MECHANICS

CORIOLIS CONCEPT: USEFUL OR NOT ?

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ICHTISAR

GAJA CORIOLIS DISINI DIURAIKAN SETJARA KWALITATIP. Perhitungannja didasarkan atas sistem salib sumbu^2 jang tetap dan lagi atas sistem salib sumbu jang bergerak (gaja coriolis).

Diterangkan bahwa tjara dengan memakai sistem sumbu jang tetap ini, adalah lebih mudah daripada tjara dengan perhitungan coriolis jang matematis komplex. Effek coriolis adalah menarik sekali, karena ini dapat memberi kesan jang salah, umpamanja untuk seorang pengamat satelit dari bumi. Djalan jang ditempuh satelit sesungguhnja terletak dalam bidang datar, tetapi sipengamat tadi melihatnja sebagai djalan lengkung.

Qualitative Description

ONE OF THE MOST MISUNDERSTOOD CONCEPTS IN dynamics is the Coriolis force (or acceleration). Almost every engineer has heard of this force, but few have a completely clear picture of what this force is and how the concept may be used in practical calculations of motion. This unfamiliarity is probably a blessing, since, as will become clear, most calculations can be performed more easily by a shift of coordinate axes than by use of the Coriolis force.

The Coriolis force arises from the fact that Newton's Law does not apply in a reference system accelerating with respect to „inertial space". As an example of this, suppose that a scientist in an elevator is investigating Newton's Law. When the elevator is at rest or moving at constant velocity, the scientist will observe that a dropped object will obey Newton's law. However, if the elevator starts from rest and moves downward at the instant that the object is dropped, the scientist will observe a departure from Newton's Law. He knows that the force of gravity has not changed, but he will observe that the time for the object to reach the floor has increased. It is obvious to anyone outside the elevator that the object falls in the same fashion whetner the elevator is at rest or accelerating. The longer time to reach the floor when the elevator accelerates downward is simply due to the fact that the elevator floor has moved downward, and so the object has a greater distance to fall. We normally solve the problem of the object in the elevator by studying the object's motion with respect to "fixed" axes, so that Newton's Law applies, and converting to axes moving the elevator in order to determine how the motion appears to the scientist in the elevator. If we insisted on computation only with respect to axes moving with the elevator, we could postulate an imaginary (or apparent) acceleration in a direction opposite to the elevator's acceleration. Then, by adding this acceleration algebraically to the acceleration of gravity and putting the net acceleration into Newton's Law, we would get the correct motion. This imaginary acceleration is analogous to the Coriolis acceleration, and the imaginary acceleration into Newton's Law is analogous to the Coriolis force.

The Coriolis force is more complicated than the imaginary force just described in that it applies to a rotating coordinate system. Since a rotating coordinate system is accelerating with respect to „inertial space", Newton's Law will not apply in such a rotating system. In order to apply Newton's Law in a rotating system, an imaginary force — the Coriolis force — must be added to the actual forces.

We would not require an imaginary force if we investigate the motion of the object in the elevator relative to fixed axes. It is important to realize that analogously the Coriolis force is not required and in fact has no meaning if, instead of using a rotating axis system, we switch to fixed (non-rotating) axes.

The concept of an imaginary force is common in dynamics. D'Alembert's force and centrifugal force are both imaginary. Many persons are surprised to hear centrifugal force called imaginary, but a moment's reflection will cause one to recall that **centripetal** force is very real, but **centrifugal** force is not. For example, we can describe the motion of an earth satellite without reference to centrifugal force by saying that the centripetal force (gravity) pulls the satellite toward the center of the earth. The satellite remains at constant altitude because the earth's surface falls away as fast as the satellite falls (see figure). If the would-be satellite goes at an appreciably smaller

velocity, it will still be acted upon by the same centripetal force, but there would soon be contact with the earth's surface, because the would-be satellite drops faster than the earth's surface falls away.

