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Parameters Optimization of Tuned Mass Damper Using Fast Multi Swarm Optimization

Richard Frans

Department of Civil Engineering, Atma Jaya Makassar, Makassar, Indonesia
E-mail: richardfrans.rf@gmail.com

Yoyong Arfiadi

Department of Civil Engineering, Atma Jaya Yogyakarta, Yogyakarta, Indonesia

Abstract

Tuned mass damper (TMD) has been used for vibration controller of building, especially for high rise building. TMD is one of passive device for reducing response of the structure which subjected to dynamic external disturbance such as wind, or earthquake load. TMD used its weight for reducing the vibration, TMD's frequency has been set with structure's frequency so that the frequency can resonate each other for reducing the response of the structure during the dynamic load. Therefore, three variables have significant effect for TMD performance which are mass ratio of TMD (μ), frequency ratio of TMD (α), and damping ratio of TMD (ζ) which lead to two important variables of TMD properties which are stiffness and damping of TMD. This paper developed an empirical equation for obtaining the optimum parameters of tuned mass damper based on H_2 norm control system and fast multi swarm optimization (FMSO). The objective function was to minimize the acceleration and displacement response of the structure. Accelerogram of El-Centro 1940 NS was chosen to be ground motion acceleration for simulating the response of the structure with and without TMD. The result shows a strong correlation both mass ratio of TMD to frequency ratio of TMD and mass ratio of TMD to damping ratio of TMD.

Keywords: *Tuned mass damper, parameters optimization, fast multi swarm optimization.*

Abstrak

Penggunaan peredam massa selaras (PMS) telah banyak digunakan sebagai alat kontrol getaran pada bangunan, khususnya bangunan-bangunan tinggi. PMS adalah salah satu kontrol pasif yang digunakan untuk mengurangi respon getaran dari struktur akibat beban-beban dinamik seperti angin, atau gempa. PMS menggunakan berat sendiri untuk mengurangi getaran, frekuensi dari PMS disesuaikan dengan frekuensi dari struktur sehingga beresonansi satu sama lain untuk mengurangi respon dari struktur selama berlangsungnya beban dinamik. Berdasarkan hal tersebut, ada tiga variabel yang mempunyai efek yang signifikan terhadap performa dari PMS, antara lain: rasio massa PMS (μ), rasio frekuensi PMS (α), dan rasio redaman PMS (ζ) yang akan berhubungan langsung dengan dua parameter penting dari PMS yaitu kekakuan dan redaman dari PMS. Tulisan ini mengusulkan persamaan empirik untuk mendapatkan properti optimum dari PMS dengan menggunakan fungsi kontrol H_2 dan algoritma fast multi swarm optimization. Fungsi objektif yang digunakan adalah untuk meminimumkan respon percepatan dan perpindahan dari struktur. Data rekaman El-Centro 1940 NS dipilih sebagai data percepatan tanah dasar untuk mensimulasikan respon dari struktur dengan dan tanpa menggunakan PMS. Hasil menunjukkan terdapat korelasi yang kuat antara rasio massa dari PMS terhadap rasio frekuensi dari PMS serta rasio massa dari PMS terhadap rasio redaman dari PMS.

Kata-kata Kunci: *Peredam massa selaras, parameter optimum, fast multi swarm optimization.*

1. Introduction

In a few decades, TMD has been adopted in structural building especially in high rise (tall) building for vibration controller. The function of TMD is to reduce the dynamic response of building during dynamic loading such as wind load or earthquake load. Dynamic response can be interpreted as displacement response, velocity response, or acceleration response. With TMD, peak of the dynamic response can be reduced. But for obtaining the optimum result, three variables of TMD must be considered wisely to obtain the optimum stiffness and damping of TMD (k_d and c_d). Therefore, it

is important to make a calculation for determining the optimum parameters of TMD. One of the technique which has good result for optimization problem is Fast Multi Swarm Optimization (FMSO). In this paper, FMSO has been applied for determining the optimum parameters of TMD via H_2 norm control system.

