_r such that any red-blue coloring of its edges contains a subgraph G in red color or a subgraph H in blue color. Furthermore, we say F *arrowing* graphs G and H, denoted by $F \rightarrow (G,H)$, if any red-blue coloring of

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the edges of F contains a subgraph G in red color or a subgraph H in blue color. In addition, a red-blue coloring of the edges of F is called (G, H)-$good$ if under this coloring, F does not contain G in red color and H in blue color. The notation $F \nrightarrow (G, H)$ means that there exists a (G, H)-good coloring in F.

The concept of the size Ramsey number was introduced by Erdös, *et al.* [2] in 1978, who also gave some results for this problem. Long before this introduction, the concept of the Ramsey number had already been established in graph theory. The restricted size Ramsey number is a direct consequence of the concept of the size Ramsey and Ramsey number in graphs. Some results on the size Ramsey number and the restricted size Ramsey number of graphs can be found in [3-6].

To find the exact values of the (restricted) size Ramsey number for a pair of graphs is challenging, even for a pair of small graphs. In 1983, Harary and Miller [7] initiated the investigation on the (restricted) size Ramsey number for a pair of small graphs. They obtained some exact values for a pair of graphs with order not more than four. However, since the proof is long and needs a tedious amount of work, they omitted the proof of some of their results. Faudree and Sheehan [8] continued the investigation and compiled the complete values for the (restricted) size Ramsey number for any pair of graphs with order not more than four. They also did not give any proof of their results. Lortz and Mengenser (1998) in [9] continued the investigation and derived the size and the restricted size Ramsey numbers for all pairs of small forests with order not more than five.

For the same reason as given by Faudree and Sheehan [8], they also did not provide proof of their results. Recently, in [6] we gave the restricted size Ramsey number for pairs of a path P_3 and any connected graph of order five. We presented the complete proof for this case. To carry on the research on the restricted size Ramsey number involving small graphs, we investigated the restricted size Ramsey number for pairs of a matching with two edges, $2K_2$, and graph with no isolates of order five.

2 Preliminaries

The list of all graphs of order five that do not have isolated vertices is given in Figure 1. In 1972, Chvátal and Harary [10] gave the Ramsey number for $2K_2$ and any graph with no isolates, as stated in Theorem 1. This theorem provides the order of graph F such that $F \rightarrow (2K_2, H)$, in finding $r^*(2K_2, H)$.

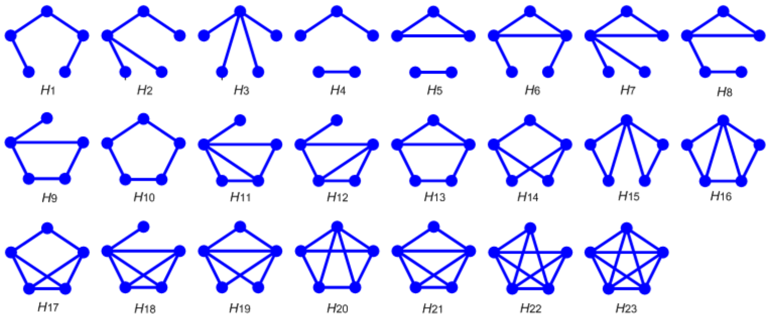

Figure 1 List of graphs with no isolates with order 5.

Theorem 1 [10] For any graph H with no isolates,

$$r(2K_2, H) = \begin{cases} v(H) + 2, & H \text{ is complete}, \\ v(H) + 1, & \text{otherwise}. \end{cases}$$

Some exact values of $r^*(2K_2, H)$ when H is a connected graph of order five are already known. From the results of Lortz and Mengenser [9] we have $r^*(2K_2, H_1) = 6$, $r^*(2K_2, H_2) = 8$, $r^*(2K_2, H_3) = 12$, and $r^*(2K_2, H_4) = 6$. Furthermore, from our previous results in [11] we have $r^*(2K_2, H_7) = 12$, $r^*(2K_2, H_{14}) = 13$, $r^*(2K_2, H_{18}) = 15$, $r^*(2K_2, H_{19}) = 13$, $r^*(2K_2, H_{20}) = 14$, $r^*(2K_2, H_{21}) = 15$, $r^*(2K_2, H_{22}) = 15$ and $r^*(2K_2, H_{23}) = 21$. For the remaining graph H_i, we will derive the exact values for $r^*(2K_2, H_i)$. To prove some of our results, we use Theorem 2.

