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1D Numerical Scheme of Dambreak Flows Over Moveable Bed Using FTCS Scheme

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Abstrak

This paper will try to simulate the modeling of sediment transport phenomena. We use a Numerical scheme for the classical Saint-Venant – Exner model. It is based on a relaxation approach and works with all sediment flux functions. The governing equation will be divided into Hydrodynamic and Morphodynamic. Morphodynamics will be using the classical Exner approach and Hydrodynamic will be using FTCS numeric scheme. This paper will validate the simulation with the previous research. From the simulation results, it can be seen that a simple numeric method such as FTCS is capable of simulating scour due to dam break flow. Several conditions must be considered to carry out this simulation, namely the purchase of the Courant Number value.

Keywords : Dambreak, FTCS, morphodynamic.

Abstract

Makalah ini akan mencoba mensimulasikan pemodelan fenomena transpor sedimen. Kami menggunakan skema numerik untuk model klasik Saint-Venant - Exner. Skema ini didasarkan pada pendekatan relaksasi dan dapat digunakan untuk semua fungsi fluks sedimen. Persamaan yang mengatur akan dibagi menjadi Hidrodinamika dan Morfodinamika. Morfodinamika akan menggunakan pendekatan Exner klasik dan Hidrodinamika akan menggunakan skema numerik FTCS. Makalah ini akan memvalidasi hasil simulasi dengan penelitian sebelumnya. Dari hasil simulasi dapat dilihat bahwa metode numerik sederhana seperti FTCS mampu mensimulasikan gerusan akibat aliran jebol bendungan. Beberapa syarat yang harus diperhatikan untuk melakukan simulasi ini, yaitu pembelian nilai Courant Number.

Kata-kata kunci : Dambreak, FTCS, morfodinamik.

1. Introduction

Dam-break flow will cause erosion and sediment transport, where the damage caused by this erosion and sediment transport may be greater than the

damage caused by the water flood itself (Spinewine, B. And Zech, Y., 2007). Erosion and transport of these sediments will also result in morphological changes which in turn can modify the flow (Qian, H., Cao, Z., Liu, H., dan Pender, G., 2018). Study of dam-break flow

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on moving base layers is needed to plan mitigation in the event of dam failure. Several laboratory experiments of dam-break flow on mobile bed layers have been carried out, eg Spinewine & Zech (2007), Bahmanpouri et al. (2013), also Qian et al. (2018). However, laboratory testing has the disadvantage that it takes a long time and costs a lot, so numerical modeling is needed to overcome these drawbacks (Zendrato, N. L. H., Harlan, D., Bagus Adityawan, M., dan Kardana Natakusumah, D., 2019). Several numerical modeling for dam-break flow over moving layers have been carried out, one of them by Harlan et al. (2019). In research conducted by Jiaheng Zhao, et al. (2018) It can be seen that in order to simulate dambreak flow and its effect on changes in channel bottom, the St. Venant-Exner. The approach taken in this study is the different volume approach. In this study, research development will be carried out using a 1D explicit finite difference numerical approach. In this study also applied a numerical filter, namely the Hansen (Mellivera, A., et al., 2020) filter to reduce the oscillations that occur in the model. In this study, a simulation will be carried out using a simple method but is able to provide simulation results that are close to the test results. The method to be used in this study is the 1D explicit FTCs finite difference method

2. Method

2.1. Hydrodynamic

The equation that will be used to calculate hydrodynamics in this study is the Saint Venant Equation, where this equation calculates continuity and momentum.

2.1.1. Continuity

The law of conservation of mass leads to the derivative form of the continuity equation. Let's say the components of velocity in the x, y, and z directions are u, v, w. Examine the fluid through the paralepipedum elements which have dimensions dx, dy, and dz. The mass of fluid passing through the surface of this element in unit time is the density of the fluid multiplied by the cross-sectional area of the surface times its normal velocity to the surface.

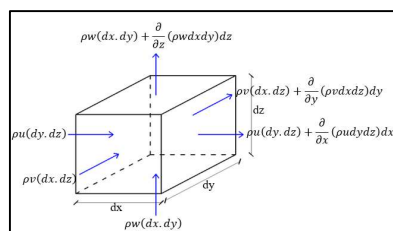


Figure 1. Continuity law mass change scheme

1 Dimensional Continuity Equation:

$$\frac{\partial A}{\partial t} + \frac{\partial Q_x}{\partial x} = 0 \quad (1)$$

2.1.2. Momentum

Newton's second law states that the net force acting on a given mass is proportional to the rate of change of the mass's linear momentum with respect to time. This law can be applied to fluids with a control volume reduced to the size of a point to obtain the equations of motion in differential form.