2. Equation of motion of structure applied TMD

In general, equation of motion of building which attached TMD as follows,

$$(M_o + M_p)\ddot{X}_s + (C_o + C_p)\dot{X}_s + (K_o + K_p)X_s = e_s \ddot{x}_g \qquad (1)$$

or

$$M_s \, \ddot{X}_s + C_s \, \dot{X}_s + K_s X_s = e_s \, \ddot{x}_g \qquad (2)$$

where, M, C, and K are mass matrix, damping matrix, and stiffness matrix respectively, X is displacement vector, e_s is motion vector relatively to structure, is ground acceleration. "o" and "p" subscript refer to structure without TMD and with TMD. Equation (2) can also be written as state space equation as follows,

$$\dot{Z} = AZ + Ew \qquad (3)$$

where,

$$Z = \begin{Bmatrix} \dot{X}_s \\ \ddot{X}_s \end{Bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ -M_s^{-1}K_s & -M_s^{-1}C_s \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ M_s^{-1}e_s \end{bmatrix}, and \quad w = \ddot{x}_g \quad (4)$$

with the output of equation (4) can be obtain with equation (5),

$$z = C_z Z \qquad (5)$$

where, C_z is output matrix which are either displacement or velocity or acceleration or combination of these three response for determining the output vector, z. The objective function which used in this paper was to minimize the displacement and acceleration response of the structure and H_2 norm has been used for the control system. Hence, the equation (3) can be rewritten as (Arfiadi, 2000),

$$\dot{Z} = AZ + Ew \qquad (6)$$

$$z = C_z Z \qquad (7)$$

$$J = [tr(C_z L_c C_z^T)]^{1/2} \longrightarrow \text{Minimum} \qquad (8)$$

$$A^T L_O + L_C A^T + EE^T = 0 \qquad (9)$$

where, J is *performance index*, *tr* is summation of diagonal value of matrix, *Lc* is *controllability Grammians*.

3. Fast multi swarm optimization (FMSO)

Fast multi swarm optimization is one of techniques that have a good result for optimization process. Fast multi swarm optimization is derived from particle swarm optimization (PSO) which was first proposed by J. Kennedy and Eberhart (1995). Theory of particle swarm optimization is based on the behavior of insects or birds swarm. Social behavior consists of individual action and the influence of other individuals within a group. For example, a bird in a flock of birds. Any individual or particles behave in a distributed manner by using its intelligence and also will influence the behavior of the collective group. Thus, if one particle or a bird finding his way or short to get to the food source, the rest of the group will also be able to immediately follow that path although they are far from the location of the group. The basic equation of

particle swarm optimization used to update the position and location of the particle is:

$$v_{i,j}(t+1) = w v_{i,j}(t) + c_1 R_1 (p_{best,i,j} - x_{i,j}(t)) + c_2 R_2 (g_{best,i,j} - x_{i,j}(t)) \quad (10)$$

$$x_{i,j}(t+1) = x_{i,j}(t) + v_{i,j}(t+1) \qquad (11)$$

where: $v_{i,j}$ (t+1) is updated velocity of particle, $x_{i,j}$ (t+1) is updated location of particle, t represent the iteration-t, $p_{best,i,j}$ is local best location of particle at t-iteration, $g_{best,i,j}$ is global best location of particle at t-iteration. R_1 and R_2 are random number from interval 0-1, c_1 and c_2 are particle acceleration constants, in this paper, $c_1 = c_2 = 2$, and w is positive inertia weight coefficient which is a function ρ_{min} and ρ_{max} as follow:

$$w_{(t)} = \rho_{max} - ((\rho_{max} - \rho_{min}).t / \max t) \qquad (12)$$

In PSO, each particle shares the information with its neighbors. PSO combines the cognition component of each particle with the social component of all the particles in a group. Although the speed of convergence is very fast, Once PSO traps into local optimum, it is difficult to jump out of local optimum. Therefore, the addition of a mutation operator to PSO will enhance its global search capacity and thus improve its performance. In order to prevent of falling to a local optimum, a new technique using combination of Cauchy mutation and crossover operation which is called fast multi swarm optimization (FMSO) was introduced by Zhang *et al* (2007). Similar to distributed genetic algorithm, multiple swarm's idea is very useful for speeding up the search. The new information of exchanging and sharing mechanisms of FMSO makes it converge fast to the global optimum.