Theorem 2 [11] For $n \geq 3$,

$$r^*(2K_2, K_n) = \begin{cases} \binom{n+2}{2}, & n \geq 4, \\ \binom{n}{2} - 1, & n = 3 \end{cases}$$

where $\binom{n}{r}$ is a combination of n objects taken r at a time.
Obviously, the following monotonicity property can be derived from the definition of the (restricted) size Ramsey number. If $G' \subseteq G$ and $H' \subseteq H$, then

$$\hat{r}(G', H') \leq \hat{r}(G, H) \tag{1}$$

and

$$r^*(G', H') \leq r^*(G, H) \tag{2}$$

Note that Chvátal and Harary [10] gave this kind of monotonicity property for the Ramsey number of a pair of graphs.

3 Main Results

In this section, we present $r^*(2K_2, H_i)$ for which the values are not yet known. Since $r^*(2K_2, K_5) = 21$ is already known, our goal is to find $r^*(2K_2, H_i)$ for all

H_i in Figure 1, except H_{23} or K_5. Using Theorem 1 we obtain $r(2K_2, H_i) = 6$ for every H_i in our consideration. For any pair of graphs G and H, it is known that $e(G) + e(H) - 1 \leq r^*(G, H) \leq \binom{r(G,H)}{2}$. Using this bound, we have $e(H_i) + 1 \leq r^*(2K_2, H_i) \leq 15$ for all H_i in our consideration. We will give $r^*(2K_2, H_i)$ for H_i a graph that contains a C_3 in Theorems 3 and 4; H_i a graph that contains a C_4 in Theorems 5, 7, and 6; H_i a graph that contains a C_5 in Theorems 8, 9, and 10; and the remaining in Theorem 11.

Lemmas 1 and 2 give the properties of graph F such that $F \to (2K_2, H)$ for any graph H without isolates. Lemma 1 is a generalization of the lemma given in [12], which they gave for $H = K_{1,n}$. Actually, the lemma holds for any graph H and the proof is similar to the proof in [12]. Lemmas 3 and 4 give the properties of F such that $F \to (2K_2, H)$ when graph H contains cycles. We will use all these lemmas in proving our theorems.

Lemma 1. Let H be a graph. $F \to (2K_2, H)$ holds if and only if the following conditions are satisfied:

1. $H \subseteq F - v$ for every $v \in V(F)$ and
2. $H \subseteq F - C_3$ for every C_3 in F.

Lemma 2. Let H be a graph with no isolates. If $F \to (2K_2, H)$ and $v(F) = r(2K_2, H)$, then $\delta(F) \geq 2$.

Proof. If $H \cong K_n$, $F \to (2K_2, H)$, and $v(F) = r(2K_2, H)$, then Theorem 2 implies $\delta(F) \geq 2$. If $H \ncong K_n$, then using Theorem 1 we obtain $v(F) = n + 1$. Suppose to the contrary that $F \to (2K_2, H)$, $v(F) = n + 1$, and $\delta(F) \leq 1$. Assume u is a vertex with $d(u) = 1$ and v is a neighbor of u. The graph $F - v$ consists of a component with order $n - 1$ and an isolate. Obviously, $H \nsubseteq F - v$ and Lemma 2 implies $F \nrightarrow (2K_2, H)$. We have a contradiction.

Lemma 3. For $n \geq 4$, let H be a graph with $v(H) = n$ and H contains a cycle of length t, C_t, for $3 \leq t \leq n$. If $F \to (2K_2, H)$, then F contains at least two C_t which do not share a vertex and are not incident to a C_3.

Proof. For $n \geq 4$, let H be a graph with $v(H) = n$ and $C_t \subseteq H$ for $3 \leq t \leq n$. Suppose to the contrary that $F \to (2K_2, H)$ and all C_t for $3 \leq t \leq n$ in F share a vertex or are incident to a C_3. If all C_t in F share a vertex v, then $H \nsubseteq F - v$ and if all C_t in F are incident to a C_3, then $H \nsubseteq F - C_3$. Lemma 1 implies $F \nrightarrow (2K_2, H)$. We have a contradiction.