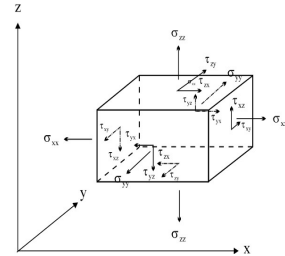


Figure 2. Newton's second law scheme (momentum)

Momentum for 1 dimension:

$$\frac{\partial Q_x}{\partial t} + \frac{\partial Q_x^2}{\partial x} + gA \frac{\partial h}{\partial x} - gA(So_x - Sf_x) = 0 \quad (2)$$

2.2. Morphodynamic

The Exner equation of the conservation of sediment mass is the basis of morphodynamic analysis. The Exner equation states that the rate of change of river bed sediment elevation z can be related to the water velocity u. This equation also describes the mass balance of sediments by suggesting that the local rate of change in sediment uplift is related to the spatial gradient of the release of solids. (Qihua Ran, 2015)

2.3. Governing equation

The phenomena of sediment flow and transport can be explained and described by mathematical equations. The mathematical equation that is commonly used is the two-dimensional shallow water equation, where this equation consists of hydrodynamic equations and sediment transport. Sketches of shallow waters can be seen in Figure 3.

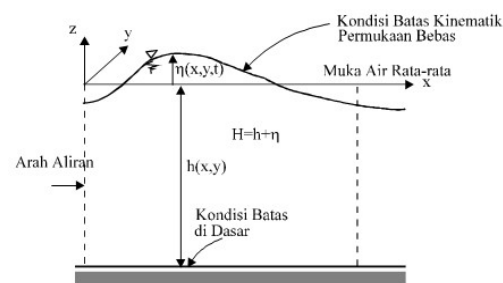


Figure 3. Shallow water sketch (Harlan, 1998)

2.3.1. St. Venant equation

2.3.1.1. Continuity

$$\frac{\partial A}{\partial t} + \frac{\partial Q_x}{\partial x} = 0 \quad (3)$$

2.3.1.2. Momentum

$$\frac{\partial Q_x}{\partial t} + \frac{\partial Q_x^2}{\partial x} + gA \frac{\partial h}{\partial x} - gA(So_x - Sf_x) = 0 \quad (4)$$

In the research conducted by Audusse E and Chalons C. (2012) on Sediment Transport Modeling: Three Layer Models, it can be seen that to simulate dambreak flow and its effect on channel bed changes, the St. Venant-Exner. The approach used in this research is a different volume numerical approach. In this study, research development will be carried out using a different to explicit 1D numerical approach.

$$\frac{\partial zb}{\partial t} + \varepsilon \frac{\partial Q_s}{\partial x} = 0 \quad (5)$$

$$\frac{\partial zb}{\partial t} + (1-p)^{-1} \frac{\partial Q_s}{\partial x} = 0 \quad (6)$$

$$\frac{zb_i^{t+1} - zb_i^t}{dt} + (1-p)^{-1} \frac{Q_{s,i+1}^t - Q_{s,i-1}^t}{2 dx} = 0 \quad (7)$$

$$\frac{zb_i^{t+1}}{dt} = \frac{zb_i^t}{dt} + (1-p)^{-1} \frac{Q_{s,i+1}^t - Q_{s,i-1}^t}{2 dx} = 0 \quad (8)$$

$$Zb_i^{t+1} = Zb_i^t - \frac{dt}{2dx} (1-p)^{-1} (Q_{s,i+1}^t - Q_{s,i-1}^t) \quad (9)$$

where zb is the base elevation, Qs is the base load, p is the porosity of the gravel base. Qs is searched using the Meyer-Peter and Muller methods given directions.

2.3.2. Meyer-Peter and Muller

The most commonly used empirical equation is the equation due to Meyer-Peter and Muller, They describe a fairly simple and fairly accurate formula for basic load shedding. They developed a basic load equation based on experiments with uniform sand particles and natural gravel and mix sizes. Since AHDR, analysis of samples of basic materials showed that the grain size distribution was 30% fine gravel and sand only, 40% fine sand 30% silt and clay (Makary A, 1982). Therefore the Mayer Peter equation can be applied to AHDR Aziz T.M.A (2017). Their equation in the flow of water takes the form

$$Qb = (d50 \left(g d50 \left(\frac{\rho_s}{\rho_w} \right) - 1 \right)^{0,5} 8 \dots \quad (10)$$

$$(\tau^* - \tau^*c)^{1,5} \quad (11)$$

$$\tau_o = \rho_s g R Sf \quad (12)$$

$$\tau^*c = 0,047 \quad (12)$$

$$\tau^* = \frac{\tau_o}{\rho g d50 \left(\frac{\rho_s}{\rho_w} - 1 \right)} = \frac{\rho_s g R Sf}{\rho g d50 \left(\frac{\rho_s}{\rho_w} - 1 \right)} = \frac{R Sf}{d50 \left(\frac{\rho_s}{\rho_w} - 1 \right)} \quad (13)$$