Whenever the particle converges, it will "fly" to the personal best position and the global best particle's position. Due to this information of sharing mechanism makes the speed of convergence of PSO is very fast. Meanwhile, because of this mechanism, PSO cannot guarantee to find global optimum value of function. In fact, the particles are usually converge to local optima. Without loss of generality, only function minimization is discussed here. Once the particles trap into a local optimum, in which $p_{best,i,j}$ can be assumed to be the same as $g_{best,i,j}$, all the particles converge to $g_{best,i,j}$. At this condition, the velocity's update equation becomes:

$$v_{i,j(t+1)} = \omega.v_{i,j(t)} \qquad (13)$$

When the iteration in the equation (13) goes to infinite, the velocity of the particle $v_{i,j}$ will be closed to 0 because of $0 \leq \omega < 1$. After that, the position of the particle $x_{i,j}$ will not change, so that PSO has no capability of jumping out of the local optimum. It is the reason that PSO often fails on finding the global minimal value. To overcome the weakness of PSO discussed at the middle of this section, the Cauchy mutation is incorporated into PSO algorithm. The basic idea is that, the velocity and position of a particle are updated not only according to equation (10) and (11), but also according to Cauchy mutation as follows:

$$x_{i,j(t+1)} = x_{i,j(t)} + (v_{i,j(t)} \exp(\delta_i))\delta \qquad (14)$$

where, δ_i, and δ denotes Cauchy random numbers.

Since the expectation of Cauchy distribution does not exist, the variance of Cauchy distribution is infinite so that Cauchy mutation could make a particle have a long jump. By adding the update equations of (14), PSO greatly increases the probability of escaping from the local optimum.

For crossover operation, for rand() < q_c the crossover operation is taken as follows:

$$x_{i,j(t+1)} = (1-\alpha).x_{i,j(t)} + \alpha.p_{best,i,j} \qquad (15)$$

$$v_{i,j(t+1)} = rand().(p_{best,i,j} - x_{i,j(t)}) \qquad (16)$$

where, α is random number with 0-1 interval, q_c is crossover rate.

4. Application

Considered one story frame with shear building assumption as shown on **Figure 1**. **Table 1** shows five variations of structural stiffness with fixed mass used.

Table 1. One story frame properties

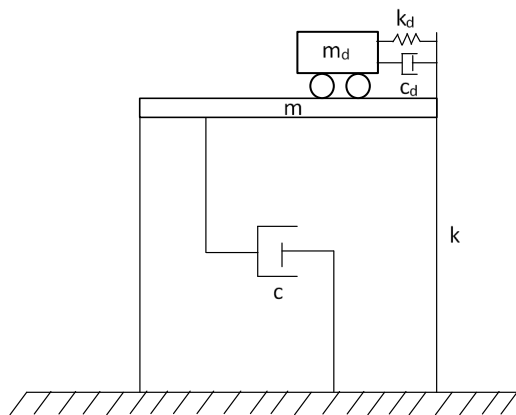| Frame number	m (ton)	k (kN/m)	c (kN-s/m)
1	115	6740.74	352.178
2	115	2778.94	226.125
3	115	1500	166.132
4	115	723.38.00	11.537
5	115	474.61	9.345

Figure 1. One story building with TMD

TMD was applied to this structure to minimize the peak displacement and peak acceleration response using H_2 norm control system and FMSO. In order to develop empirical equation for determining the optimum parameters, the relationship between three variables have been considered. Optimization parameters of FMSO can be seen on **Table 2**.

Table 2. Optimization parameters

Swarm population	20
Maximum velocity	10
Maximum iteration	200
Rho maximum (ρ_{max})	00.09
Rho minimum (ρ_{min})	00.04
Crossover rate	00.08

The program has been run for four times with different lower and upper bound to see the consistency of the result. Lower bound value and upper bound value are interval value in domain area. The units are kN/m for damper's stiffness and kN-s/m for damper's damping. Lower and upper bound value can be found on **Table 3**.

Table 3. Lower and upper bound of each run

Run	Lower Bound	Upper Bound
1	0	1000
2	0	10
3	0	100
4	30	100

Mass ratio was taken 1.2%, 1.8%, 2.6%, 3.4%, and 4.2% from mass of structure. The result will be compared to find out the behavior of TMD with different mass ratio and to obtain the relationship between the mass ratio to frequency ratio and mass ratio to damping ratio. Fitness value was considered as a number to define the H_2 norm value (J) which the objective function is to minimize the J value. The lower of fitness value, the higher of TMD performance to reduce the response of structure during dynamic external disturbance. To obtain the optimum fitness value, the program was run for 200 iterations with each different lower and upper bound. If each run produces similar result (consistency), it can be concluded that the optimum fitness value has obtained. **Figure 2** shows decreasing of fitness of each iteration for 1.8% of mass ratio with minimization of peak displacement while **Figure 3** shows the minimization of peak acceleration using H_2 norm control function. The

Table 4. Mass ratio and natural frequency of structure with k = 6740.74 kN/m and c = 35.2178 kN-s/m

Mass ratio	$\omega_{structure}$ (rad/s)		
	Undamped	Minimum displacement	Minimum acceleration
0.012	76.561	71.684	72.123
0.018	76.561	70.487	71.080
0.026	76.561	69.131	69.918
0.034	76.561	67.943	68.916
0.042	76.561	66.866	68.019

consistency of result can be seen, there is no different of result in spite of different on lower and upper bound. Hence, the result is convergent.

