A Mathematical Model of Black Rhino Translocation Strategy

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Abstract. A deterministic mathematical model of the black rhino population in South Africa will be discussed. The model is constructed by dividing the black rhino population into multiple patches. The impact of human intervention on different translocation strategies is incorporated into the model. It is shown that, when implemented correctly, translocation can accelerate the growth rate of the total black rhino population. Equilibrium points are shown with their local stability criteria.

1   Introduction

The black rhino population in the 1970s had a bleak future, since there were only about 2500 left in South Africa. Prior to this, there were about 70,000 black rhinos in South Africa [1,2]. This alarming decrease in the population number arose from poaching rhinos for their horns, which are marketed in Asia and Europe. To overcome poaching, the South African government has tried to prevent illegal rhino hunting. However, these attempts have been largely unsuccessful. In order to increase the population numbers of the black rhino, a translocation strategy has been implemented.

The translocation strategy involves relocating rhinos in an area populated fairly densely with rhinos to another area which is less dense and has a suitable habitat to support population growth [3]. The involvement of the government in the translocation strategy can be considered to be fairly successful since the current black rhino population reached 4860 rhinos in 2011 [4]. The translocation

method is implemented in densely populated areas such as the Hluhluwe-Umfolozi National Park.

In the relocating process there are many factors to be considered: rhino size, age, sex and climate variability are the main influencing factors [5,6]. The population density is also of critical importance. The carrying capacity is mostly influenced by environmental factors such as rainfall. With sufficient rainfall, food and water will be abundant and thus the rhino population growth will be higher due to a higher reproduction rate. It should be noted that harvesting, which is the removal and relocation of rhinos, should be conducted in a way that ensures that the donor population will not be negatively affected. The aim of the translocation strategy is to increase the growth rate of the total black rhino population in South Africa.

A simple mathematical model was developed in this study in order to describe the effect of translocation on the total rhino population in South Africa. The translocation rate, which was defined as an intervention variable, was assumed to be time-dependent. The object of this research was to find better strategies for optimizing the growth rate of the black rhino population. The donor population should not be severely affected in a negative way and the receiver population must benefit from the harvest.

A mathematical model to describe this situation in a population of black rhinos in South Africa will be discussed in this paper. This paper is constructed as follows. In the next section, the construction of the mathematical model will be discussed. In the third section, a mathematical analysis of the equilibrium points and their local stability criteria will be given. A numerical simulation with a number of different translocation strategies will be given in section 4. The conclusion will be given in the last section.

2   Mathematical Model

The logistic mathematical model is implemented in three different patches, namely: the variable $y_1(t)$ denotes the population number of the resource or donor population, $y_2(t)$ denotes the first translocation or receiver population, and $y_3(t)$ the second translocation population. In each population group, the recruitment rate, which depends on environmental factors and thus differs for each population, is a combination of the rate at which rhinos are born, μ, which may decrease due to competition between rhinos. The carrying capacity, $k,$ which also depends on climatic conditions such as rainfall variability, will differ for each population group. The recruitment rates and carrying capacities are assumed to be known parameters. A fraction of rhinos from population group $y_1(t)$ will be translocated to populations $y_2(t)$ and $y_3(t)$ at a rate of

$h_1(t)$ with proportions of p and $(1-p)$ respectively. We include the parameter p so that the total rate at which rhinos are taken from y_1 is $h_1(t)$ and because we do not want to restrict the fraction of rhinos being received from populations $y_2(t)$ and $y_3(t)$ to be the same. The value of parameter p will depend on the state of the receiving populations. For instance, it may happen that one population is in a more desperate need of donated rhinos. When population $y_2(t)$ reaches a state in which it can function independently, a fraction of this population will be translocated to $y_3(t)$ at a rate of $h_2(t)$. We implement the possibility of population $y_2(t)$ becoming a donor population once it can function unaided by harvesting, since we want to maximize the growth rate of all three populations. In other words, we want to ensure that we reach a state in which all three populations can function independently. The three decision variables in this model are $p, h_1(t), h_2(t)$. The model illustrated in Figure 1.
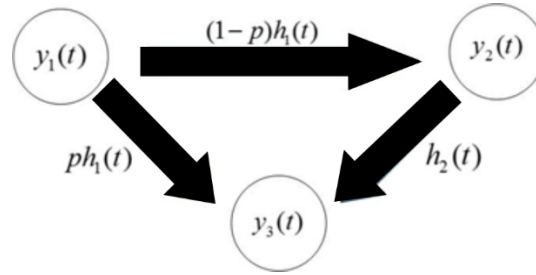

Figure 1 Diagram showing the translocation strategy for the black rhino population.