So,

$$Qb = (d50 \left(g d50 \left(\frac{\rho_s}{\rho_w} \right) - 1 \right)^{0,5} 8 \left(\frac{R Sf}{d50 \left(\frac{\rho_s}{\rho_w} - 1 \right)} - 0,047 \right)^{1,5} \quad (14)$$

The MPM equation is still an empirical equation that

w2is capable of calculating at one distance unit and time unit so it needs to be given a direction according to the number of nodes in the domain. This direction includes values at t+1 and t-1. So if:

For $Q_i^{t+1} > 0$:

$$u_i^{t+1} = \frac{Q_i^{t+1}}{A_i^{t+1}} \quad (15)$$

$$R_i^{t+1} = \frac{A_i^{t+1}}{(B+2h_i^{t+1})} \quad (16)$$

$$Sf_i^{t+1} = \frac{n^2 (u_i^{t+1})^2}{(R_i^{t+1})^3} \quad (17)$$

For $Q_i^{t-1} > 0$

$$u_i^{t-1} = \frac{Q_i^{t-1}}{A_i^{t-1}} \quad (18)$$

$$R_i^{t-1} = \frac{A_i^{t-1}}{(B+2h_i^{t-1})} \quad (19)$$

$$Sf_i^{t-1} = \frac{n^2 (u_i^{t-1})^2}{(R_i^{t-1})^3} \quad (20)$$

The Exner equation become:

For t+1

$$Qb1 = (d50 \left(g d50 \left(\frac{\rho_s}{\rho_w} \right) - 1 \right)^{0,5} 8 \left(\frac{\frac{A_i^{t+1}}{(B+2h_i^{t+1})} \frac{n^2 (u_i^{t+1})^2}{(R_i^{t+1})^3}}{d50 \left(\frac{\rho_s}{\rho_w} - 1 \right)} - 0,047 \right)^{1,5} \quad (21)$$

For t-1

$$Qb1 = (d50 \left(g d50 \left(\frac{\rho_s}{\rho_w} \right) - 1 \right)^{0,5} 8 \left(\frac{\frac{A_i^{t-1}}{(B+2h_i^{t-1})} \frac{n^2 (u_i^{t-1})^2}{(R_i^{t-1})^3}}{d50 \left(\frac{\rho_s}{\rho_w} - 1 \right)} - 0,047 \right)^{1,5} \quad (22)$$

$$\Delta Qb = Qb1 - Qb1 \quad (23)$$

$$Zb_i^{t+1} = Zb_i^t - \frac{dt}{2dx} (1-p)^{-1} \Delta Qb \quad (24)$$

2.4. FTCS numerical scheme

The finite difference method used to solve a partial differential equation, in this case is the St.Venant equation. The acronym FTCS was first used by Patrick Roache. According to this scheme, the time derivative is approximated by a forward difference first order, while the space derivative is approximated by a central difference.

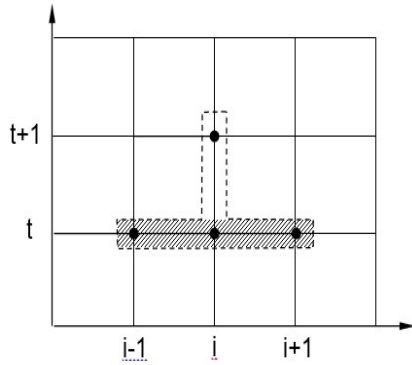


Figure 4. FTCS Numerical Scheme

2.5. Previous research

Transport Model With Improved Discretization Of Flux And Source Terms, Journal Of Hydrology, The domain is 2.5 m long and 0.1 m wide. A dam is installed at a height of 1.25 m. The upstream water depth is initially $h_0 = 0.1$ m, and with a dry bottom downstream, four boundaries are set to be solid limits, where there will be hydraulic jumps near the dam site during the flow process. A sediment layer with a constant thickness of about 5.6 cm is located within the boundary domain, the sediment diameter is reported to be $d = 0.0035$ m, and the density is $s = 1540$ kg/m³, bed porosity is $p = 0.3$, Manning coefficient $n = 0.025$ s m^{-1/3}, angle of repose $r = 30^\circ$, and erosion calibration parameter = 2.4. The domain is discretized with 1710 triangular cells, the whole experiment runs for 2 seconds. The following is the result of the test.