Natural frequency of structure with certain stiffness which without and with TMD can be seen on **Table 4** until **Table 8**. As shown on Table 4 until Table 8, increasing of mass ratio lead decreasing of the natural frequency of structure which attached TMD for all cases of stiffness variations and for all objective function (minimization of displacement and minimization of acceleration).

Table 9 and **10** show peak displacement and peak acceleration result of structure subjected to accelerogram El-Centro 1940 NS respectively using time history analysis in order to minimize the displacement and acceleration response of the structure.

Based on the result, the peak displacement and peak acceleration decreased significantly when TMD applied. It should be noted that there needs to be further review about the maximum mass ratio which potentially increasing the total mass of building. The other variable which must considered further is building which has long period because this paper only

Table 5. Mass ratio and natural frequency of structure with k = 2778.94 kN/m and c = 22.6125 kN-s/m

Mass ratio	$\omega_{structure}$ (rad/s)		
	Undamped	Minimum displacement	Minimum acceleration
0.012	49.158	46.027	46.309
0.018	49.158	45.258	45.639
0.026	49.158	44.388	44.893
0.034	49.158	43.624	44.249
0.042	49.158	42.933	43.674

Table 7. Mass ratio and natural frequency of structure with k= 723.38 kN/m and c = 11.537 kN-s/m

Mass ratio	$\omega_{structure}$ (rad/s)		
	Undamped	Minimum displacement	Minimum acceleration
0.012	25.052	23.483	23.627
0.018	25.052	23.091	23.285
0.026	25.052	22.647	22.904
0.034	25.052	22.257	22.576
0.042	25.052	21.905	22.282

Table 6. Mass ratio and natural frequency of structure with k = 1500 kN/m and c = 16.6132 kN-s/m

Mass ratio	$\omega_{structure}$ (rad/s)		
	undamped	Minimum displacement	Minimum acceleration
0.012	36.116	33.816	34.023
0.018	36.116	33.251	33.530
0.026	36.116	32.611	32.982
0.034	36.116	32.050	32.509
0.042	36.116	31.543	32.087

Table 8. Mass ratio and natural frequency of structure with k = 474.61 kN/m and c = 9.345 kN-s/m

Mass ratio	$\omega_{structure}$ (rad/s)		
	Undamped	Minimum displacement	Minimum acceleration
0.012	20.315	19.021	19.138
0.018	20.315	18.704	18.861
0.026	20.315	18.344	18.553
0.034	20.315	18.028	18.287
0.042	20.315	17.743	18.049

