INFRARED SURVEY OF GIANT M STARS

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ABSTRACT

A spectroscopic survey in an area of 5 square degrees, located in the general direction of the Palomar-Groningen Variable-Star Field 2, yields 263 giant M stars. A test of completeness shows that the survey was complete down visual magnitude 16 for early giant M stars and to visual magnitude 17 for late giant M stars. The space density distribution of the stars is presented.

ICHTISAR

Survey setjara spektroskopis bintang raksasa M, didaerah seluas 5 deradjat persegi, kearah Palomar-Groningen Variable-Star Field no. 2, menghasilkan 263 bintang² raksasa kelas M.

Sebuah test statistik menundjukkan bahwa survey tersebut lengkap sampai magnitude 16, untuk bintang M-awal, dan sampai magnitude 17, untuk bintang² M-tua.

Distribusi bintang² tersebut didalam ruang djuga dihitung dan dibitjarakan.

INTRODUCTION

An infrared spectroscopic technique for detecting giant M stars has proved to be very useful in the study of the galactic structure. This technique has made possible to search for faint giant M stars. There is, however, a problem which arises in such a survey, if it is carried out in the fields near the galactic plane. The problem which occurs in such a survey is how completely have the stars of a given type been discovered. The parameters which determine the completeness are the magnitudes of the stars themselves, the spectral types and the frequency per unit area of the spectral images on the spectroscopic plates. In crowded regions the chance of the detection of a particular type of spectrum, within a certain limiting magnitude, is less than in an uncrowded fields.

In order to test for completeness, a region of 1 square degree within Field 2 of the Palomar-Groningen Variable-Star Field (R. A, 1950 = $17^h07^m.5$; Dec. 1950 = — 19°24′) where the most crowding can be observed, is chosen as a sample field.

OBSERVATIONAL MATERIAL

The Schmidt-type telescope of the Warner and Swasey Observatory has been used to collect spectroscopic as well as the photometric plates. A 4°-objective prism was attached to the telescope in order to obtain the spectra of the stars. The prism yields a disperson of 1700 A/mm at the atmospheric A band.

Kodak 1-N plates were exposed behind a Wratten 89 or RG 8 filter to secure the spectral regions between 6800A-8800A. In almost all cases the photographic plates were ammonia sensitized in order to gain exposure-time by a factor of 4 (Sanduleak, 1961).

The visual and infrared magnitudes of the stars were determined by means of photographic photometry method. Combinations of

Kodak plate 103 a-D and filter GG 11,
Kodak plate 1-N and filter RG 8,

were used to obtain the respective visual and infrared magnitudes. The instrumental magnitudes were derived by means of a photoelectrically observed sequence stars in the area. These sequence stars have been observed on the UBV-photometry system of Johnson and Morgan (1953) by Hidajat and Wehinger (1968). The I-magnitude was obtained by employing the relationship of the instrinsic colors $(V\text{-}I)_0$ and $(B\text{-}V)_0$ given by Blanco (1964). The average B-V color excesses of the sequence stars were obtained from the absorption law in the region derived by Hidajat (1967).

Pascu (1964) and Blanco (1964) have shown that there exist a linear relation between the color excesses in B-V and V-I. Using this relation, for stars whose $(B\text{-}V)_0$ are less than 1.2, the desired apparent infrared magnitude can be obtained from

$$I = V - (V\text{-}I)_0 + 1.20E_{B-V}. \qquad (1)$$

The probable error associated with I can be estimated from

$$e_I^2 = e_V^2 + e^2_{(V-I)_0} + 1.44e^2_{E_{B-V}}, \qquad (2)$$

where the e's are the errors corresponding to the quantities denoted by the subscripts. The error in $(V\text{-}I)_0$ made in transforming $(B\text{-}V)_0$ to $(V\text{-}I)_0$ in Blanco's relationship has been estimated by Pascu (1964) and is $\pm$ 0.037. The error e_{B-V} is estimated as $\pm$ $0^m.10$. Inserting these values into Eq. (2) together with $e_V = \pm 0^m.030$ we obtain $e_I = \pm 0^m.13$ The relevant data of the sequence stars in the field will be published elsewhere by Hidajat and Wehinger (1968).