Using the assumptions stated above and Figure 1, the system of differential equations for the black rhino translocation strategy is given by

$$\frac{dy_1(t)}{dt} = \mu_1 y_1(t)\left(1 - \frac{y_1(t)}{k_1(t)}\right) - h_1(t)y_1(t) \tag{1}$$

$$\frac{dy_2(t)}{dt} = \mu_2 y_2(t)\left(1 - \frac{y_2(t)}{k_2(t)}\right) + ph_1(t)y_1(t) - h_2(t)y_2(t) \tag{2}$$

$$\frac{dy_3(t)}{dt} = \mu_3 y_3(t)\left(1 - \frac{y_3(t)}{k_3(t)}\right) + (1-p)h_1(t)y_1(t) + h_2(t)y_2(t), \tag{3}$$

where μ_i, $i = 1,2,3$, is the recruitment rate for each population and the initial conditions $y_i(0)$ are given. All parameters are non-negative and we have that $h_i(t) \in [0,1]$ for $i = 1,2$ and also $p \in [0,1]$.

To accommodate seasonal rainfall variability and the geography of each environment, the carrying capacity for each population is time-dependent. In this model, we assume the carrying capacity in each patch is presented as a

sinusoidal function given by $k_i(t) = k_{0i} + a_i \, sin\left(\frac{\pi t}{b_i}\right)$ for $i = 1,2,3$ where k_{0i} is the initial carrying capacity for each i while a and b are constants to account for the fluctuations of k_{0i}.

In the next section we assume that all translocation rates and carrying capacities are constant. We let $h_i(t) = h_i$, $i = 1,2$ and $k_i(t) = k_i$, $i = 1,2,3$. The stability analysis of the model will be discussed in the next section.

3   Analysis of the Model

There are four equilibrium points for Eq. (1-3), which are given by

$$E_0 = (0,0,0)$$

$$E_1 = (0,0,k_3)$$

$$E_2 = \left(0, \frac{k_2(\mu_2 - h_2)}{\mu_2}, \frac{1}{2}\left(k_3 + \sqrt{k_3\left(1 + \frac{4k_2 h_2}{\mu_3}\left(1 - \frac{h_2}{\mu_2}\right)\right)}\right)\right)$$

$$E_3 = \left(\frac{k_1(\mu_1 - h_1)}{\mu_1}, \frac{1}{2}\left(k_2\left(1 - \frac{h_2}{\mu_2}\right) + \sqrt{\left(\frac{k_2}{\mu_2}\right)^2 (\mu_2 - h_2)^2 + \frac{4h_1 k_1 k_2 p}{\mu_2}\left(1 - \frac{h_1}{\mu_1}\right)}\right),\right.$$

$$\left.\frac{1}{2}\left(k_3 + \sqrt{k_3^2 + \frac{4h_1 k_1(\mu_1 - h_1)k_3(1-p)}{\mu_1 \mu_3} + \frac{4h_2 y_2^* k_3}{\mu_3}}\right)\right),$$

where y_2^* is the equilibrium point of y_2 in E_3.

Equilibrium point E_0 represents the situation where all populations are extinct and equilibrium point E_1 represents the situation where only the receiver population exists. Both cases may occur if the translocation strategy is not successful. An example of a possible failure of the translocation strategy would be to harvest rhinos at a rate that is greater than the natural recruitment rate $(h_i > \mu_i)$. On the other hand, E_2 represents the situation where the main source population becomes extinct because the harvesting rate is greater than the natural recruitment rate $(h_1 > \mu_1)$. Trivially, E_2 will always be non-zero if the harvesting rate is smaller than the natural recruitment rate $(h_2 < \mu_2)$.