**Table 9. Peak displacement of structure without and with TMD subjected to El Centro 1940-NS**

Mass Ratio	Peak Displacement (m)				
	Frame Number				
	1	2	3	4	5
without TMD	0,936111111	0,858333333	1,272222222	2,233333333	2,328472222
0.012	0,691666667	0,803472222	1,109722222	2,101388889	1,910416667
0.018	0,677777778	0,793055556	1,041666667	2,045138889	1,880555556
0.026	0,664583333	0,783333333	1,039583333	1,976388889	1,842361111
0.034	0,652777778	0,777083333	1,040277778	1,9125	1,807638889
0.042	0,640972222	0,772222222	1,038194444	1,854166667	1,775
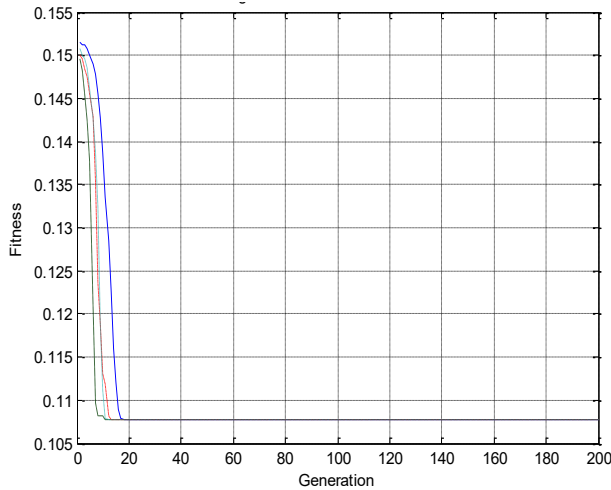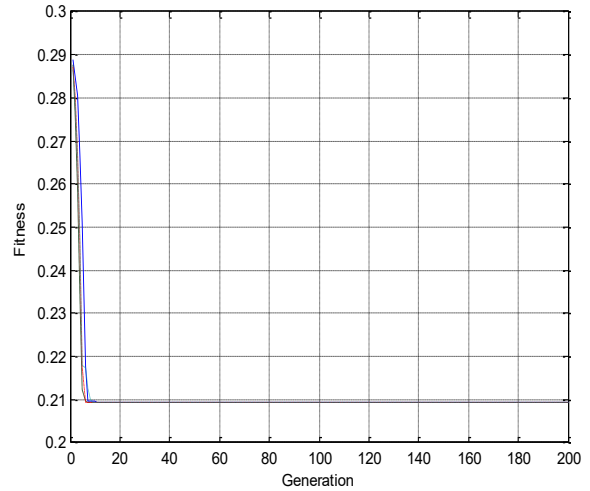
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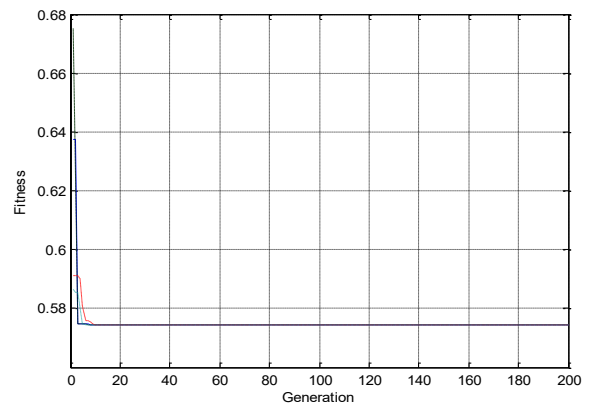
(a) k=6740.74 kN/m, c=35.2178 kN-s/m
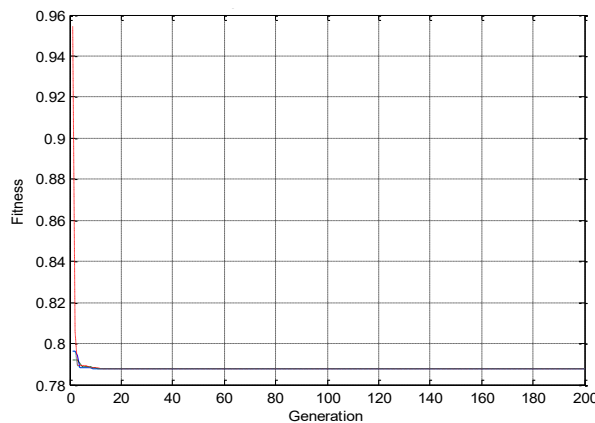
(b) k=2778.94 kN/m, c=22.6125 kN-s/m
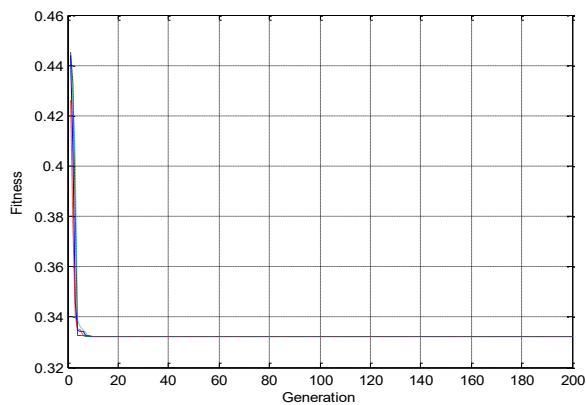
(c) k=1500 kN/m, c=16.6132 kN-s/m

(d) k=723.38 kN/m, c=11.537 kN-s/m

(e) k=474.61 kN/m, c=9.345 kN-s/m

Figure 2. Decreasing of fitness each iteration with 1.8% of mass ratio
(objective function: minimization of peak displacement)

considered building with short period of natural frequency. Due to space limitation, only the displacement response versus time graph of case 1 shown (**Figure 4**).