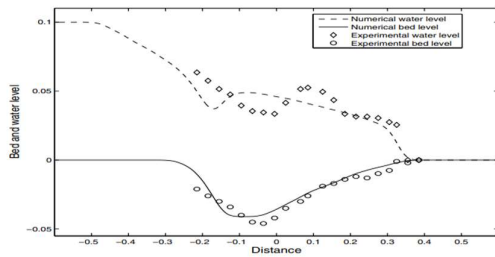


Figure 5. Simulation result in previous research

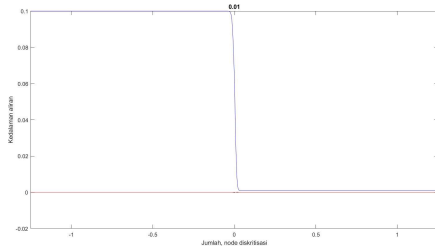


Figure 6. Initial condition

3. Result

3.1. Discretization of equations

The form of the discrete St. Venant equation which is solved by FTCS is as follows:

3.1.1. Continuity

$$\frac{\partial A}{\partial t} + \frac{\partial Q_x}{\partial x} = 0 \quad (25)$$

$$\frac{A_i^{t+1} - A_i^t}{\Delta t} + \frac{Q_{i+1}^t - Q_{i-1}^t}{2 \Delta x} = 0 \quad (26)$$

$$A_i^{t+1} = A_i^t - \frac{\Delta t}{2 \Delta x} (Q_{i+1}^t - Q_{i-1}^t) \quad (27)$$

3.1.2. Momentum

$$\frac{\partial Q_x}{\partial t} + \frac{\partial Q_x^2}{\partial x} + g A \frac{\partial h}{\partial x} - g A (S_{0x} - S_{fx}) = 0$$

$$\frac{Q_i^{t+1} - Q_i^t}{\Delta t} + \frac{(Q_{i+1}^t)^2 / A_{i+1}^t - (Q_{i-1}^t)^2 / A_{i-1}^t}{2 \Delta x} +$$

$$g A_i \frac{h_{i+1}^t - h_{i-1}^t}{2 \Delta x} - g A_i (S_{0x} - S_{fx}) = 0 \quad (28)$$

$$Q_i^{t+1} = Q_i^t - \frac{\Delta t}{2 \Delta x} \left(\frac{(Q_{i+1}^t)^2}{A_{i+1}^t} - \frac{(Q_{i-1}^t)^2}{A_{i-1}^t} \right) -$$

$$g A_i \frac{\Delta t}{2 \Delta x} (h_{i+1}^t - h_{i-1}^t) + g A_i \Delta t (S_{0x} - S_{fx}) \quad (29)$$

$$\text{where } \rightarrow S_{fx} = \frac{n^2 (u_i^t)^2}{(R_i^t)^{4/3}} = \frac{n^2 (Q_i^t)^2}{(A_i^t)^{4/3}} \quad (30)$$

3.1.2. Exner

$$\frac{\partial z}{\partial t} + \xi \frac{\partial Q_b}{\partial x} = 0 \quad (31)$$

$$\frac{z_i^{t+1} - z_i^t}{\Delta t} + (1-p)^{-1} \frac{Q_{b,i+1}^t - Q_{b,i-1}^t}{2 \Delta x} = 0$$

$$z_i^{t+1} = z_i^t - \frac{\Delta t}{2 \Delta x} (1-p)^{-1} (Q_{b,i+1}^t - Q_{b,i-1}^t) \quad (32)$$

With Q_b calculated by the MPM approach,

$$Q_b = (d50 (g d50 (\frac{\rho_s}{\rho_w}) - 1)^{0.5})^8 \left(\frac{R S_f}{d50 ((\frac{\rho_s}{\rho_w}) - 1)} \right) \dots - 0.047)^{1.5} \quad (33)$$

3.2. Simulations

The simulation results with different Courant Number are shown in Figure 7 to Figure 10.

From the simulation, it can be seen that the maximum Courant Number value that can be simulated on the FTCS numerical scheme is 0.1245 or at Δx 0.005 meters and Δt 0.0005 seconds.