SPECTRAL CLASSIFICATION

The preliminary classification of the M stars was on the Case system by Nassau and van Albada (1949) and Nassau and Velghe (1964). In this system of classification the temperature criterium was established on the basis of the relative strength of the TiO bands at 7054A, 7589A and 8432A. These bands increase in strength as the temperature decreases. For late M stars, the VO band at 7900A (Keenan and Shroder, 1952) was used to differentiate the giant from the dwarf M stars, since this band has not been observed in dwarf M stars. Visual inspection of the spectral plates was the procedure of classifying the spectra.

The final classification of the M stars were given on the Mt. Wilson system described by Adams **et al** (1926). Blanco (1964) pointed out that there is a systematic difference between the two systems of classification. The transformation of one system to the other is posible by the use of the table provided by Blanco. In the present study the final classification was also made by direct comparison with the spectra of Mt. Wilson standards secured with the same instrumental system.

For comparison, the author's classification in the test area has been compared with the classification made by Blanco and Mawridis (unpublished). There is no systematic difference between the two classifications.

Spectroscopically, it was not possible to differentiate between early giant and dwarf M stars with small dispersion observation in the near infrared spectral regions. Our assumption that we are dealing with early giant M stars rests upon the following assumptions:

1. The giant M stars, besides being very luminous, are strongly concentrated in the galactic disk.
2. Because of the low luminosity the dwarf M stars cannot be observed at great distances.

The above considerations indicate that in the present survey of M stars, in low galactic latitude, down to the limiting magnitude of the present study, we would expect to find more giant than dwarf M stars. The expected number of dwarf M stars in the field covered here was estimated by assuming a uniform distribution of the dwarfs within the element of volume covered in the survey. Table I shows the limiting magnitudes reached in the present survey, the absolute infared magnitudes as given by Blanco (1963) and the space densities of the dwarf M stars in the solar neighborhood. These space densities were derived from the Gliese's Catalogue (1957) of stars nearer than 20 pc by Blanco (1963).

Table I

The expected number of dwars M stars, deduced from the given abs. magn., space density in the solar neighborhood and the limiting magnitudes of this study.

M Spec. Type	M_{IR}	D stars per $10^6 pc^3$	lim. magn.	r (pc)	N stars
2	8.1	4.0	12.2	66	0.56
3	8.4	7.0	1.23	60	0.76
4	9.2	10.0	12.4	44	0.42
5	10.5	12.0	12.6	26	0.11
6	12.5	14.0	12.9	13	0.01
				Total	1.87

The expected total number of dwarf M stars is obtained by summing the number of dwarfs in each spectral type (under the heading "N"). Thus, the total number of dwarf M stars is 1.87 stars, which is indeed exceeded considerably by the total number of early M stars actually found in the survey (224 stars). We deduce, therefore, that the early M stars found in this survey are likely to be giant M stars.

TEST OF COMPLETENESS

The search for M stars in the test area was tested for completeness. This area was chosen due to its apparent smoothness of the stellar surface distribution.

Van Gent (1933) (see also Plaut, 1965), devised a statistical method to test for completeness of a survey of variable stars. The method is modified here for our purpose of testing the completeness of the spectral survey. To achieve this, the identification of M stars was carried in the following way.

The M stars of a given type were searched for by scanning the spectral plates; for example: the first scan was for stars of class M2, the second for class M3 and so forth. Three plates, having approximately the same photographic qualities, were scanned independently in such a way. The number of repeated detection in the different plates can be used to estimate how completely were the stars of a given spectral discovered.

We now adapt the van Gent's method to the discovery of M stars on 3 plates. Let:

w be the probability that a particular type of stars can be detected in a scan of 1 plate. The value w was assumed to be the same for all plates when we deal with the same type of stars and plates of approximately the same quality.

N be the number of stars of particular type in the field.

Then the number of stars found 0, 1, 2. and 3 times when 3 plates are scanned can be written as :

$a_0 = N(1-w)^3$ number of stars missed,

$a_1 = N(1-w)^2 3w$ „ „ „ found once,

$a_2 = N(1-w)\ 3w^2$ „ „ „ „ twice,

$a_3 = Nw^3$ „ „ „ „ in all plates,

where $a_0 + a_1 + a_2 + a_3 = N.$ (3)