Empirical equations have been developed for determining the damping ratio and frequency ratio of TMD in order to obtain the optimum stiffness and optimum damping of TMD. **Figure 5** shows relationship between mass ratio vs damping ratio of TMD. Power regression used for developing empirical equation. The correlation coefficient was 1 which mean there is strong correlation between the mass ratio and damping ratio of TMD. While the relationship between
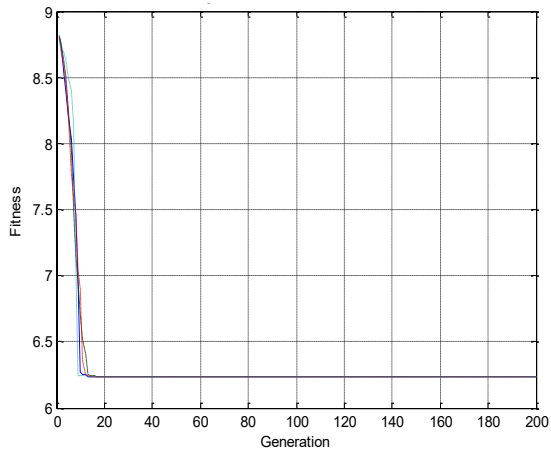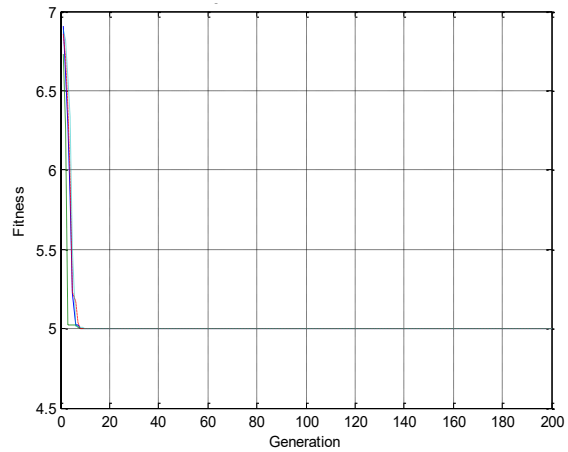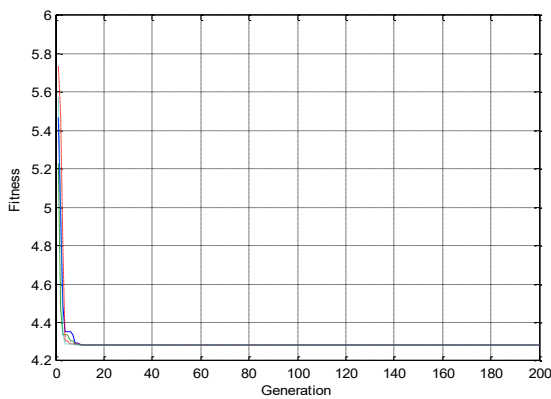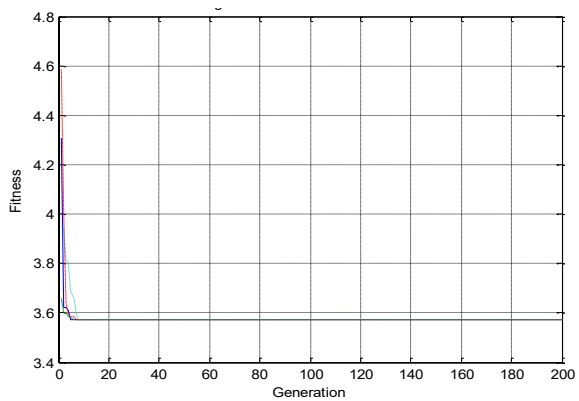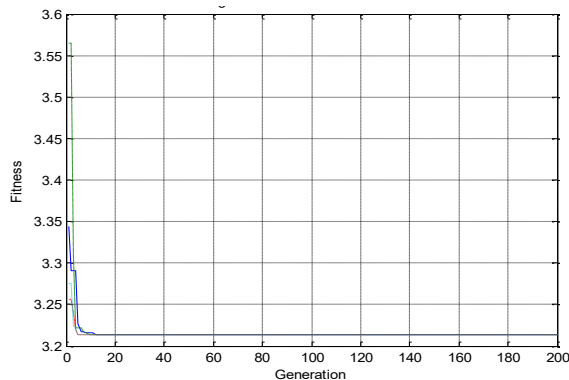
(a) k=6740.74 kN/m, c=35.2178 kN-s/m