The last equilibrium point, denoted by E_3, depicts the situation where all populations exist. The point E_3 will always be a positive equilibrium point if $h_i < \mu_i$ for $i = 1,2$. Therefore, as long as the translocation rate is always less than the natural recruitment rate, E_3 will always be positive.

The Jacobian matrix of system (1-3) is given by

$$J = \begin{bmatrix} \mu_1\left(1-\frac{y_1}{k_1}\right)-\frac{\mu_1 y_1}{k_1}-h_1 & 0 & 0 \\ ph_1 & \mu_2\left(1-\frac{y_2}{k_2}\right)-\frac{\mu_2 y_2}{k_2}-h_2 & 0 \\ (1-p)h_1 & h_2 & \mu_3\left(1-\frac{y_3}{k_3}\right)-\frac{\mu_3 y_3}{k_3} \end{bmatrix}. \quad (4)$$

According to matrix (4), E_0 has three different eigenvalues, namely, $\lambda_1 = (\mu_1 - h_1)$, $\lambda_2 = (\mu_2 - h_2)$ and $\lambda_3 = \mu_3$. Because $\mu_3 > 0$, E_0 will always unstable. In a similar way, we have three eigenvalues for E_1, i.e. $\lambda_1 = (\mu_1 - h_1)$, $\lambda_2 = (\mu_2 - h_2)$ and $\lambda_3 = -\mu_3$. We can see that E_1 will always be locally asymptotically stable if $h_i > \mu_i$ for $i = 1,2$. From an ecological point of view, this situation must be avoided. It may happen if the translocation rate is larger than the natural recruitment rate in each population.

The third equilibrium point, E_2, will be locally asymptotically stable if $h_1 > \mu_1$ and $h_2 < \mu_2$ since it has three different eigenvalues given by $\lambda_1 = (\mu_1 - h_1)$, $\lambda_2 = -(\mu_2 - h_2)$ and $\lambda_3 = \frac{\mu_3(k_3 - 2y_3^*)}{k_3}$, where y_3^* is the equilibrium state for y_3 in E_2. This situation represents the failure of the translocation strategy in the y_1 population because the translocation rate is larger than the natural recruitment rate. The Jacobian matrix of equilibrium point E_3 is omitted due to its complexity. The eigenvalues of E_3 are given by

$$\lambda_1 = h_1 - \mu_1$$
$$\lambda_2 = -\frac{\sqrt{\left(\frac{1}{4}\mu_1(h_2-\mu_2)^2 k_2 + ph_1 k_1 \mu_1(\mu_1-h_1)\right)\mu_1 k_2}}{\mu_1 k_2},$$
$$\lambda_3 = \frac{\mu_3(k_3 - 2y_3^*)}{k_3}$$

where x_3^* is the equilibrium point of x_3 in E_3. The first eigenvalue λ_1 will be negative if $(h_1 < \mu_1)$. (This is already fulfilled by the positiveness criteria of E_3). The second eigenvalue will always be negative also due to the positiveness requirement for E_3, i.e $(h_i < \mu_i)$. The last eigenvalue will always be negative if $\frac{k_3}{2x_3^*} < 1$. From equilibrium point E_3, it is easily verified that this condition is indeed satisfied.

A summary of the positiveness and local stability criteria for each equilibrium point is presented in Table 1.

Table 1 Positiveness and local stability criteria for each equilibrium point.

Equilibrium Point	Positive criteria	Local stability criteria
$E_0 = (0,0,0)$	-	Always unstable
$E_1 = (0,0,y_3^*)$	-	$h_1 > \mu_1$, $h_2 > \mu_2$
$E_2 = (0,y_2^*,y_3^*)$	$h_2 < \mu_2$	$h_1 > \mu_1$, $h_2 < \mu_2$
$E_2 = (y_1^*,y_2^*,y_3^*)$	$h_1 < \mu_1$, $h_2 < \mu_2$	$h_1 < \mu_1$, $h_2 < \mu_2$