In the above equations the desired unknows are w and N. To determine these quantities the two equations can be combined as follows. The actual number of stars found in the survey of 3 plates is

$$A_1 = a_1 + a_2 + a_3 \qquad (4)$$

The average number of times, that each star was found is

$$G_1 = \frac{a_1 + 2.a_2 + 3.a_3}{A_1} \tag{5}$$

while the average number of times that any star, including the undiscovered ones, was found is

$$G_0 = \frac{0.a_0 + 1.a_1 + 2.a_2 + 3.a_3}{N} = 3w \tag{6}$$

From (5) and (6) G_1 can be computed is equal to

$$G_1 = \frac{3w}{1 - (1 - w)^3} \, . \tag{7}$$

where G_1 is computed from (5). Thus w can be obtained. The value of N is then

$$N = \frac{A_1 G_1}{3w} \, . \tag{8}$$

In such a survey the problem of duplication, that is the assigning in the different scans different spectral types to the same star, may be serious. We have minimized such a possible error by studying the stars in the natural subgroups with negligible classification error, except for M2-M4. Stars of type M1 may occasionally be included in this group.

Table II gives the results of the completeness study. Because of the duplication problem the results for M2-M4 group should be regarded as marginal. It can be seen from the table that the value of A_1, the number of stars found, and N, the number expected, are close to each other. This indicates that the survey is probably complete up to visual magnitude 16 for early M stars and to 17 for stars in the M7-M10 group.

THE STELLAR SURFACE DISTRIBUTION

A total of 263 giant M stars, in the region of 5 square degrees, have been discovered in the present survey. In order to know whether the stars are distributed uniformly over the searched area, we have divided the region into squares of 0.25 square degrees. The sides of these squares are paralel to the galactic coordinate sytem and are $0^\circ.5$ long.

As the region is relatively small and is free from partial interstellar obscuration (Plaut, 1959), we would expect to observe equal number of stars in each subareas. However, some low surface density areas can be observed. In general, the surface densities for squares above the galactic latitude $+11^\circ$ are comparable, whereas the densities in the lower subareas show rather large variation.

It is, therefore, interesting to investigate whether the distribution of stars in the areas above and below the +11° -line is different or not. We will denote the areas above and below the line respectively areas I and II.

Table II

Spec / m_v	M_2 — M_4 Found 1x	2x	3x		M_5 — M_6 Found 1x	2x	3x		M_7 — M_{10} Found 1x	2x	3x		Total
	1	2	7	$A_1 = 10$	1								
1200				$g_1 = 2.600$									
				w = .866									
				N = 10.0			1				1		10.0
	3	2	4	$A_1 = 9$									
12.01-13.00				$g_1 = 2.286$									
				w = .681									
				N = 9.3									9.3
	2	6	6	$A_1 = 14$			1				1		
13.01-14.00				$g_1 = 2.286$									
				w = .749									
				N = 14.2									16.2
	2	11	22	$A_1 = 45$	0	4	8	$A_1 = 12$	0	2	2	$A_1 = 4$	
14.01-15.00				$g_1 = 2.00$				$g_1 = 2.666$				$g_1 = 2.500$	
				w = .63				w = .88				w = .830	
				N = 48.0				N = 12.1				N = 4.0	64.1
	3	2	5	$A_1 = 10$	2	7	12	$A_1 = 21$	0	0	4	$A_1 = 4$	
15.01-16.00				$g_1 = 2.20$				$g_1 = 2.476$				$g_1 = 3$	
				w = .73				w = .82				w = .998	
				N = 10				N = 21.0				N = 4.0	35.0
	2	2	6	$A_1 = 10$	1	4	10	$A_1 = 15$	1	2	6	$A_1 = 9$	
16.01-17.00				$g_1 = 2.400$				$g_1 = 2.600$				$g_1 = 2.555$	
				w = .794				w = .866				w = .850	
				N = 11.1				N = 16.1				N = 9.0	36.2
17.01	1	1		$A_1 = 2$	1	1		$A_1 = 2$	1	2	1	$A_1 = 4$	
				$g_1 = 1.50$				$g_1 = 1.50$				$g_1 = 2.000$	
				w = .38				w = .38				w = .633	
				N = 3.0				N = .42				N = 4.2	10.2
													181.0

The average densities for these two areas are 13.3 stars per unit area (i.e. per 0.25 square degrees) and 13.0 stars per unit area, respectively. Assuming that the stars in the two areas have random distribution (from two different parent populations) and that the stars have been detected independently, the estimated variance of the stellar surface distribution in each area can be computed from

$$s^2 = \frac{1}{n-1} \left\{ \Sigma \, (x_i - \bar{x})^2 \right\} \tag{9}$$

where n = number of areas (which is 10 in each cases),

x_i = surface stellar density in each area,

$\bar{x}$ = average surface stellar density in the area above or below the $+11^{\circ}$-line.