Quantitative Application

Now that a qualitative discussion of Coriolis force has given an idea of what this force is, is is desirable to investigate the quantitative application of Coriolis force. For our rotating system, we will take the most common system in our experience — the earth. The earth rotates once each 24 hours, which is 15 degrees each hour and 15 nautical miles per minute at the equator. If motions are investigated with respect to non-rotating axes, there is no Coriolis force. However, if the axes are fixed with respect to the earth and so rotate with it, the Coriolis force describes the difference between the motion observed by a person on the earth and the motion that would be predicted by Newton's Law.

The Coriolis force acts normal to the path of motion and produces an acceleration of 2 omega dr/dt, where omega is the angular rotation of the axis system, axis system, r is Uw is the distance from the earth's axis, and t is time. For the earth, the value of omega is 7.272×10^{-5} radion per second.

As a concrete example, take a satellite in a circular orbit at an altitude of 400,000 feet and exactly above the North Pole. The problem is to compute the "drift" of the satellite as it crosses the equator. As viewed from fixed axes, the satellite is moving in a plane containing the center of the earth. Since the satellite goes over the North Pole and the orbital plane passes through the center of the earth, the path of the satellite, relative to fixed axes, must lie at a right angle to the equator. The period of a satellite at the assumed altitude is 86.9 minutes. During the time required to go from the North Pole to the equator (86.9/4 minutes), a point on the equator moves

$$\textit{drift (earth's surface)} = \frac{(86{,}9)\ (15)}{4} = 326 \textit{ nautical miles}$$

This means that, instead of passing over a point on the equator that is due south of its path as it passes over the North Pole, the satellite will pass over a point 326 nautical miles farther west.

Fixed axes are implicit in the above computation of drift. The next step is to perform the same calculations using a rotating axis system and Coriolis acceleration. We see that such a computation is not easy, since dr/dt varies from satellite velocity, which is 25,700 feet per second, when the satellite is above the North Pole to zero when the satellite is above the equator. In other words, dr/dt is 25,700 sin gamma, where gamma is the latitude angle (zero at the equator and pi/2 at the North Pole). Since the latitude angle varies linearly with time for a circular orbit, the time-average value of dr/dt as the satellite goes from North Pole to equator is given by

$$\frac{dr/dt}{25{,}700} = \frac{\int_0^{\pi/2} \sin\alpha \, d\alpha}{\int_0^{\pi/2} d\alpha} = \frac{\left[\cos\alpha\right]_{\pi/2}^{0}}{\pi/2} = 2/\pi$$

The drift is given by the familiar equation relating distance, time, and acceleration:

$$\text{drift} = \tfrac{1}{2}\,at^2$$

where a is the Coriolis acceleration. Thus

$$\text{drift} = \left(\tfrac{1}{2}\right)(2)(7.272)(10^{-5})\left(\tfrac{2}{\pi}\right)(25{,}700)\left(\tfrac{86.9}{4}\right)^2(60)^2/6080$$

$$= 332.5 \text{ nautical miles}$$

where 6080 is the number of feet in a nautical mile. The drift of 332.5 nautical miles is at an altitude of 400,000 feet. The drift when projected on the earth's surface is

$$\text{drift (earth's surface)} = 332.5\left(\frac{3441}{3441 + \frac{400.000}{6080}}\right)$$

$$= 326 \text{ nautical miles}$$

where 3441 is the radius of the earth in nautical miles.

The computation based on Coriolis acceleration yields, as would be expected, the same result as the computation based on fixed axes. However, the above computation based on Coriolis acceleration is much more complicated than the calculation based on fixed axes. It is strongly recommended, as stated in the first paragraph, that if computations are to be made for a rotating system, a shift in axes be made to a fixed system in order to eliminate the Coriolis effect. The results can then be converted back to a rotating system by a second shift of axes. The shifting of axes is geometric and in almost every case simpler and less liable to calculation error than handling the Coriolis effect.