In Figure 2(a) we give an example of a graph that contains more than one C_3 but all share a vertex v. By removing v, it means that by coloring all edges incident to v red (edges in dotted line), $C_3 \nsubseteq F - v$. In Figure 2(b) we give an example of a graph that contains more than one C_3 but all are incident to a C_3 (let's call it C_3'). By removing C_3', it means that by coloring all edges belonging to C_3' as red (edges in dotted line), $C_3 \nsubseteq F - C_3'$.

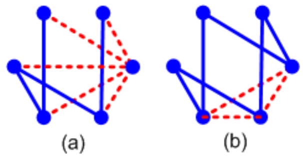

Figure 2 Examples for Lemma 3.

Lemma 4. For $n \geq 4$, let H be a graph with $v(H) = n$ and H contains a cycle with length $n-1$. If $F \rightarrow (2K_2, H)$ and $v(F) = r(2K_2, H)$, then $\delta(F) \geq 3$.

Proof. For $n \geq 4$, let H be a graph with $v(H) = n$ and $C_{n-1} \subseteq H$. If $H \cong K_n$, $F \rightarrow (2K_2, H)$, and $v(F) = r(2K_2, H)$, then Theorem 2 implies $\delta(F) \geq 3$. If $H \ncong K_n$, then using Theorem 1 we obtain $v(F) = n+1$. Suppose to the contrary that $F \rightarrow (2K_2, H)$, $v(F) = n+1$, and $\delta(F) \leq 2$. Lemma 2 implies $\delta(F) = 2$. Suppose u is a vertex with degree 2 and v is a neighbor of u. The degree of u in $F - v$ is 1. Since $v(F) = n+1$, it is clear that $C_{n-1} \nsubseteq F - v$. Hence, Lemma 1 implies $F \nrightarrow (2K_2, H)$. We have a contradiction.

Theorem 3. $r^*(2K_2, H_5) = r^*(2K_2, H_8) = 10$.

Proof. We know that $r(2K_2, H_5) = r(2K_2, H_8) = 6$. Note that $C_3 \subseteq H_5 \subseteq H_8$. To show the upper bound, consider $F = K_6 - (C_4 \cup K_2)$. All vertices in F have degree either 3 or 4. The graph $F - v$ with $d(v) = 3$ is a wheel without a spoke and $F - v$ with $d(v) = 4$ is a graph containing two triangles that share a vertex. It is clear that $H_8 \subseteq F - v$ for both kind of vertices. Furthermore, all C_3 in F are isomorphic, involving two vertices with degree 3 and a vertex with degree 4. It is easy to verify that $H_8 \subseteq F - C_3$ for every C_3. Hence, Lemma 1 implies $F \rightarrow (2K_2, H_8)$, so $r^*(2K_2, H_8) \leq 10$. Since $H_5 \subseteq H_8$, by (2) we obtain $r^*(2K_2, H_5) \leq 10$.

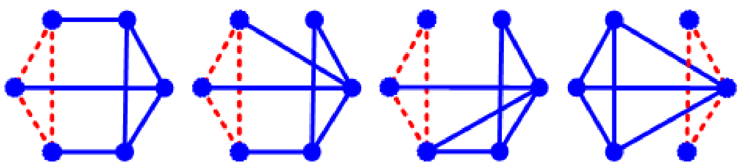

Figure 3 Graphs F that satisfy the conditions for the lower bound of Theorem 3.

To show the lower bound, we must consider all graphs F with $v(F) = 6$ and $e(F) = 9$. According to Lemma 2 and 3, $\delta(F) \geq 2$ and F must contain at least two C_3 which do not share a vertex and are not incident to a C_3. There are four graphs satisfying these conditions, as shown in Figure 3, each with a red-blue coloring that is $(2K_2, H_5)$-good (dotted line in red color). For all F, $F \nrightarrow (2K_2, H_5)$, so $r^*(2K_2, H_5) \geq 10$. Since $H_5 \subseteq H_8$, by (2) we obtain $r^*(2K_2, H_8) \geq 10$.

Theorem 4. $r^*(2K_2, H_6) = 9$.

Proof. We know that $r(2K_2, H_6) = 6$. Note that $C_3 \subseteq H_5$. To show the upper bound, let $F = M_6$ with M_6 be a Möbius ladder with six vertices. Observe that F is a 3-regular graph and contains two isomorphic C_3. It is easy to verify that $H_6 \subseteq F - v$ for every $v \in V(F)$ and $H_6 \subseteq F - C_3$ for every C_3 in F. Hence, Lemma 1 implies $F \rightarrow (2K_2, H_6)$, so $r^*(2K_2, H_6) \leq 9$.