The test statistic is to show whether the variances observed in the areas I and II are different or not, that is whether $S_I^2 = S_{II}^2$ is true. Here S_I and S_{II} are the variances of the stellar population in areas I and II respectively. The "F-test" (see for example: Brownlee, 1960),

$$F = \frac{s_I^2}{s_{II}^2}$$

can be performed in order to accept or to reject the hypotheses of the equality of the variances. The hypotheses will be rejected if

$$F > F_{a/2,\ f_I,\ f_{II}} \tag{10}$$

where a is the level of significance; a/2 simply denotes that are performing an equal-tail test,

f_I and f_{II} are the degree of freedom for areas I and II respectively. In both cases they are equal to 9.

The value of the right hand side of Eq. (10) is read from a table of percentage points of the "F-distribution" (Brownlee, 1960, page 550).

In practice, it would not be possible to select an arbitrary value of "a". It is best to be done if we would decide at what level of significance "a" that our hypothese can be accepted.

Using Eq. (9), we found s_I^2 and s_{II}^2 are 14.9 and 56.6, respectively. These values yield $F = s_{II}^2 / s_I^2$ equal to 3.79. The tabulated values, for the significance level of 5% and 10% are 4.03 and 3.18 respectively.

If we were to choose the 5% level of significance then we cannot reject the hypotheses that the variance in II is the same with the variance in I. However, at 10% level of significance the computed value of F is greater than the tabulated value. Thus, at this level of significance the test shows that we cannot accept the hypotheses of the equality of variances. In this case, the larger variability which is observed in area II is probably caused by grouping of giant M stars. This grouping would make some areas more densely populated than other areas.

The "t-test" can be used to show whether the average surface densities in I and II are comparable or not. Let d be the difference between the surface densities of the populations in areas I and II, the "t-test" can be written as

$$t = \frac{\bar{x}_I - \bar{x}_{II} - d}{s_o\,(1/n_I + 1/n_{II})^{2}} \tag{11}$$

where d, in this case, is equal to zero. $\bar{x}_I - \bar{x}_{II} = 0.3$ and

$$s_o = \frac{\sum\limits_{i=1}^{10} (x_{iI} - \bar{x}_I)^2 + \sum\limits_{i=1}^{10} (x_{iII} + \bar{x}_{II})^2}{(n_I + n_{II} - 2)} = 35.78.$$

Inserting these values into Eq. (11) we obtain $t = 0.209$.

Using the 5% level of significance the computed t is found to be less than $t_{0.025,18}$, which is equal to 2.101. Thus, at this level of significance the difference of the average surface densities in I and II is not significant. We may, therefore, conclude that the distribution of giant M stars in areas I and II are essentially the same.

THE DENSITY ANALYSIS

The fundamental equation of stellar statistics is

$$A(m)\,dm = w.dM \int_0^\infty r^2.\,D(r).\,L\,(m + 5 - 5\log r)\,dr \tag{12}$$

where A(m) dm = number of stars observed in solid angle w. having magnitude between m — ½ dm and m + ½ dm.

D(r) = number of stars per unit volume at a distance r from the sun.

L(M) dM = luminosity function or the fraction of stars in a unit volume having absolute magnitude between M — ½ dM and M + ½ dM.

Schwarzschild (see Bok, 1937) has shown that Eq. (12) can be integrated directly to yield the analytical relation between A(m) and the apparent magnitude provided D(r) is of the form

$$D(r) = \exp \{ H + K \log r + L \; (\log r)^2 \} \qquad (13)$$

where H, K and L are constants, and

$$L(M) = \frac{1}{\sigma\sqrt{2\pi}} \exp. \frac{1}{2\sigma^2} (M - M_0)^2 \qquad (14)$$

Here M_0 and σ are the absolute magnitude and dispersion, respectively. Then the resulting equation is

$$\log A(m) = a + bm + cm^2 \qquad (15)$$

where A(m) is the number of stars per square degree having magnitude between m— ½ dm and m+ ½ dm. The relation between A(m) and m, obtained in the present study is shown in Table III.