As shown from the analytical results regarding positivity and local stability for each equilibrium point, it can be seen that only the natural recruitment rate and translocation rate play a role in the stability for each point. The carrying capacities do not play any role in the stability of the system.
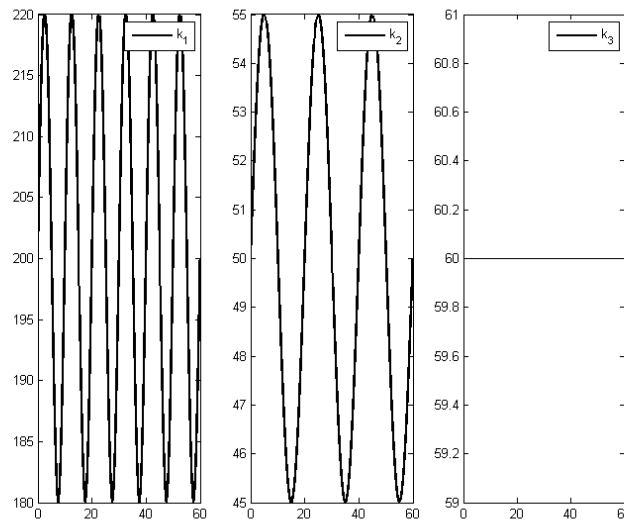
Figure 2 Carrying capacity for each population with $k_1 = 200 + 20 \, sin\left(\frac{\pi t}{5}\right)$, $k_2 = 50 + 5 \, sin\left(\frac{\pi t}{10}\right)$, $k_3 = 10$. We define the carrying capacities to be sinusoidal functions as they have seasonal fluctuations. The carrying capacity for y_3 is given as a constant.

Numerical simulations to show the dynamics of each population group for the cases given in Table 1 are shown below. The carrying capacities of each black rhino population depend on climatic conditions and also the geography of each area. We let the carrying capacities for $y_1(t)$ and $y_2(t)$ be sinusoidal functions given by $k_1 = 200 + 20 \, sin\left(\frac{\pi t}{5}\right)$, $k_2 = 50 + 5 \, sin\left(\frac{\pi t}{10}\right)$, while we set the carrying capacity for $y_3(t)$ to a constant parameter because we assume that this receiver population exists in a sufficiently small region (for an example, a

private game farm). In Figure 2, plots of the carrying capacities with respect to time (in years) are given.
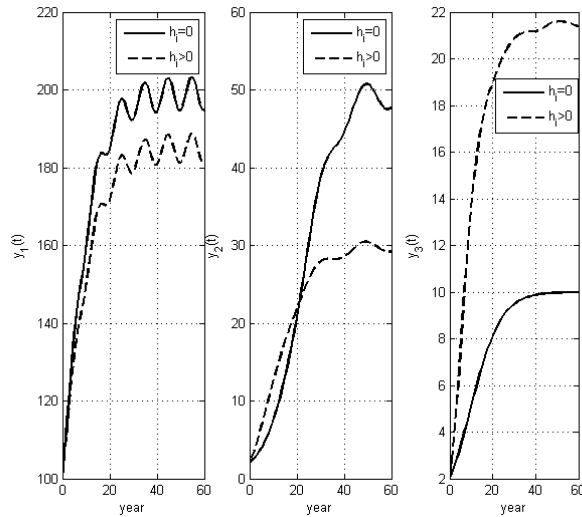

Figure 3 Dynamics of $y_1(t), y_2(t)$ and $y_3(t)$ (for $\mu_i > h_i$) with parameter values $\mu_i = \frac{1}{7}, p = 0.7, h_1 = 0.01, h_2 = 0.1$.
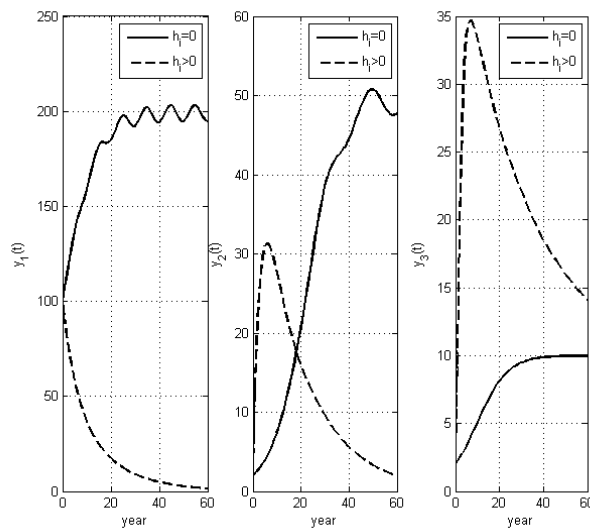