(b) k=2778.94 kN/m, c=22.6125 kN-s/m

(c) k=1500 kN/m, c=16.6132 kN-s/m

(d) k=723.38 kN/m, c=11.537 kN-s/m

(e) k=474.61 kN/m, c=9.345 kN-s/m

Figure 3. Decreasing of fitness each iteration with 1.8% of mass ratio
(objective function: minimization of peak acceleration)

mass ratio vs frequency ratio had one empirical equation caused by optimization result for two objective function was similar (displacement minimization and acceleration minimization), the relationship between mass ratio vs frequency ratio had two empirical equations due to difference of optimization result for each objective function (**Figure 6**). Second polynomial order used to develop the

empirical equation. From Figure 6, it can be seen the correlation coefficient was also 1.

5. Conclusions

One story fixed mass building with various building's stiffness considered. From the result, building which used TMD has a better result compared with building

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Table 10. Peak acceleration of structure without and with TMD subjected to El Centro 1940-NS

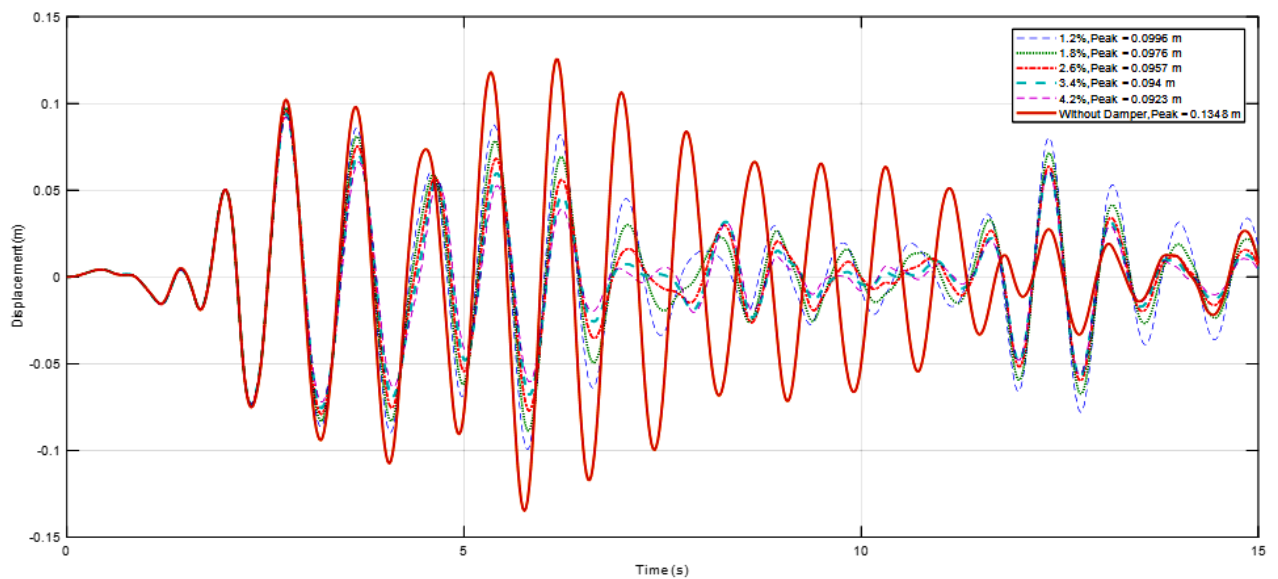| Mass Ratio	Peak Acceleration (m/s^2)				
	Frame Number				
	1	2	3	4	5
without TMD	75.491	50.232	40.532	37.478	30.440
0.012	58.218	28.029	20.610	19.017	11.357
0.018	56.497	27.378	19.237	18.492	11.158
0.026	55.132	26.651	19.032	17.856	10.906
0.034	53.855	26.050	18.869	17.275	10.668
0.042	52.625	25.518	18.698	16.740	10.442