Table III

The number of stars in 0.5 magnitude intervals per 5 square degrees.

m_v	M2-M4	M5-M6.5	M7-M10
9.5	0.6	—	—
10.0	1.8	—	—
10.5	4.0	—	—
11.0	7.0	—	—
11.5	9.3	1.1	—
12.0	12.7	1.4	—
12.5	16.5	3.0	0.2
13.0	26.1	6.4	1.0
13.5	42.5	5.3	2.3
14.0	20.0	7.0	3.8
14.5	24.0	8.0	6.3
15.0	7.0	10.7	9.2
15.5	—	12.9	16.9

In practice the constants a, b and c in Eq. (15) are obtained by fitting a quadratic curve to the observed log A(m)-m distribution. Further we proceed to solve Eq. (13) to obtain the density distribution.

The density distribution obtained this way must be corrected for interstellar absorption. The relation between the fictitous and true densities is

$$D(r) = D(r_0) \; 10^{0.6\, a(r)} \left\{ 1 + \frac{r \; a'(r)}{5 \text{ Mod}} \right\} \qquad (16)$$

where $D(r_0)$ = the fictitous density at distance r_0

$a(r)$ = the absorption up to the point r, in magnitude,

$a'(r)$ = the derivatives of the absorption function at point r, in magnitude per parsec.

The relation between r and r_0 is

$$r = r_0^{-0.2\ A_v}$$

where A_v is the total visual absorption, which is adopted as 3.2 E_{B-V}. The color-excess-distance relation is taken from the results given by Hidajat (1967) and Hidajat and Wehinger (1968).

Table III shows that there is a drop in frequency at magnitudes 14.5 for early M stars (M2-M4). The drop is believed to be real and is a reflection of the scarcity of the early M stars beyond apparent magnitude 14.5 In the other regions of the frequency distribution curve the run is smooth.

The analytical solution outlined above was carried out by a computer program kindly communicated by Dr. N.B. Sanwal (unpublished). In the calculation of the density distibution we have adopted the absolute magnitudes and dispersions given by Blanco (1965), after having been averaged on the basis of the apparent magnitude frequency distribution found in this study. Table IV shows these adopted quantities.

Table IV

The adopted absolute magnitudes and dispersions of giant M stars

Group	M_v	σ
M2-M4	—1.0	± 0.7
M5-M10	—0.9	± 0.7

For the purpose of comparison the density of the stars later than M5 was computed by using the absolute magnitude differences of $\pm\ 0.^m5$ from the adopted value. It was found that the absolute magnitude effect is large at distances of r less than 2.0 kpc. In these regions the densities are halved or doubled by changes of $0.^m5$ in M_0. Beyond 2 kpc the effect of absolute magnitude becomes smaller and gives variation in densities of about 25% — 30%.

The results of the analytical solution was checked by means of a numerical method such that described by Schalen (1928). In this method a modification is made such that the stars of a given apparent magnitude are assumed to have a Gaussian disribution of absolute magnitude

$$L_m(M) = \frac{1}{\sigma_m \sqrt{2\pi}} \exp. \frac{1}{2\,\sigma_m^2} (M - M_m)^2 \tag{17}$$

where
$$\overline{M}_m = M_o - \frac{\sigma_o^2}{\log e} \cdot \frac{d \log A(m)}{dm}$$

$$\sigma_m^2 = \sigma_0^2 + \frac{\sigma_0^4}{\log e} \cdot \frac{d^2 \log A(m)}{dm^2}$$

Here $\overline{M}_m$ = the mean absolute magnitude for stars of apparent magnitude m.

M_o = mean absolute magnitude per unit volume

σ_m = dispersion in absolute magnitude for stars selected by volume. Since the second derivative of the log A(m)-m distribution is a small quantity σ_m was always taken as σ_o.

We have computed the number of stars found between $M \pm \frac{1}{2} dM$ as follows. Let $\triangle = dM$. It is convenient to let $\triangle$ also equal to dm, the apparent magnitude interval within which the stars counts are made. Then the number of stars found in the interval of $\overline{M}_m \pm \triangle$ is

$$N_m\left(\frac{\triangle}{2}\right) = A_m \frac{\triangle}{\sigma_m \sqrt{2\pi}} \exp. \left\{ - \frac{1}{2\,\sigma_m^2} \left(\frac{\triangle}{2}\right)^2 \right\}$$

In general, the number of stars within the intervals
$M_m + (n - 1) \triangle$ to $M_m + n \triangle$ and $M_m - (n - 1) \triangle$ to $M_m - n \triangle$ is

$$N_m\left(\frac{n \triangle}{2}\right) = A_m \frac{\triangle}{\sigma_m \sqrt{2\pi}} \exp. \left\{ - \frac{1}{2\,\sigma_m^2} \left(\frac{n \triangle}{2}\right)^2 \right\}$$

or
$$= A_m \, f(\triangle, \sigma_m), \tag{18}$$

where n = 1, 3, 5, 7, ...