Figure 4 Dynamic of $y_1(t), y_2(t)$ and $y_3(t)$ (for $h_i > \mu_i$) with parameter values $\mu_i = \frac{1}{7}, p = 0.7, h_1 = 0.2, h_2 = 0.3$.
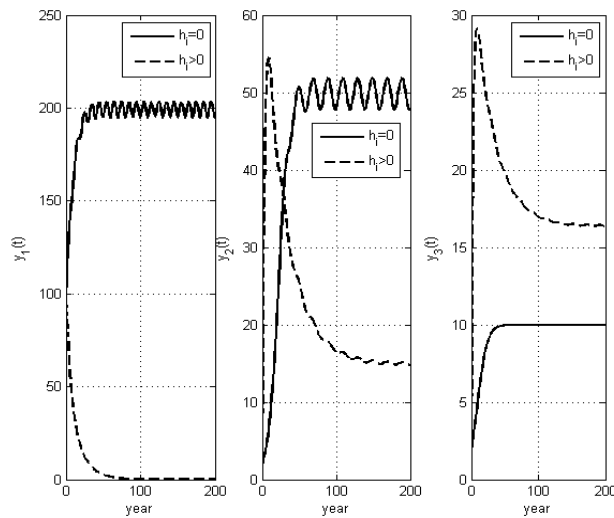
Figure 5 Dynamic of $y_1(t), y_2(t)$ and $y_3(t)$ (case $h_1 > \mu_1, h_2 < \mu_2$) with parameter values $\mu_i = \frac{1}{7}, p = 0.7, h_1 = 0.2, h_2 = 0.1$.

The first simulation shows that when the translocation rates are smaller than the natural recruitment rates ($\mu_i > h_i$ for $i = 1,2$) with $t \in [0,60]$, all populations tend to the non-zero equilibrium point (E_3). Populations y_1 and y_2 fluctuate about their carrying capacities due to the sinusoidal nature of the carrying capacities (see Figure 3). The next simulation, shown in Figure 4, depicts the opposite situation, where the translocation rates are larger than the natural recruitment rates. As shown in Table 2, this scenario will reach the equilibrium point E_1 where only $y_3(t)$, as the main receiver population, will be positive while the other populations tend to zero. The dynamics of each population are shown in Figure 4. The last simulation, shown in Figure 5, considers the situation where the translocation rate from $y_1(t)$ is larger than its natural recruitment rate while the translocation rate from $y_2(t)$ is smaller than its natural recruitment rate. This situation will give us equilibrium point E_2, where all populations are zero, except $y_1(t)$.

If we cannot ensure successful implementation of the translocation strategy, one or more rhino populations may become extinct. Therefore, in the next section we will show that there are some translocation strategies that will lead to an increase in the total black rhino population.