Figure 4. Displacement response of first case structure subjected to El Centro 1940-NS using time history analysis
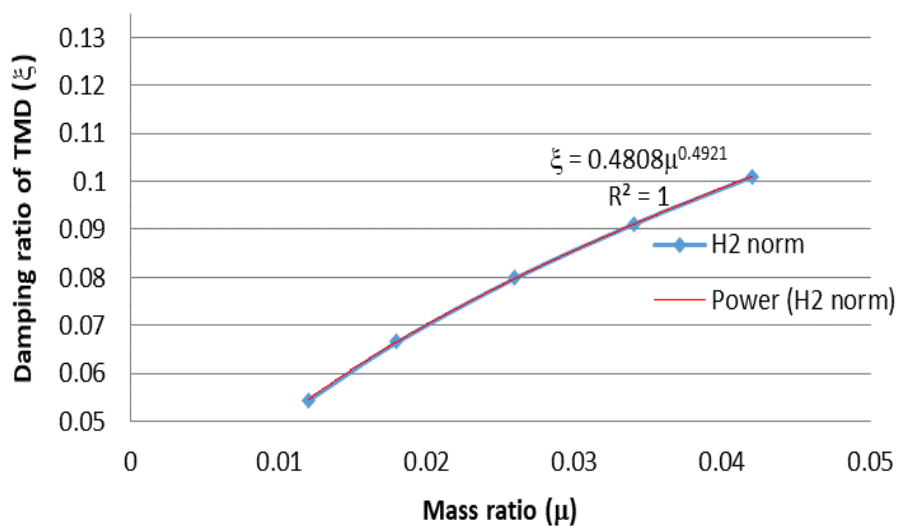

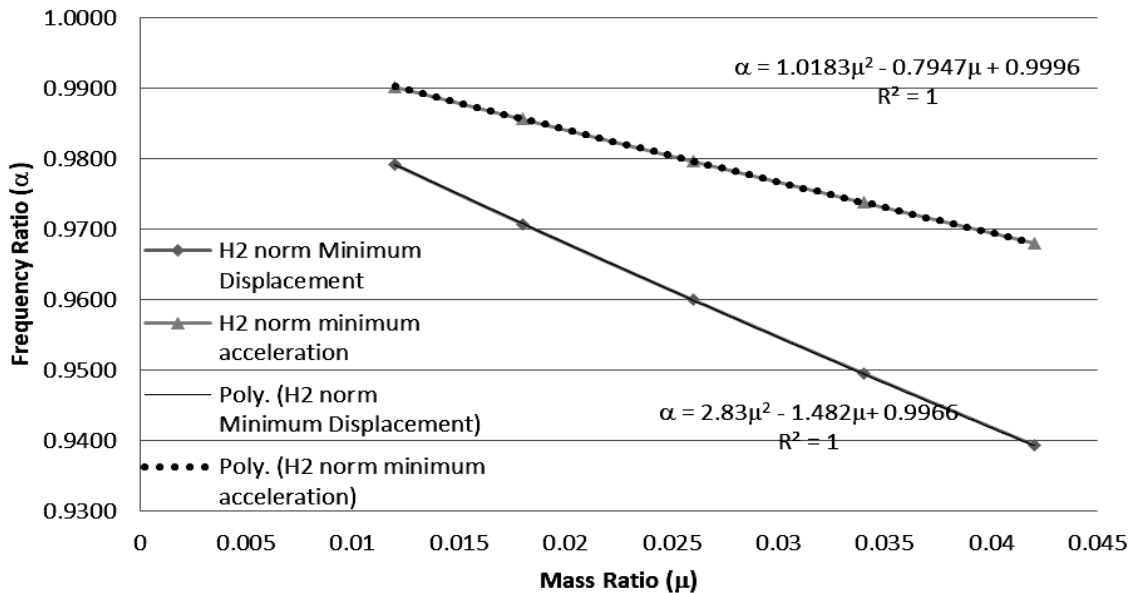**Figure 5. Relationship between mass ratio vs damping ratio of TMD**

Figure 6. Relationship between mass ratio vs frequency ratio of TMD

without TMD. Empirical equations have been developed for easily determining the optimum parameters (stiffness and damping) of TMD. One empirical equation for obtaining the damping ratio of TMD ($\zeta=0.4808\mu^{0.4921}$), while two empirical equations were built for obtaining the frequency ratio of TMD which depend of the minimization case. If the case is to minimize the peak displacement of the structure then $\alpha=2.83\mu^2-1.482\mu+0.9966$, if the case is to minimize peak acceleration of structure then $\alpha=1.0183\mu^2-0.7947\mu+0.9966$. It should be noted that the structure which considered in this paper was structure with short natural period, hence must a further review about building structure with long period.

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