Table V shows the values of $f(\triangle, \sigma_m)$ for $\triangle = 0^m\!.5$ and $\sigma_m = 0^m\!.7$, corresponding to the values which we used in the analysis of giant M stars.

Table V

The adopted values of f $(\triangle, \sigma_m)$

for $\sigma_m = 0^m\!.7$

n	f
1	0.261
3	0.160
5	0.060
7	0.014
9	0.002
11	0.000

Then an $(m-\overline{M})$-table can be constructed. The entry in each cell of the table is the value found in Eq. (18). The $(m,\overline{M})$-table can readily be transformed into an (m,r)-table, since the entry in each cell of the $(m,\overline{M})$-table is the number of stars within the distances r_i and r_{i+1}, where the values of r's are computed from $5 \log^r 10 = m\text{-}M$. The total number of stars between the distances r_i and r_{i+1} is the sum of the entries over the magnitudes. Let the volume bounded by the shells of radii r_i and r_{i+1} be V, then the space density of stars at the distance

$\frac{r_i + r_{i+1}}{2}$ is computed from $D = \dfrac{N_m\left(n\dfrac{\Delta}{2}\right)}{V}$.

In the present study we have taken the averages of $\frac{d \log A'(m)}{dm}$ since the slope of the frequency distribution does not change very much within the magnitudes limits. Also we have taken σ_m always equal to the dispersions tabulated in Table IV. This is justified due to the fact that the second derivative of the frequency distribution is found to be nearly zero in the whole range of magnitude covered by the present survey.

RESULTS AND DISCUSSION

The results of the two methods of solving the fundamental equation of the stellar statistics are shown in Table VI.

Table VI

Space density of giant M stars

(stars per 10^6 pc^3)

r	M2 — M4		M5 — M10	
(pc)	(1)	(2)	(1)	(2)
750	—	3.0	—	—
1000	5.8	8.0	(0.5)	(0.5)
1500	6.1	5.8	(0.6)	(0.5)
2000	4.4	3.8	0.5	0.5
2500	3.6	2.8	0.4	0.5
3000	2.7	2.3	0.4	0.5
3500	1.7	1.1	0.3	0.3
5000	1.0	—	0.2	0.2
6000	0.7	—	0.2	—
8000	0.3	—	0.1	—

Note: (1) result of analytical solution
(2) ,, numerical ,,

The results presented in Table VI suggests that there is no systematic difference between the mehods of solution. This result supports Crowder's study (1959) in which he has shown that the Schwarzschild density analysis does not differ from the solution by means of the m-log π method (Bok, 1937) if the run of log A(m) is smooth. The latter method does not employ any assumed analytic relation between log A(m) and the apparent magnitude, and is free from a "smoothing" process in star counts.

Care must be taken regarding the distance within which the results of the analysis can be considered reliable. These distance limits can be computed from the distances of the brightest and faintest stars observed in the survey. The parentheses in Table VI denote the values which should be considered as having lower weight, due to the above mentioned limitation and the scarcity of the stars at the brighter and fainter ends of the sequence.

Taking the results at face value we note that there is a high concentration of M2-M4 stars at the distance of approximately 1.5 kpc from the sun. Albers (1962) pointed out the concentration of M stars in the Saggitarius arm. The line of sight of the present study is probably crossing the extension of the arm observed by Albers. The density found in the present study is, however, lower than the stellar density found in Albers' study. This fact may be caused by the location of the field, which is at higher galactic latitude compared to the Alber's field.

The run of the stellar density distribution for M5-M10 group decreases systematically with the distance from the sun. The results tend to confirm an earlier finding that the late M stars are disk population (Nassau, 1958).

A detailed discussion of the results of the present study will be outlined, in term of the general galactic structure, in a subsequent paper together with results obtained in other fields (Hidajat and Blanco, 1968).

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