4   Numerical Simulation of Translocation Strategies

In this section, numerical results are given in order to analyze the response of the black rhino populations to different translocation strategies. The first simulation shows the behavior of the populations when different carrying capacities for k_3 are considered. The simulations include the assumption that the government or private farm owners can enable the population of $y_3(t)$ to have a large carrying capacity. We use the same parameter values as in Figure 3, except for the translocation rates ($h_1 = 0.01, h_2 = 0.01$), while the initial conditions are given as $y_1(0) = 180, y_2(0) = 2, y_3(0) = 2$. A constant translocation rate does not address the actual physical situation but is an important approximation for scientific justification. As shown in Figure 6, with $k_3 = 60$ (first row), $k_3 = 120$ (second row) and $k_3 = 250$ (third row), these translocation strategies succeed to increase total black rhino population.
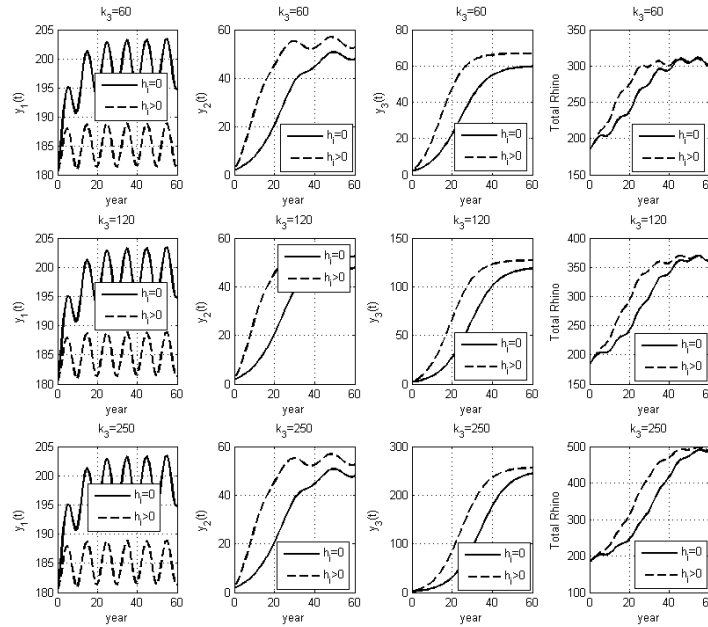

Figure 6 Dynamics of the black rhino populations with different k_3 ($k_3 = 60$ (first row), $k_3 = 120$ (second row) and $k_3 = 250$ (third row)).

Although in Figure 7 we have shown that these translocation strategies succeed in increasing the total number of the black rhino population, the higher the value of k_3, the better the result. This means that one of the biggest challenges for the government or private farm owners is to provide a suitable environment that promotes rhino population growth before implementing the translocation strategy.
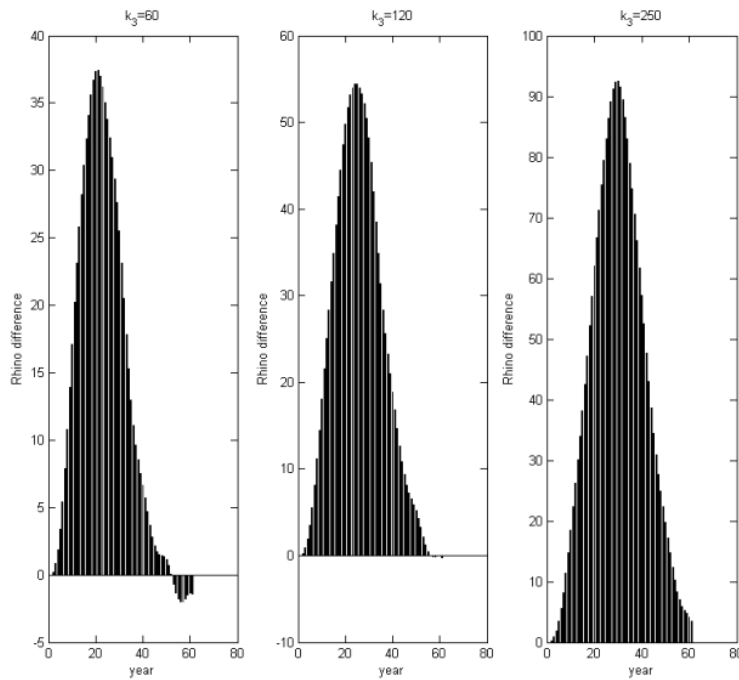
Figure 7 Difference of total rhino population when $k_3 = 60$ (left), $k_3 = 120$ (center) or $k_3 = 250$ (right).