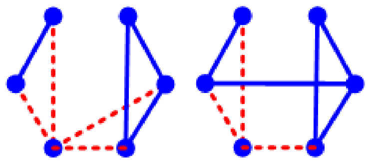

Figure 4 Graphs F that satisfy the conditions for the lower bound of Theorem 4.

To show the lower bound, we must consider all graphs F with $v(F) = 6$ and $e(F) = 8$. According to Lemma 2 and 3, $\delta(F) \geq 2$ and F must contain at least two C_3 which do not share a vertex and are not incident to a C_3. There are only two graphs satisfying these conditions, as shown in Figure 4, with red-blue coloring that are $(2K_2, H_6)$-good (the red color in dotted line). Hence, $F \nrightarrow (2K_2, H_6)$ for both F, so $r^*(2K_2, H_6) \geq 9$.

Theorem 5. $r^*(2K_2, H_9) = 8$.

Proof. We know that $r(2K_2, H_9) = 6$. Note that $C_4 \subseteq H_9$. To show the upper bound, let $F' = M_6$ with M_6 be a Möbius ladder with six vertices. Consider $F = F' - e$ with e is an edge belonging to C_6 in F'. Observe that F is a C_3-free and all vertices in F have degree either 2 or 3. Suppose u is a vertex with $d(u) = 2$ and v with $d(v) = 3$. It is clear that $H_9 \subseteq F - v \subseteq F - u$. Hence, Lemma 1 implies $F \rightarrow (2K_2, H_9)$, so $r^*(2K_2, H_9) \leq 8$.

To show the lower bound, we must consider all graphs F with $v(F) = 6$ and $e(F) = 7$. According to Lemma 2 and 3, $\delta(F) \geq 2$ and F must contain at least two C_4 which do not share a vertex and are not incident to a C_3. However, there is no graph satisfying these conditions, so $r^*(2K_2, H_9) \geq 8$.

Theorem 6. $r^*(2K_2, H_{12}) = 12$.

Proof. We know that $r(2K_2, H_{12}) = 6$. Note that $C_4 \subseteq H_{12}$. To show the upper bound, consider $F = K_6 - 3K_2$. Observe that F is a 4-regular graph and all C_3 in F are isomorphic. It can be verified that $H_{12} \subseteq F - v$ for every $v \in V(F)$ and $H_{12} \subseteq F - C_3$ for every C_3 in F. Hence, Lemma 1 implies $F \to (2K_2, H_{12})$, so $r^*(2K_2, H_{12}) \leq 12$.

To show the lower bound, we must consider all graphs F with $v(F) = 6$ and $e(F) = 11$. According to Lemmas 2 and 3, $\delta(F) \geq 2$ and F must contain at least two C_4 which do not share a vertex and are not incident to a C_3. There are four graphs F satisfying these conditions, namely, F is isomorphic to $K_6 - (C_3 \cup K_2)$, $K_6 - P_5$, $K_6 - (P_4 \cup K_2)$, or $K_6 - 2P_3$. If $F = K_6 - (C_3 \cup K_2)$ or $F = K_6 - P_5$, then $H_{12} \nsubseteq F - v$ with v is a vertex with $d(v) = 5$. If $F = K_6 - (P_4 \cup K_2)$ or $F = K_6 - 2P_3$, then $H_{12} \nsubseteq F - C_3$ for any C_3 in each F. Hence, Lemma 1 implies $F \nrightarrow (2K_2, H_{12})$ for all F, so $r^*(2K_2, H_{12}) \geq 12$.

Theorem 7. $r^*(2K_2, H_{11}) = 12$.

Proof. We know that $r(2K_2, H_{11}) = 6$. To show the upper bound, consider $F = K_6 - K_{1,3}$. In F there is a vertex u with $d(u) = 5$ and $H_{11} \subseteq F - u \subseteq F - v$ for any v in F. Furthermore, there is a K_4 in F and all vertices $v \in V(K_4)$ are adjacent to u and one is adjacent to vertex x with $d(x) = 2$. There are five kinds of C_3 in F, namely, C_3 involving u and two v not adjacent to x, C_3 involving u and two v one is adjacent to x, C_3 involving three v not adjacent to x, C_3 involving three v one is adjacent to x, and C_3 involving x. It can be verified that for every kind of C_3, $H_{12} \subseteq F - C_3$. Hence, Lemma 1 implies $F \to (2K_2, H_{11})$, so $r^*(2K_2, H_{11}) \leq 12$.