Manifestations

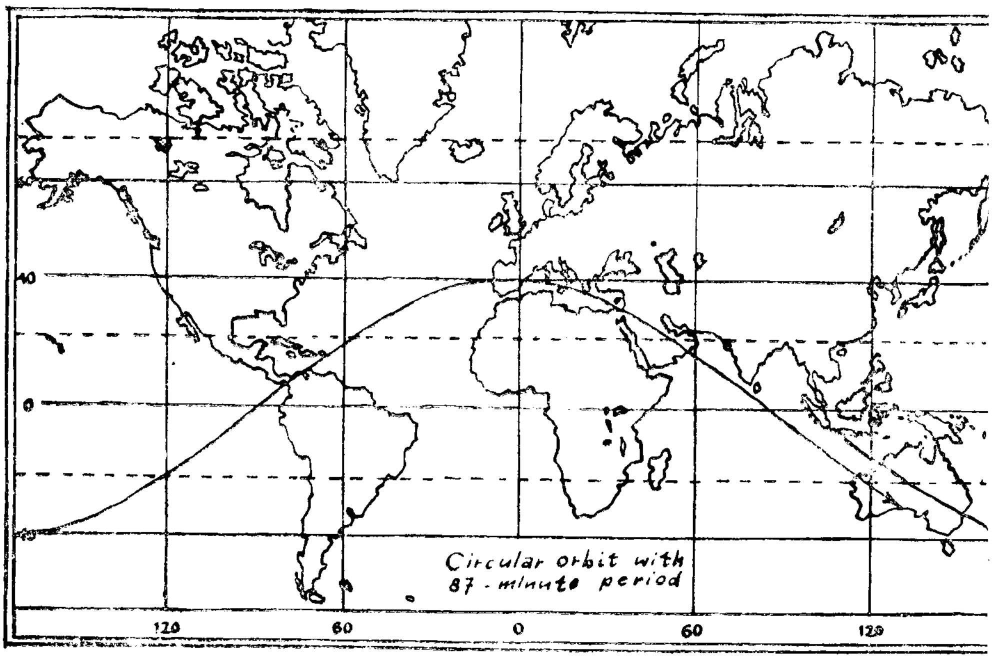
The Coriolis effect is interesting from the point of view of tricks it can play on us, since all of us, except for a few space pioneers, are forced to live on a rotating planet. The Coriolis effect is not commonly experienced in our everyday life. Since the effect is proportional to dr/dt (velocity normal to the earth's axis), there are two ways to achieve an observable Coriolis effect during conventional travel on the earth's surface or in its atmosphere. One way is to go very rapidly in roughly a northsouth direction anywhere except near the equator (dr/dt is zero for travel in any direction at the equator and is zero anywhere for travel in an east-west direction). The second way is to travel at even moderate speed in the vicinity of one of the poles (dr/dt being equal to ground speed at either the North or South Pole). Since until recently humans have traveled slowly and since the majority of the earth's population lives near the equator, the effect of Coriolis acceleration on navigation is commonly masked by other larger navigation errors.

Even at the latitude of New York, the distance to the earth's axis is 76 percent of this distance at the equator, so dr/dt changes three times as much between New York and the North Pole as between the equator and New York. Consequently, it is necessary to be very close to the North Pole before the Coriolis effect hampers navigation. Let us calculate the effect on an explorer who starts from the North Pole and walks in a direction his sense of inertia indicates to be a straight line. If he goes at the rate of 5 kilometers per hour, after one hour his lateral movement from the straight line he intended to walk would be

$$\begin{aligned} \text{drift} &= (\tfrac{1}{2})\,(2)\,(7{,}272)\,(10^{-5})\,(5)\,(1)^{2}\,(3600) \\ &= 1.31 \text{ Kilometers} \end{aligned}$$

This deviation of over a kilometer from a straight line is very significant and would be very perplexing to a polar explorer if in fact ability to walk in a straight line is based on an inertial sense as assumed in order to make the above calculation.

The explorer's drift at the North Pole would be to the right. At the South Pole, the drift would be of the same magnitude, but to the left. In fact, the Coriolis drift is always to the right in the northern hemisphere and always to the left in the southern hemisphere. This can be seen from the figure, in which the path relative to the earth (rotating axes) in shown for a satellite launched from Bandung southwesterly into a circular orbit.



Summary

In computations of trajectories, it is usually more convenient to switch to a fixed set of axes than to try to handle the mathematical complexity of the Coriolis acceleration. The Coriolis effect is interesting, since it can give a false impression, as for example to a ground observer viewing a satellite trajectory. The actual trajectory lies in a plane, but the ground observer sees a curved path.