The next simulations were generated with strategies using different translocation rates. The translocation rates differ from being constant to being a periodic discrete rate. Each strategy is implemented for 50 years. Figure 8 shows the periodic translocation rate strategies that were used.
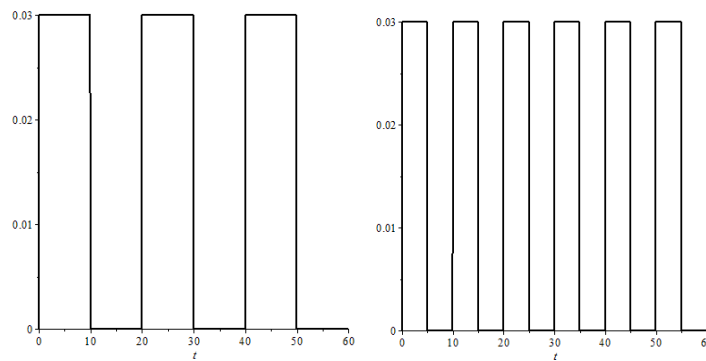

Figure 8 Periodic translocation rate strategies with a 10-year gap (left) and a 5-year gap (right).
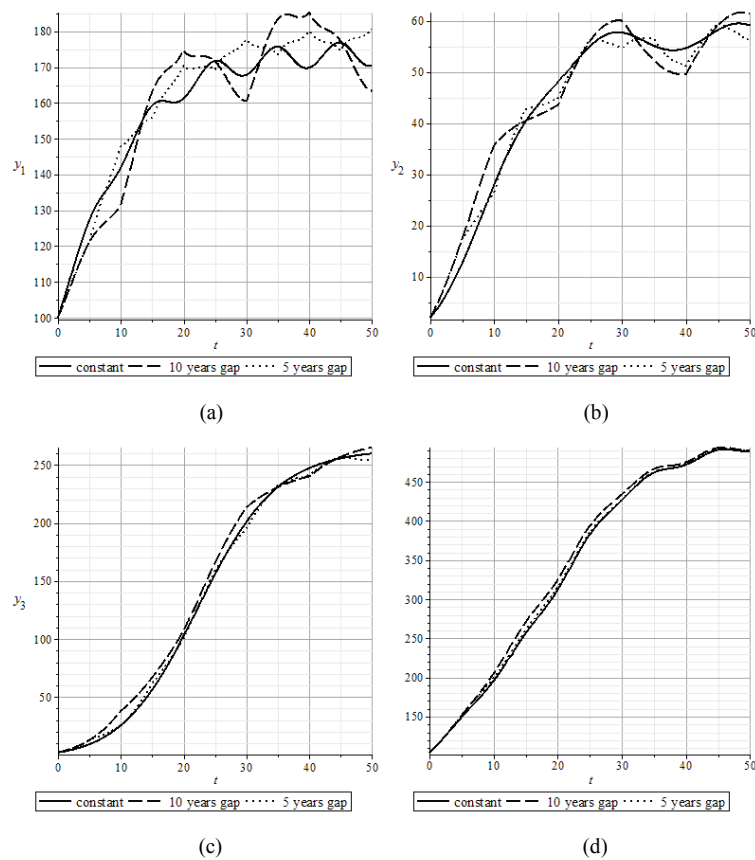
(a)

(b)

(c)

(d)

Figure 9 Dynamics of the black rhino populations with different translocation strategies. All parameter values are the same as used in Figure 2. The figures show the dynamics of $y_1(t), y_2(t), y_3(t)$ and the total rhino population.

It can be seen from Figure 9 that the gap of the translocation strategies does not play an important role. As long as the translocation rate fulfills the criteria in Table 1, the total population of black rhinos will still increase and reach equilibrium state E_3.

5 Conclusions

A deterministic mathematical model of the black rhino population with a translocation strategy has been developed in this paper. Translocation strategies involve relocating a fraction of one population to one or more other populations. The idea of translocation strategies is to optimize the growth rate of a population that is still far from its carrying capacity.

The total number of black rhinos will increase with the implementation of successful translocation strategies. The higher the carrying capacities of the receiver populations, the better the result. This is dependent on the government and private farm owners, who have to provide or choose a suitable environment so that the receiver populations are able to increase significantly in size.

Discrete translocation strategies are more suitable to describe the real physical situation than using a constant translocation rate. The reason for this lies within the recovery time given to a population to acclimatize to the changes implemented due to translocating rhinos. For instance, a rest period where no translocation is implemented is accounted for in this model.

Further development of the model would involve formulating the translocation strategies as an optimal control problem. It is important to see how the translocation strategies should be applied in an optimal way. This depends on the carrying capacities and also on the initial conditions of the populations.

Acknowledgments

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