Before proving the lower bound, consider $F' = K_6 - 2P_2$. It is clear that $H_{11} \nsubseteq F' - v$ for vertex v with $d(v) = 5$. To show the lower bound, we must consider all graphs F with $v(F) = 6$ and $e(F) = 11$. According to Lemma 2, $\delta(F) \geq 2$. However, all F that satisfy these conditions are subgraphs of F'. Therefore, Lemma 1 implies $F \nrightarrow (2K_2, H_{11})$ for all F. Hence $r^*(2K_2, H_{11}) \geq 12$.

Theorem 8. $r^*(2K_2, H_{10}) = r^*(2K_2, H_{13}) = 11$.

Proof. We know that $r(2K_2, H_{10}) = r(2K_2, H_{13}) = 6$. Note that $H_{10} = C_5 \subseteq H_{13}$. To show the upper bound, consider $F = K_6 - (P_4 \cup K_2)$. All vertices in F have degree either 3 or 4. For u is a vertex with $d(u) = 4$, $H_{13} \subseteq F - u \subseteq F - v$ for every v in F. Furthermore, there are two different C_3 in F, namely, C_3 involving three vertices with degree 4 and C_3 involving two vertices with degree 3 and a vertex with degree 4. It can be verified that for both kinds of C_3,

$H_{13} \subseteq F - C_3$. Hence, Lemma 1 implies $F \to (2K_2, H_{13})$, so $r^*(2K_2, H_{13}) \le 11$. Since $H_{10} \subseteq H_{13}$, by (2) we obtain $r^*(2K_2, H_{10}) \le 11$.

To show the lower bound, we must consider all graphs F with $v(F) = 6$ and $e(F) = 10$. According to Lemma 4, $\delta(F) \ge 3$. There are four graphs F satisfying these conditions, namely F is isomorphic to $K_6 - (C_3 \cup P_3)$, $K_6 - (C_4 \cup K_2)$, $K_6 - C_5$, or $K_6 - 2P_6$. For all F, $H \nsubseteq F - C_3$ for any C_3 in each F. Thus, Lemma 1 implies $F \nrightarrow (2K_2, H_{10})$ for all F, so $r^*(2K_2, H_{10}) \ge 11$. Since $H_{10} \subseteq H_{13}$, by (2) we obtain $r^*(2K_2, H_{13}) \ge 11$.

Theorem 9. $r^*(2K_2, H_{16}) = 12$.

Proof. We know that $r(2K_2, H_{16}) = 6$. Note that $C_5 \subseteq H_{16}$. To show the upper bound, consider $F = K_6 - 3K_2$. Observe that F is a 4-regular graph and all C_3 in F are isomorphic. It is easy to verify that $H_{16} \subseteq F - v$ for every $v \in V(F)$ and $H_{16} \subseteq F - C_3$ for every C_3 in F. Hence, Lemma 1 implies $F \to (2K_2, H_{16})$, so $r^*(2K_2, H_{16}) \le 12$.

To show the lower bound, we must consider all graphs F with $v(F) = 6$ and $e(F) = 11$. According to Lemma 4, $\delta(F) \ge 3$. Furthermore, $\Delta(F) \le 4$ as if there is a vertex v with $d(v) = 5$, then $e(F - v) = 11 - 5 = 6 < e(H_{16})$. The only graph satisfying the above conditions is F isomorphic to either $K_6 - (P_4 \cup K_2)$ or $K_6 - 2P_3$. Note that $\Delta(H_{16}) = 4$. For both F, $H_{16} \nsubseteq F - v$ since $\Delta(F - v) = 3$ for $v \in V(F)$ with $d(v) = 4$. Hence, Lemma 1 implies $F \nrightarrow (2K_2, H_{16})$, so $r^*(2K_2, H_{16}) \ge 12$.

Theorem 10. $r^*(2K_2, H_{17}) = 13$.