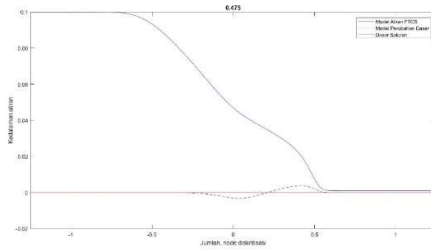


Figure 7. FTCS Coeff. 0,99 with dt0,00001

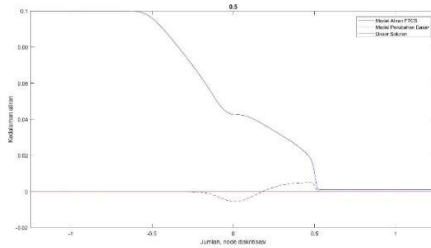


Figure 8. FTCS Coeff. 0,99 with dt0,00005

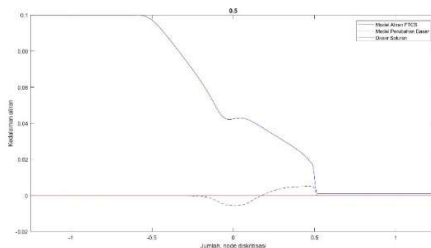


Figure 9. FTCS Coeff. 0,99 with dt0,0001

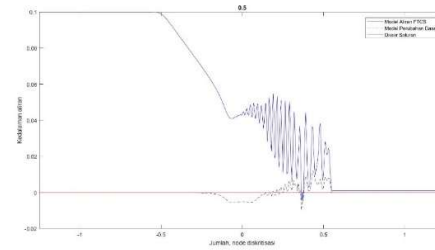


Figure 10. FTCS Coeff. 0,99 with dt0,0005

4. Discussion

The simulation results will be validated with the previous study to examine the value of RMSE.

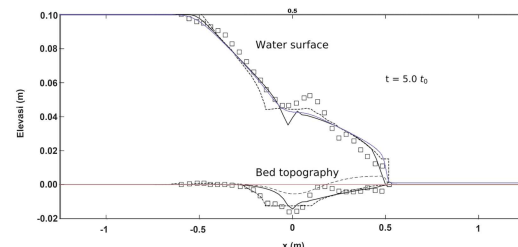


Figure 11. FTCS simulation results with previous research

From the simulation results it can be seen that a simple numerical scheme such as FTCS is capable of simulating scour due to dam break flow. Several conditions must be considered to carry out this simulation, namely the purchase of the Courant Number value. FTCS is recommended to use with CN in the range of 0.02 until 0.05 to give good results. Because after that there will be oscillations in the simulation. Numerical filters are also capable of keeping simulations of oscillations especially for schemes that require order-1.

Table 1. RMSE value of hydrodynamics and morphodynamics

No	dt	Courant Number	RMSE Hydrodynamic	RMSE Morfodynamic
1	0.00001	0.0015	0.0018	0.0043
2	0.000025	0.004	0.0046	0.0042
3	0.00005	0.0081	0.0094	0.0041
4	0.000075	0.0122	0.0144	0.0041
5	0.0001	0.0162	0.0194	0.0041
6	0.00025	0.0423	0.0498	0.0041
7	0.0005	0.1245	0.1033	0.0041

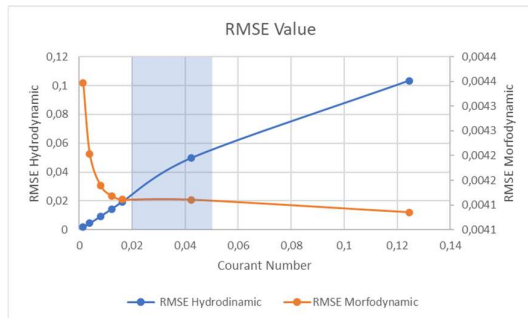


Figure 12. FTCS simulation results with previous research

5. Conclusion

This study demonstrates that a simple numerical scheme like FTCS (Forward Time Central Space) is capable of effectively simulating scour phenomena caused by dambreak flows. The model employs the Saint-Venant–Exner approach, where hydrodynamics are solved using the FTCS scheme and morphodynamics are based on the classical Exner equation. The simulation results were validated against previous studies and showed that the method can adequately represent the behavior of flow and channel bed changes, especially when the Courant Number (CN) is maintained within a specific range (recommended between 0.02 and 0.05). Higher CN values tend to cause oscillations in the simulation output. To maintain stability and accuracy, numerical filters such as the Hansen filter were applied to reduce oscillations.

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