Proof. We know that $r(2K_2, H_{17}) = 6$. Note that $C_5 \subseteq H_{17}$. To show the upper bound, consider $F = K_6 - 2K_2$. All vertices in F have degree either 4 or 5. For u is a vertex with $d(u) = 5$, $H_{17} \subseteq F - u \subseteq F - v$ for every v in F. Furthermore, there are two different C_3 in F, namely, C_3 involving two vertices with degree 5 and a vertex with degree 4 and C_3 involving two vertices with degree 4 and a vertex with degree 5. It can be verified that for both kinds of C_3, $H_{17} \subseteq F - C_3$. Hence Lemma 1 implies $F \to (2K_2, H_{17})$, so $r^*(2K_2, H_{17}) \le 13$.

To show the lower bound, we must consider all graphs F with $v(F) = 6$ and $e(F) = 12$. According to Lemma 4, $\delta(F) \ge 3$. There are four graphs F satisfying these conditions, namely F is isomorphic to $K_6 - 3K_2$, $K_6 - (P_3 \cup K_2)$, $K_6 - 2P_4$, or $K_6 - C_3$. If $F = K_6 - 3K_2$, then $H_{17} \nsubseteq F - C_3$ for any C_3. If $F = K_6 - (P_3 \cup K_2)$, then $H_{17} \nsubseteq F - C_3$ with C_3 involving three vertices with degree 4. If $F = K_6 - P_4$, then $H_{17} \nsubseteq F - v$ with v of degree 5. If $F = K_6 - C_3$, then $H_{17} \nsubseteq F - C_3$ with C_3 involving three vertices with degree 5. Hence, Lemma 1 implies $F \nrightarrow (2K_2, H_{17})$ for all F, so $r^*(2K_2, H_{17}) \geq 13$.

Theorem 11. $r^*(2K_2, H_{15}) = 14$.

Proof. We know that $r(2K_2, H_{14}) = 6$. To show the upper bound, consider $F = K_6 - K_2$. All vertices in F have degree either 4 or 5. For u is a vertex with $d(u) = 5$, $H_{15} \subseteq F - u \subseteq F - v$ for every $v \in V(F)$. Furthermore, there are two different C_3 in F, namely, C_3 involving three vertices with degree 5 and C_3 involving two vertices with degree 5 and a vertex with degree 4. It can be verified that for both kinds of C_3, $H_{15} \subseteq F - C_3$. Hence, Lemma 1 implies $F \rightarrow (2K_2, H_{15})$, so $r^*(2K_2, H_{15}) \leq 14$.

To show the lower bound, we must consider all graphs F with $v(F) = 6$ and $e(F) = 13$. The only graph satisfying these conditions is F isomorphic to either $K_6 - P_3$ or $K_6 - 2K_2$. If $F = K_6 - P_3$, then $H_{15} \nsubseteq F - C_3$ with C_3 involving three vertices with degree 5. If $F = K_6 - 2K_2$, then $H_{15} \nsubseteq F - C_3$ with C_3 involving two vertices with degree 4 and a vertex with degree 5. Hence, Lemma 1 implies $F \nrightarrow (2K_2, H_{15})$ for both F, so $r^*(2K_2, H_{15}) \geq 14$.

We compile the restricted size Ramsey number for $2K_2$ versus any graph of order five with no isolates in Table 1.

Table 1 Compilation of $r^*(2K_2, H)$ with H is a graph that has no isolates of order five.

r^*	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8
$2K_2$	6 [9]	8 [9]	12 [9]	6 [9]	10 Th.3	9 Th.4	12 [11]	10 Th.3
r^*	H_9	H_{10}	H_{11}	H_{12}	H_{13}	H_{14}	H_{15}	H_{16}
$2K_2$	8 Th.5	11 Th.8	12 Th.7	12 Th.6	11 Th.8	13 [11]	14 Th.11	12 Th.9
r^*	H_{17}	H_{18}	H_{19}	H_{20}	H_{21}	H_{22}	H_{23}	
$2K_2$	13 Th.10	15 [11]	13 [11]	14 [11]	15 [11]	15 [11]	21 [11]	

4 Conclusion

In this paper we gave the complete list of the exact values of the restricted size Ramsey number for $2K_2$ versus any graph of order five with no isolates. For further research:

1. Find the size Ramsey number of $\hat{r}(2K_2, H)$ for all H in Figure 1 except H_{23}.

2. Find the restricted size Ramsey number $r^*(2K_2, H)$ with H is a graph of order six for which $r^*(2K_2, H)$ is not yet given in [5].

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